



# Model predictive control: past, present and future<sup>☆</sup>

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## Abstract

More than 15 years after model predictive control (MPC) appeared in industry as an effective means to deal with multivariable constrained control problems, a theoretical basis for this technique has started to emerge. The issues of feasibility of the on-line optimization, stability and performance are largely understood for systems described by linear models. Much progress has been made on these issues for non-linear systems but for practical applications many questions remain, including the reliability and efficiency of the on-line computation scheme. To deal with model uncertainty ‘rigorously’ an involved dynamic programming problem must be solved. The approximation techniques proposed for this purpose are largely at a conceptual stage. Among the broader research needs the following areas are identified: multivariable system identification, performance monitoring and diagnostics, non-linear state estimation, and batch system control. Many practical problems like control objective prioritization and symptom-aided diagnosis can be integrated systematically and effectively into the MPC framework by expanding the problem formulation to include integer variables yielding a mixed-integer quadratic or linear program. Efficient techniques for solving these problems are becoming available. © 1999 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

The intention of this paper is to give an overview of the origins of model predictive control (MPC) and its glorious present. No attempt is made to categorize and comprehensively review the literature which includes several books (Bitmead, Gevers & Wertz, 1990; Soeterboek, 1992; Clarke, 1994; Berber, 1995; Camacho & Bordons, 1995; Martín Sánchez & Rodellar, 1996) and hundreds of papers (Kwon, 1994). The review should give the novice reader an impression which practical objectives have been pursued, which theoretical problems have been formulated and what progress has been made without undue mathematical complexity. All citations are only ememplary and should point the reader in a direction where more details are available. There is more emphasis on the future of MPC than on its past.

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MPC brings out new needs in related areas like system identification, state estimation, monitoring and diagnostics, etc. We show that many important practical and theoretical problems can be formulated in the MPC framework. Pursuing them will assure MPC of its stature as a vibrant research area, where theory is seen to support practice more directly than in most other areas of control research.

## 2. The past

Though the ideas of receding horizon control and model predictive control can be traced back to the 1960s (García, Prett & Morari, 1989), interest in this field started to surge only in the 1980s after publication of the first papers on IDCOM (Richalet, Rault, Testud & Papon, 1978) and dynamic matrix control (DMC) (Cutler & Ramaker, 1979, 1980) and the first comprehensive exposition of generalized predictive control (GPC) (Clarke, Mohtadi & Tuffs, 1987a,b). At first sight, the ideas underlying the two methods are similar.

The objectives behind the developments of DMC and GPC were very different, however. DMC was conceived

to tackle the multivariable constrained control problems typical for the oil and chemical industries. In the pre-DMC era these problems were handled by single loop controllers augmented by various selectors, overrides, decouplers, time-delay compensators, etc. For the DMC task a time-domain model (finite impulse or step response model) was natural. GPC was intended to offer a new adaptive control alternative. In the tradition of much of the work in adaptive control input/output (transfer function) models were employed. Stochastic aspects played a key role in GPC from the very beginning, while the original DMC formulation was completely deterministic and did not include any explicit disturbance model.

The GPC approach is not suitable or, at the very least, awkward for multivariable constrained systems which are much more commonly encountered in the oil and chemical industries than situations where adaptive control is needed. Essentially all vendors have adopted a DMC-like approach (Qin & Badgwell, 1996). Because of these reasons and because of the type of applications of interest to the readers of this journal, GPC will not be discussed any further. The interested reader is referred to several recent books on this subject (Bitmead, et al., 1990; Soeterboek, 1992; Martín Sánchez & Rodellar, 1996).

DMC had a tremendous impact on industry. There is probably not a single major oil company in the world, where DMC (or a functionally similar product with a different trade name) is not employed in most new installations or revamps. For Japan some statistics are available (Ohshima, Ohno & Hashimoto, 1995). The initial research on MPC is characterized by attempts to understand DMC, which seemed to defy a traditional theoretical analysis because it was formulated in a non-conventional manner. One example was the development of internal model control (IMC) (García & Morari, 1982) which failed to shed light on the behavior of constrained DMC but led to some insights on robust control (Morari & Zafriou, 1989).

### 3. The present

#### 3.1. Linear model predictive control

Nowadays in the research literature MPC is formulated almost always in the state space. The system to be controlled is described by a linear discrete time model.

$$x(k+1) = Ax(k) + Bu(k), \quad x(0) = x_0, \quad (1)$$

where  $x(k) \in \mathcal{R}^n$  and  $u(k) \in \mathcal{R}^m$  denote the state and control input, respectively. A receding horizon implementation is typically formulated by introducing the following open-loop optimization problem (see García, et al., (1989)).

$$J_{(p,m)}(x_0) = \min_{u(\cdot)} \left[ x^T(p)P_0x(p) + \sum_{i=0}^{p-1} x^T(i)Qx(i) + \sum_{i=1}^{m-1} u^T(i)Ru(i) \right] \quad (2)$$

subject to

$$Ex + Fu \leq \psi \quad (3)$$

( $p \geq m$ ) where  $p$  denotes the length of the *prediction horizon* or *output horizon*, and  $m$  denotes the length of the *control horizon* or *input horizon*. (When  $p = \infty$ , we refer to this as the *infinite horizon problem*, and similarly, when  $p$  is finite, we refer to it as a finite horizon problem). For the problem to be meaningful we assume that the origin ( $x=0, u=0$ ) is in the interior of the feasible region.

Eqs. (1)–(3) define a quadratic program for which many algorithms and commercial software exist. Let  $u_{(p,m)}^*(i | x(k))$ ,  $i=0, \dots, m-1$  be the minimizing control sequence for  $J_{(p,m)}(x(k))$  subject to the system dynamics (Eq. (1)) and the constraint (Eq. (3)). A receding horizon policy proceeds by implementing *only the first* control  $u_{(p,m)}^*(0 | x(k))$  to obtain  $x(k+1) = Ax(k) + Bu_{(p,m)}^*(0 | x(k))$ . The rest of the control sequence  $u_{(p,m)}^*(i | x(k))$  is discarded and  $x(k+1)$  is used to update the optimization problem (Eq. (2)) as a new initial condition. This process is repeated, each time using only the first control action to obtain a new initial condition, then shifting the cost ahead one time step and repeating, hence the name receding horizon control. In the special case when  $p=m=N$ , then  $J_{(p,m)} = J_N$  as defined in (Eq. (2)).

We note that as the control horizon and the prediction horizon both approach infinity and when there are no constraints we obtain the standard linear quadratic regulator (LQR) problem, which was studied extensively in the 1960s and 1970s (Kwakernaak & Sivan, 1972). The optimal control sequence is generated by a static state feedback law where the feedback gain matrix is found via the solution of an algebraic Riccati equation (ARE). This feedback law has some well known nice properties, in particular, it guarantees closed-loop stability for any positive semi-definite weighting matrix  $Q$  and any positive definite  $R$ .

With constraints an infinite dimensional optimization problem results, which is—at least at first sight—not a very practical proposition. On the other hand, by choosing both the control and the output horizons to be finite, the quadratic program is finite dimensional and can be solved relatively easily on-line at every time step.

Three practical questions are immediate: (1) When is the problem formulated above feasible, so that the algorithm yields a control action which can be implemented? (2) When does the sequence of computed control actions lead to a system which is closed-loop

stable? (3) What closed-loop performance results from repeated solution of the specified open-loop optimal control problem?

### 3.1.1. Feasibility

The constraints stipulated in (3) may render the optimization problem infeasible. Input saturation constraints cannot be exceeded, while constraints involving outputs can be violated, albeit with undesirable consequences for the controlled system. It may happen, for example, because of a disturbance, that the optimization problem posed above becomes infeasible at a particular time step. It may also happen, that the algorithm which minimizes an *open-loop* objective, inadvertently drives the closed-loop system outside the feasible region. (This difference between open-loop objective and closed-loop behavior is addressed below).

Obviously a real-time control algorithm must not fail in this trivial fashion. Therefore in all commercial algorithms (Qin & Badgwell, 1996) the hard constraints are softened by introducing slack variables which are kept small by introducing a corresponding penalty term in the objective (Zheng & Morari, 1995b). There are many variations on this theme to suit different tastes. At issue are magnitude of the violation versus duration and if the solution of the problem with softened constraints may lead to a constraint violation though a feasible solution without constraint violation exists (Scokaert & Rawlings, 1996a,b).

If the system is unstable then, in general, the system cannot be stabilized globally, when there are input saturation constraints. Algorithms for precalculating a feasible region of non-zero initial conditions within which stabilization is possible were proposed by Gilbert and Tan (1991) and Zheng and Morari (1995a). Needless to say, these constraints on the allowed states imposed by the stabilization requirement cannot be relaxed and infeasibility can only be dealt with by a modification of the plant itself.

### 3.1.2. Closed loop stability

In either the infinite or the finite horizon constrained case it is not clear under what conditions the closed loop system is stable. Much recent research on linear MPC has focused on this problem. Two approaches have been proposed to guarantee stability: one based on the original problem (1), (2), and (3) and one where a ‘contraction constraint’ is added (Polak & Yang, 1993a,b). With the contraction constraint the norm of the state is *forced* to decrease with time and stability follows trivially independent of the various parameters in the objective function. Without the contraction constraint the stability problem is more complicated. General proofs of stability for constrained MPC based on the monotonicity property of the value function have been proposed by Keerthi and Gilbert (1988) and Bem-

porad, Chisci and Mosca (1994). The most comprehensive and also most compact analysis was presented by Nevistić and Primbs (1997) and Primbs and Nevistić (1997) whose arguments we will sketch here.

To simplify the exposition we assume  $p = m = N$ , then  $J_{(p,m)} = J_N$  as defined in Eq. (2). The key idea is to use the optimal finite horizon cost  $J_N$ , the value function, as a Lyapunov function. One wishes to show that  $J_N(x(k)) - J_N(x(k+1)) > 0$  for  $x \neq 0$ . Rewriting  $J_N(x(k)) - J_N(x(k+1))$  gives:

$$\begin{aligned} & J_N(x(k)) - J_N(x(k+1)) \\ &= [x^T(k)Qx(k) + u_N^{*T}(x(k))Ru_N^*(x(k))] \\ & \quad + [J_{N-1}(x(k+1)) - J_N(x(k+1))] \end{aligned} \quad (4)$$

If it can be shown that the right hand side of Eq. (4) is positive, then stability is proven. Assuming  $Q > 0$ , the first term  $[x^T(k)Qx(k) + u_N^{*T}(0|x(k))Ru_N^*(0|x(k))]$  is positive. In general, it *cannot* be asserted that the second term  $[J_{N-1}(x(k+1)) - J_N(x(k+1))]$  is nonnegative.

Several approaches have been presented to assure that the right hand side of Eq. (4) is positive:

- Primbs and Nevistić (1997) showed that the second term approaches zero as  $N \rightarrow \infty$  and that there exists a finite  $N^*$  such that for  $N > N^*$  the first term dominates over the second. In general, the solution of a non-convex min–max problem is necessary in order to determine  $N^*$ . However, when the system is open-loop stable and when the constraints involve the control inputs only, the solution of a somewhat conservative version of this problem requires eigenvalue computations only, which is quite remarkable.
- When an end constraint  $x(k+N) = 0$  (Kwon & Pearson, 1977) is imposed it can be argued in a straight forward manner that  $J_N$  is non-increasing as a function of  $N$ , which trivially guarantees stability. Another option is to add a constraint that forces the terminal state to be inside a positively invariant region, for instance, the maximal output admissible set in the sense of Gilbert and Tan (1991).
- When the system is open-loop stable and  $P_0$  is chosen as the solution to the Lyapunov equation  $A^T P_0 A + Q = P_0$  (Rawlings & Muske, 1993), then  $J_N$  is again non-increasing and stability follows. This choice of  $P_0$  amounts to using an infinite output horizon (with input horizon of  $N$ ). For unstable systems, the unstable modes must be zeroed at the end of the input horizon for the objective function to be finite. Then the outlined approach can be applied to the remaining stable modes. The constraint horizon must be chosen large enough so that satisfying the constraints within the finite horizon implies the same for the infinite horizon.
- Rather than assuming the control moves to be zero after the end of the control horizon one can intro-

duce a stabilizing local controller  $u(k+i) = Lx(k+i)$  for  $i \geq N$  (as opposed to  $u(k+i) = 0$ ). The idea is essentially the same, but the terminal cost and the positively invariant region need to be defined with respect to the system  $x(i+1) = (A+BL)x(i)$  rather than  $x(i+1) = Ax(i)$ . In addition, the positive invariance is to be defined with respect to the input constraint as well as the state constraint. The local feedback can be chosen as the infinite horizon unconstrained LQR (Chmielewski & Manousiouthakis, 1996; Scokaert & Rawlings, 1998).

- In the special case when there are no constraints and  $N$  is finite, the FARE conditions for guaranteeing stability (Bitmead, et al., 1990) follow directly.

Some remarks are in order. Despite the fact that there exist now techniques to test for stability of constrained systems with finite  $p$ ,  $p$  is not recommended for tuning for the following reasons. The system behavior is relatively insensitive to changes in both  $p$  and  $m$  over a wide range of values. Therefore  $Q$  and  $R$  are the preferred tuning parameter to affect performance. Moreover, Soeterboek (1992) has shown that for a finite  $p$  the effect of the control weighting  $R$  may be 'non-monotonic', i.e. increasing  $R$  may lead to instability, which is counter-intuitive. This type of behavior was not observed for the infinite horizon case, though no proof exists.

The various constraints introduced to guarantee stability (end constraint for all states, end constraint for unstable modes, terminal region, etc.) may lead to feasibility problems. For instance, the terminal equality constraint may become infeasible unless a sufficiently large horizon is used. The alternative formulation based on a locally stabilizing controller gives a more relaxed constraint that can be satisfied with fewer moves.

For systems with integrators, the terminal equality constraints can always be satisfied given a sufficiently long horizon (Zheng & Morari, 1995b). However, the difficulty in implementing these ideas on-line is knowing a priori how many moves are needed to satisfy these stability constraints.

It is a common practice in the process industries to augment the model with integrators for offset-free tracking of constant setpoints and rejection of constant disturbances. This is done, for instance, by writing the model in terms of differenced inputs and outputs, thus creating integrating modes for the outputs (Lee, Gelormino & Morari, 1992). In this case the objective function (Eq. (2)) includes a penalty term on  $\delta u$  rather than  $u$ , which effectively adds integral action to the controller. When implementing an infinite horizon based MPC algorithm on an augmented system, the integrating modes must be zeroed at the end of the control horizon in order for the infinite horizon cost to be bounded. For FIR systems, this amounts to setting  $p = m + t_s$  and adding the constraint  $y(m + t_s) = 0$

(where  $y$  represents the output of the FIR system and  $t_s$  the number of time steps it takes for the system to settle) (Lee, 1996).

The requirement gives rise to some complications since zeroing the integrating modes is not always possible for two reasons: first, the chosen control horizon may not be sufficiently large, in other words, there may not be enough degrees of freedom available to force the integrating modes to zero at the end of the horizon; second, one may have hard constraints on the input  $u$ , which translate into constraints on the *integrated*  $\delta u$ , the input to the augmented system. Both problems can be overcome by employing a bi-level optimization, i.e. steady-state error minimization followed by dynamic error minimization, as suggested by Lee (1996). The asymptotic stability is preserved, if the constraints on  $u$  are such that returning the integrating modes to the origin is possible. It is interesting to note that a similar bi-level optimization has been a standard feature in popular commercial algorithms (Qin & Badgwell, 1996).

An alternative to this approach is the reference governor philosophy proposed by Bemporad and Mosca (1994), Gilbert, Kolmanovsky and Tan (1995) and Bemporad, Casavola and Mosca (1997). The main idea is to separate the stabilization problem from the constraint fulfilment problem. The first is left to conventional linear controllers, for instance pre-existent PID controllers. This primal controller is assumed to be designed without taking care of the existing operating constraints.

Constraints are then enforced by manipulating at a higher level the desired set-points through a reference governor. This is a predictive controller based on the primal closed-loop linear model, which generates set-points rather than command inputs, and basically smoothes out the reference trajectory when abrupt set-point changes would lead to constraint violations. The advantages of this scheme are that short input horizons are possible (typically one degree of freedom suffices), with consequent computational benefits, without sacrificing stability properties and performance. This scheme has been extended to uncertain and non-linear systems by Bemporad (1998) and Bemporad and Mosca (1998).

### 3.1.3. Open-loop performance objective versus closed loop performance

In receding horizon control only the first of the computed control moves is implemented; the remaining ones are discarded. Therefore the sequence of actually implemented control moves may differ significantly from the sequence of control moves calculated at a particular time step. Consequently the finite horizon objective which is minimized may have only a tentative connection with the value of the objective function as it is obtained when the control moves are implemented.

As mentioned above, it is even conceivable that the sequence of calculated control moves leads the system outside the feasible region. When both input and output horizons are infinite, there is no difference between the sequence determined at a time step and the implemented sequence. As the control horizon is lengthened we should expect the difference to diminish. A measure introduced by Primbs and Nevistić (1997) quantifies this difference and can be used to decide on the horizon length.

By choosing the output horizon long relative to the input horizon short-sighted control policies and potential problems with stability and feasibility are avoided but the mismatch criticized above is not eliminated. Thus, it was proposed to set both horizons to infinity which also reduces the number of tuning parameters to be selected (Scokaert & Rawlings, 1998). The computational effort increases but apparently not unduly.

#### 3.1.4. Research issues

A major problem is the stability analysis of constrained finite horizon systems. The computations suggested by Primbs and Nevistić (1997) are rather difficult except when the state dimension is low.

It was proven (Meadows & Rawlings, 1995) that if an exponentially converging observer is combined with a stable MPC algorithm where access to all the states is assumed, then this observer-controller system is stable, though the controller is non-linear and the separation principle obviously does not hold. A Kalman filter could serve as the observer. Guidelines for selecting the noise/tuning parameters and efficient implementation schemes were discussed by Lee, Morari and García (1994).

In all these deterministic formulations ‘certainty equivalence’ was assumed tacitly. It has been argued (Rawlings, Meadows & Muske, 1994) that performance gains could be achieved by accounting more accurately for the characteristics of this non-linear stochastic system. It is unclear how much could be gained from tackling this difficult theoretical problem.

### 3.2. Non-linear model predictive control

The same receding horizon idea which we discussed in detail above is also the principle underlying non-linear MPC, with the exception that the model describing the process dynamics is non-linear. Various model forms (differential equations, differential-algebraic equations, discrete time algebraic descriptions, Wiener models, neural nets, etc.) have been tried and some specific theoretical results for some of them are available (Li & Biegler, 1988; Bhat & McAvoy, 1990; Patwardhan, Rawlings & Edgar, 1990; Eskinat, Johnson & Luyben, 1991; Hernandez, 1992; Tulleken, 1993; Koulouris, 1995; Maner, Doyle, Ogunnaike & Pearson,

1996; Norquay, Palazoglu & Romagnoli, 1996). Also see Bequette (1991) for a review of non-linear process control, which includes an extensive list of different methods for solving non-linear model predictive control problems. Not to be led astray by these specifics, we will focus on general issues common to all non-linear MPC algorithms independent of the model form. We will also not go into a discussion of continuous vs. discrete time which can bring up a wealth of hairy technicalities but no new concepts.

Closed-loop stability of these algorithms has been studied extensively and addressed satisfactorily from a theoretical point of view, if not from a practical (implementation) point of view. Contrary to the linear case, however, feasibility and the possible mismatch between the open-loop performance objective and the actual closed loop performance are largely unresolved research issues in non-linear MPC. An additional difficulty is that the optimization problems to be solved on line are generally non-linear programs without any redeeming features, which implies that convergence to a global optimum cannot be assured. For the quadratic programs arising in the linear case this is guaranteed. As most proofs of stability for constrained MPC are based on the monotonicity property of the value function, global optimality is usually not required, as long as the cost attained at the minimizer decreases (which is usually the case, especially when the optimization algorithm is initialized from the previous shifted optimal sequence). However, although stability is not altered by local minima, performance clearly deteriorates.

We will discuss some of the ideas in non-linear MPC and their implications for the issues listed above. The intention is to summarize, complement and update the excellent survey by Mayne (1995).

#### 3.2.1. Infinite horizon/terminal constraint

The idea of using infinite prediction and control horizons or, alternatively, to set up the optimization problem to force the state to zero at the end of the prediction horizon was analyzed by Keerthi and Gilbert (1988) for the discrete time and by Mayne and Michalska (1990) for the continuous time case. Just as outlined for the linear case, in the proof the value function is employed as a Lyapunov function. A global optimum must be found at each time step to guarantee stability. When the horizon is infinity, feasibility at a particular time step implies feasibility at all future time steps. Unfortunately, contrary to the linear case, the infinite horizon problem cannot be solved numerically. The optimization problem with terminal constraint can be solved in principle, but equality constraints are computationally very expensive and can only be met asymptotically. In addition, one cannot guarantee convergence to a feasible solution even when a feasible solution exists, a disconcerting fact. Furthermore,

specifying a terminal constraint which is not met in actual operation is always somewhat artificial and may lead to aggressive behavior. Finally, to reduce the complexity of the optimization problem it is desirable to keep the control horizon small, or, more generally, characterize the control input sequence with a small number of parameters. For instance, Bemporad (1998) represented the sequence as the output of a stabilizing controller, whose set-point level is the only variable to be optimized. However, a small number of degrees of freedom may lead to quite a gap between the open-loop performance objective and the actual closed loop performance.

### 3.2.2. Variable horizon/hybrid model predictive control

These techniques were proposed by Michalska and Mayne (1993) to deal with both the global optimality and the feasibility problems, which plague non-linear MPC with a terminal constraint. Variable horizon MPC also employs a terminal constraint, but the time horizon at the end of which this constraint must be satisfied is itself an optimization variable. In hybrid MPC the terminal constraint is replaced by a ‘terminal region’ which must be reached at the end of a variable horizon. It is assumed that inside this region another controller is employed for which it is somehow known that it asymptotically stabilizes the system. With these modifications a global optimum is no longer needed and feasibility at a particular time step implies feasibility at all future time steps. The terminal constraint is somewhat less artificial here because it may be met in actual operation. However, a variable horizon is inconvenient to handle on-line, an exact end constraint is difficult to satisfy, and the exact determination of the terminal region is all but impossible except maybe for low order systems. In order to show that this region is invariant and that the system is asymptotically stable in this region, usually a global optimization problem needs to be solved.

### 3.2.3. Quasi-infinite horizon model predictive control

The technique recently introduced by Chen and Allgöwer (1996, 1998) uses an infinite horizon and overcomes both the global optimization and the feasibility problems without making use of artificial terminal constraints, terminal regions and controller switching. Because the infinite horizon costs cannot be evaluated for non-linear problems, an upper bound is employed, which can be calculated relatively easily and which is minimized by the control algorithm. The open-loop optimal control problem is formulated as

$$\min_{\bar{u}} J(x(t), \bar{u}(\cdot))$$

with

$$\begin{aligned} J(x(t), \bar{u}(\cdot)) \\ = \int_t^{t+T_p} (|\bar{x}(\tau; x(t), t)|_{\mathcal{Q}}^2 + |\bar{u}(\tau)|_{\mathcal{K}}^2) d\tau \\ + |\bar{x}(t+T_p; x(t), t)|_{\mathcal{P}}^2 \end{aligned}$$

subject to

$$\bar{x}(t+T_p; x(t), t) \in \Omega, \quad (5)$$

where the penalty term on the final state  $\bar{x}(t+T_p)$ , the second term in the objective function, is determined to bound the infinite horizon cost:

$$\begin{aligned} |\bar{x}(t+T_p; x(t), t)|_{\mathcal{P}}^2 \leq \int_{t+T_p}^{\infty} (|\bar{x}(\tau; x(t), t)|_{\mathcal{Q}}^2 + |\bar{u}(\tau)|_{\mathcal{K}}^2) d\tau \\ \forall \bar{x}(t+T_p; x(t), t) \in \Omega. \end{aligned}$$

This bound is established by controlling the non-linear model fictitiously by linear optimal state feedback within the region  $\Omega$  after  $t+T_p$ . The control sequence computed at time  $k$  is feasible at all future times and only ‘improvement’ is necessary from time step to time step to guarantee stability. A similar technique was also proposed by Nicolao, Magni and Scattolini (1996, 1998).

The method holds much promise. The main unresolved difficulty at this point is the determination of the positively invariant region  $\Omega$  which appears to require that some global test is satisfied which again may not be trivial except for academic examples. Recently, a similar technique that removes the need for this inequality constraint has been proposed for open-loop stable systems (Chen & Allgöwer, 1997). The method still requires the  $\Omega$  region to be defined, however, for determining the terminal weighting matrix and prediction horizon.

### 3.2.4. Contractive model predictive control

The idea of contractive MPC was mentioned by Yang and Polak (1993), the complete algorithm and stability proof were developed by De Oliveira and Morari (1999). In this approach a constraint is added to the usual formulation which forces the *actual* and not only the *predicted* state to contract at discrete intervals in the future. From this requirement a Lyapunov function can be constructed easily and stability can be established. The stability is independent of the objective function and the convergence of the optimization algorithm as long as a solution is found which satisfies the contraction constraint. The feasibility at future time steps is not necessarily guaranteed unless further assumptions are made. Because the contraction parameter implies a specific speed of convergence, its choice comes natural to the operating personnel.

### 3.2.5. Model predictive control with linearization

All the methods discussed so far require a non-linear program to be solved on-line at each time step. The effort varies somewhat because some methods require only that a feasible (and not necessarily optimal) solution be found or that only an ‘improvement’ be achieved from time step to time step. Nevertheless the effort is usually formidable when compared to the linear case and stopping with a feasible rather than optimal solution can have unpredictable consequences for the performance. The computational effort can be greatly reduced when the system is linearized first in some manner and then the techniques developed for linear systems are employed on-line. Three different approaches have been proposed.

- Nevistić and Morari (1995) apply first feedback linearization and then use MPC in a cascade arrangement for the resulting linear system. The optimization problem becomes ‘almost’ a quadratic program and conditions for global stability can be established. The method is limited to low order systems which fulfill the conditions required for feedback linearization.
- In the first detailed industrial account of an application of non-linear MPC García (1984) uses at each time step a different *linear* model derived from a local (Jacobian) linearization, and employs standard linear DMC. Gattu and Zafriou (1992) and later Lee and Ricker (1994) proposed to add the extended Kalman filter to deal with unstable dynamics and to improve disturbance estimation. De Oliveira (1996) develops this idea further, imposes contraction constraints and derives explicit stability conditions which show the dependence on the quality of the linear approximation and various tuning parameters like the contraction constant.
- Nevistić (1997) shows excellent simulation results when a linear time varying (LTV) system approximation is used which is calculated at each time step over the predicted system trajectory (see also Lee & Ricker (1994)). The time-invariant MPC algorithm can be easily modified to accommodate LTV systems.
- Zheng (1997, 1998) focuses on incorporating a closed-loop control strategy into the MPC formulation and on reducing the on-line computational demand. The following approach is taken. The non-linear MPC control law is approximated with a linear controller (by linearizing the non-linear model and assuming no constraints). This linear controller is used to compute all the future control moves. The online computation effort is significantly reduced in this manner since only the first control move is computed by solving the optimization problem.

### 3.2.6. Research issues

This area is wide open for future research and all proposed approaches are little more than initial steps in more or less promising directions. Though the theoretical purists tend to stay away from linearization approaches, linearization is the only method which has found any wider use in industry beyond demonstration projects. For industry there has to be clear justification for solving non-linear programs on-line in a dynamic setting and there are no examples to bear that out in a convincing manner. In some sense and with further development quasi-infinite MPC may be ‘tuneable’ to use non-linear MPC only when really needed (far away from equilibrium) and linear MPC otherwise, thus combining the best of the ‘exact’ and the ‘linearization’ methods.

### 3.3. Robust model predictive control

When we say that a control system is robust we mean that stability is maintained and that the performance specifications are met for a specified range of model variations (uncertainty range). To be meaningful, any statement about ‘robustness’ of a particular control algorithm must make reference to a specific uncertainty range as well as specific stability and performance criteria. Although a rich theory has been developed for the robust control of linear systems, very little is known about the robust control of linear systems with constraints.

In the main stream robust control literature ‘robust performance’ is measured by determining the *worst* performance over the specified uncertainty range. In direct extension of this definition it is natural to set up a new ‘robust’ MPC objective where the control action is selected to minimize the worst value the objective function can attain as a function of the uncertain model parameters. This describes the first attempt toward a robust MPC algorithm which was proposed by Campo and Morari (1987). They showed that for FIR models with uncertain coefficients and an  $\infty$ -norm objective function the optimization problem which must be solved on-line at each time step is a linear program of moderate size. Unfortunately it is well known now that robust stability is not guaranteed with this algorithm (Zheng & Morari, 1993).

Zafriou (1990) used the contraction principle to derive some necessary and some sufficient conditions for robust stability. The conditions are conservative and difficult to verify. Genceli and Nikolaou (1993) showed how to determine weights such that robust stability can be guaranteed. However, weights may not exist even though robust stabilization is possible for a set of FIR models. Also, they assume independent uncertainty bounds on the FIR coefficients which can be very conservative.

The Campo algorithm fails to address the fact that only the first element of the optimal input trajectory is implemented and the whole min–max optimization is repeated at the next time step with a feedback update. In the subsequent optimization, the worst-case parameter values may change because of the feedback update. In the case of a system with uncertainties, the open-loop optimal solution differs from the feedback optimal solution, thereby violating the basic premise behind MPC. This is why robust stability cannot be assured with the Campo algorithm.

A true bound on the worst-case cost can be determined when the uncertain parameters are arbitrarily time varying within specified bounds. For this case Lee and Yu (1997) have defined a dynamic programming problem (thus accounting for feedback) to determine the control sequence minimizing the worst case cost. They show that with the horizon set to infinity this procedure guarantees robust stability. However, the approach suffers from the ‘curse of dimensionality’ and the optimization problem at each stage of the dynamic program is non-convex. Thus, in its generality the method is unsuitable for on-line (or even off-line) use except for low order systems with simple uncertainty descriptions.

Most other papers in the literature aim at explicitly or implicitly approximating the problem above by simplifying the objective and uncertainty description, and making the on-line effort more manageable, but still guarantee at least robust stability. For example, Lee and Yu (1997) use a 2-norm and Zheng and Morari (1994) an  $\infty$ -norm *open-loop* objective function. Both assume FIR models with uncertain coefficients. A similar but more general technique has also been proposed for state-space systems with a bounded input matrix (Lee & Cooley, 1997).

These formulations may be conservative for certain problems leading to sluggish behavior because of two reasons. First of all, arbitrarily time-varying uncertain parameters are usually not a good description of the model uncertainty encountered in practice, where the parameters may be either constant or slowly varying but unknown. Second, the computationally simple open-loop formulations neglect the effect of feedback. Third, the worst-case error minimization itself may be a conservative formulation for most problems. Zheng and Morari (1994) and Zheng (1995) propose to optimize *nominal* rather than robust performance and to achieve robust stability by enforcing a *robust contraction constraint*, i.e. requiring the worst-case prediction of the state to contract. With this formulation robust global asymptotic stability can be guaranteed for a set of linear time-invariant stable systems. The optimization problem can be cast as a quadratic program of moderate size for a broad class of uncertainty descriptions.

Badgwell (1997) suggested recently to replace the state contraction constraint by a ‘cost contraction constraint’. This approach leads to a convex optimization problem but is applicable only to a discrete set of plants and cannot be used for systems with integrating modes, e.g. step response models.

To account for the effect of feedback Kothare, Balakrishnan and Morari (1996) propose to calculate at each time step not a sequence of control moves but a state feedback gain matrix which is determined to minimize an upper bound on robust performance. For fairly general uncertainty descriptions, the optimization problem can be expressed as a set of linear matrix inequalities for which efficient solution techniques exist.

Lastly, it is also possible to adopt a stochastic uncertainty description (instead of a set-based description) and develop an MPC algorithm that minimizes the expected value of a cost function. In general, the same difficulties that plagued the set-based approach are encountered here. One notable exception is that, when the stochastic parameters are independent sequences, the true closed-loop optimal control problem can be solved analytically using dynamic programming (Lee & Cooley, 1998). In many cases, the expected error may be a more meaningful performance measure than the worst-case error. A contraction constraint can be added to guarantee robust stability for a model set corresponding to a specified probability level.

#### 4. Future—what’s needed?

As we saw in the previous section, the theory of MPC has matured considerably. However, according to the practitioners, what limits the performance and applicability of MPC are not the deficiencies of the control algorithm, but difficulties in modeling, sensing, state estimation, fault detection/diagnosis, etc. MPC points out new needs in these areas and also suggests new approaches: For example, in the past, tasks like fault detection were dealt with at the supervisory level in the form of a ‘fuzzy’ or ‘knowledge-based’ decision maker. As we will point out, there exist now new formulations of MPC involving integer variables, which hold promise for a combined approach to control and diagnosis. Similarly, there is the possibility to include *qualitative knowledge* in a systematic manner in the control decision process.

##### 4.1. Improved identification

Model development is by far the most critical and time-consuming step in implementing a model predictive controller. It is estimated that, in a typical commissioning project, modeling efforts can take up to 90% of the cost and time (Andersen & Kummel, 1992). Quite



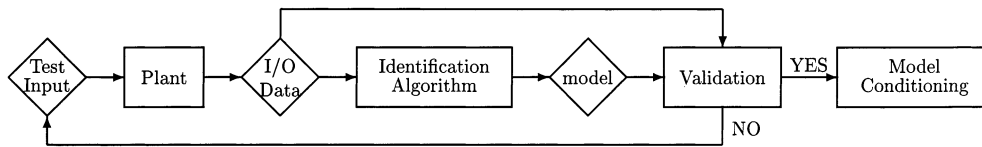


Fig. 1. Conventional model identification practice.

commonly MPC applications in industry involve dozens of inputs and outputs. The need to develop multivariable models of such sizes through plant tests puts unprecedented demands on model identification techniques. The conventional steps to arrive at models for MPC applications are illustrated in Fig. 1. Each of the steps can be improved greatly, as discussed below:

- **Test input signal design**

Conventionally, models used in MPC applications are identified through a series of step tests. In some cases, PRBS tests instead of step tests are used and impulse response coefficients are fitted through least squares or through ridge regression (Cutler & Yocum, 1991). In most cases, input channels are perturbed *one at a time*, leading to SISO identification. While this practice is simple and easy to implement, it emphasizes the accuracy of individual SISO models and may not yield a multivariable model of required accuracy. There are many practical examples where the open-loop responses (either step responses or frequency responses) for all the SISO systems are fitted almost perfectly, but the prediction based on the combined multivariable model when several inputs are changed simultaneously is extremely poor (Li & Lee, 1996b). Implementing a controller designed with such a model can lead to poor closed-loop performance and instability. One can experience the same problem with MISO/MIMO identification, as long as perturbations introduced to various input channels are independently designed. This is because, in a highly interactive process, gain directionality of the process causes the responses of output channels to exhibit strong correlation, even to the extreme of co-linearity. This can lead to problems like a poor signal-to-noise ratio and strong bias in the low gain direction(s) (Andersen & Kummel, 1992).

- **Identification algorithm**

In most cases, model fitting is done using SISO or MISO methods. Because the model for each output is fitted separately in these methods, correlations among different outputs cannot be captured or exploited. A MIMO identification algorithm on the other hand fits a single model for all the outputs simultaneously (usually in the form of a combined deterministic/stochastic system) while accounting for

any existing correlation. Not only can this lead to an improved identification of the deterministic part, but the stochastic part of the model can potentially be useful in the prediction. The latter is particularly true in designing a model predictive control system for those applications where some of the controlled variables are either not measured or measured with large delays and must be inferred from secondary process measurements for satisfactory control (see Amirthalingam & Lee (1997) for an example application).

- **Model validation**

Usually model validations amount to examining the prediction errors of individual SISO models with some additional data. As we mentioned earlier, this can lead to misleading conclusions about model quality. SISO models that are very accurate can constitute a very poor MIMO model when viewed together. What is needed is a more rigorous model (uncertainty) analysis scheme that quantifies the achievable closed-loop performance.

There are results in the literature that provide promising directions or partial solutions to the above-mentioned challenges. For instance, a number of remedies have been proposed against the gain directionality problem, including: correlated design based on SVD analysis (Koung & MacGregor, 1994), closed-loop identification (Jacobsen, 1994; Li & Lee, 1996a,b), and iterative/adaptive input design (Cooley & Lee, 1996). The recently introduced subspace identification method (Van Overschee & De Moor, 1994) may fill the need for a practical MIMO identification algorithm. In addition, several investigators have developed methods to obtain frequency-domain uncertainty bounds, albeit mostly in the SISO context (Goodwin, Gevers & Ninness, 1992; Wahlberg & Ljung, 1992; Cooley & Lee, 1997).

These tools pave the way toward *integrated identification and control*, which is depicted in Fig. 2 (Cooley & Lee, 1997). This integrated methodology includes: (1) optimal test signal generation based on the collected plant information, closed-loop objectives and plant constraints; (2) quantification of model uncertainty; and (3) rigorous analysis of stability and achievable performance on the basis of the model and its uncertainty. The tools and theories discussed above represent just a few pieces of the whole puzzle, however.

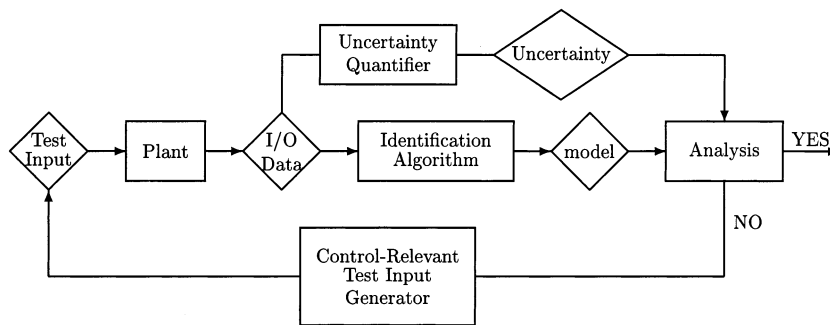


Fig. 2. Integrated identification and control methodology.

#### 4.2. Performance monitoring and diagnosis

Many model predictive controllers that perform well when first commissioned deteriorate over time, some leading to eventual shutdowns (Studebaker, 1995). In an industrial setting, easy maintainability of control systems is key to long-term success. In order to sustain the benefits of model predictive controllers over a long period of time, a mechanism to detect an abnormality and diagnose its root cause is needed. The results can be communicated to engineers and can also be used to adapt control parameters.

Recent publicity of the maintenance problem for industrial control loops has stimulated research in the area of control system performance monitoring and diagnosis. Thus far most researchers have concentrated on developing performance measures for existing loops (Stanfelj, Marlin & MacGregor, 1993; Harris, Boudreau & MacGregor, 1995; Kozub, 1996; Tyler & Morari, 1996a). Very few have examined the problem specifically for model-based control systems. For model-based control systems, Kesavan and Lee (1997) proposed to monitor the prediction error to detect an abnormal trend and run a few simple diagnostic tests to gain insights into the source.

The problem of fault diagnosis in the model-based setting has been studied by researchers in many disciplines and there is a wealth of literature on the subject (Wilsky, 1976; Isermann, 1984). For instance, with fault states created in the model, it can be viewed as a state estimation problem. It is, however, an unconventional kind in that joint-Gaussian statistics poorly describe the characteristics of most fault signals. A better choice is a Gaussian-sum model, which leads to multiple filter estimation (Tugnait & Haddad, 1979; Kesavan & Lee, 1997).

Some MPC vendors have recognized the importance of self-managing abnormal situations and have launched major research and development efforts on the subject. The next generation of commercial MPC algorithms is sure to be equipped with self-diagnostic

features and schemes to manage abnormal situations in an autonomous fashion. However, there is yet to be a consensus on what specific approaches are to be taken. Many believe that a synergistically combined variety of tools (e.g. analytical redundancy, pattern recognition, hardware redundancy) will be needed.

#### 4.3. Non-linear system identification

In practice, it is seldom feasible, technically or economically, to develop detailed first principles models. One of the important reasons for MPC's success in industry has been the ability of engineers to construct the required models efficiently from plant tests. Unlike the linear case, however, there is no established method to construct a non-linear model through a plant test. Recognition of the need has made empirical modeling of non-linear systems a focal research topic within the process control community.

In spite of vigorous research, many fundamental issues remain unresolved in the non-linear system identification area. We list some of them here.

- Model structure determination

This is by far the most difficult and pressing issue. Typically, an input output model of the following form is identified:

$$y(k) = F(\phi(k), \theta) + \varepsilon(k) \quad (6)$$

where  $\phi$  is the regressor vector containing the delayed input and output terms and  $\theta$  is a vector containing the unknown parameters. Depending on what  $\phi$  contains and what parameterization of  $F$  is used, we get different model structures (Lee, 1998). The questions regarding the structure determination include: (1) What are the intrinsic differences between various structures like NARX, NARMAX, NMA, Hammerstein, Wiener, etc. and what prior knowledge and/or plant tests are needed to determine the correct structure? (2) How do we determine how many delayed input and/or

output terms to include in  $\phi$  given a data set (Rhodes & Morari, 1998)? (3) How do we choose among the various basis functions and connection structures available today?

- **Test input signal design**

Another difficult issue is the test signal design. Unlike the linear case, conditions for parameter convergence have not been established, except in some special cases. In addition, the need to integrate the closed-loop robustness considerations into the experiment design is even more compelling than in the linear case, since non-linear system dynamics are much more general and the characteristics of the resulting model are very much shaped by those of the data. A similar approach to the one discussed earlier for linear system identification can be envisioned for non-linear system identification as well.

- **MIMO model fitting algorithm**

Most literature on non-linear system identification has focused on SISO systems while most systems of practical interest involve multiple inputs and outputs. As mentioned before, identifying the individual SISO models separately and combining them into a multivariable model is generally not effective. The non-linear time series approach is theoretically possible but unlikely to yield a practical answer due to problems like loss of identifiability and the need to solve an ill-conditioned, non-convex optimization problem. A more promising approach for non-linear multivariable system identification is to define ‘states’ from input output data through appropriate non-linear projection and build a state-space model. Some initial ideas along this line are described by Lee (1998).

- **Uncertainty quantification for robust control**

Since non-linear models derived from input-output data will inevitably contain significant bias and variance, the uncertainties need to be quantified and used in the controller design and analysis. The theory for doing this is still at the developmental stage, even for linear systems. However, the need for systematic tools to deal with them is clear as insights and heuristics developed for linear controllers do not apply to non-linear controllers in general.

Among the variety of model structures suggested and studied in the literature, two seem to be best developed or most in line with the current industrial practice. The first is the Volterra kernel, which can be viewed as an immediate high-order extension of the FIR model currently employed in most commercial MPC algorithms.

Identification of the Volterra kernel has been well studied and conditions on the input test signals for asymptotic convergence of the parameters under prediction error minimization have been established (Koh & Powers, 1985; Pearson, Ogunnaike & Doyle, 1993, 1996). A stumbling block for embracing this model type

as the choice for general non-linear control problems is the large number of parameters which explodes with the system’s input dimension. Volterra models beyond second order seem impractical.

The second is a *piece-wise* linear model, which can be obtained, for instance, by fitting so called *hinging-hyperplanes* (Breiman, 1993). This model has a nice local linear interpretation and is conducive to dynamic scheduling of linear models within the existing MPC algorithms (Chikkula, Lee & Ogunnaike, 1998). An approach related to this is to linearly interpolate several a priori constructed models in the state space (Johansen & Foss, 1994; Arkun, Ogunnaike, Banarjee & Pearson, 1995). The interpolation parameters can be determined a priori on the basis of off-line data and prior knowledge (Johansen & Foss, 1994) or can be estimated online (Arkun, et al., 1995).

#### 4.4. Model predictive control for batch processes

Control problems in batch processes are typically posed as tracking problems for time-varying reference trajectories defined over a finite time interval. During the course of a typical batch, process variables swing over wide ranges and process dynamics go through significant changes due to the non-linearity, making the task of finding an accurate process model very difficult. Because of this, a conventional model-based control system is likely to lead to significant tracking errors. This may explain why there have been so few applications of MPC to batch processes.

A unique aspect of batch operations that can be exploited is that they are repetitive. Hence, errors in one batch are likely to repeat in the subsequent batches. A framework to use the past batch data along with the real-time data is clearly needed. As a step toward this, Lee and co-workers (Lee & Lee, 1996, 1997) took the idea of iterative learning control (popular in robot arm training) and developed an MPC algorithm tailored to the specific needs and characteristics of the batch process control problem. Their work is based on a transition model of the error trajectory from one batch to the next that includes stochastic components. Previous batches are remembered through state estimation and used in the predictive control computation. The method can also be applied to processes that undergo the same transitions repeatedly. It should be mentioned that the idea of run-to-run learning has also been used in the context of batch optimization (Zafriou & Zhu, 1990; Zafriou, Chiou & Adomaitis, 1995).

Another aspect of batch system control that deserves further investigation is quality control. Quality variables can be controlled in a cascade control fashion, i.e. by adjusting the reference trajectories fed to the track-

ing controllers for process variables like the temperature and the pressure. However, direct feedback-based on-line adjustments are seldom feasible as most quality variables cannot be measured on-line. The standard industrial practice is to use the statistical monitoring charts (for off-line quality measurements available after the batches) to make adjustments only when significant and prolonged deviations are observed. Not only is this approach ineffective in reducing often-significant batch-to-batch variations, it also results in large amounts of off-spec products due to the delay. A promising alternative is to build a statistical correlation model between the process variables and the quality variables and control the quality variables in an inferential manner. Such an approach has been found to be highly effective in studies involving a pulp digester and a Nylon autoclave (Kesavan & Lee, 1998; Russell, Kesavan Lee & Ogunnaike, 1998).

The above-mentioned concepts and methods need to be tested on practical problems. After some refinements on the basis of practical trials, a general software package could be built for batch systems.

#### 4.5. Moving horizon estimation

In most practical problems, states of the system are not directly accessible and must be estimated. The quality of state estimates has important bearings on the overall performance of a model predictive controller, especially of one based on a non-linear model. Unlike the linear case, however, there is no established method for non-linear state estimation. The most popular method is the extended Kalman filter, which simply relinearizes the non-linear model at each time step and updates the gain matrix and the co-variance matrix on the basis of linear filtering theory. Motivated by the success of MPC, a similar optimization-based state estimation technique has been studied by several investigators recently (Robertson, Lee & Rawlings, 1994; Michalska & Mayne, 1992). The idea is to formulate the estimation problem within a finite moving window and to find the values of the unknown sequences (e.g. initial condition, state noise, measurement noise) in some least squares sense. Constraints can be added to the least squares problem to express a priori known bounds on the system states as well as the external signals. Once the unknowns are estimated, the states can be reconstructed using the model. In the linear case with no constraints, it can be shown that moving horizon estimation is equivalent to the Kalman filter for certain choices of weighting matrices (Robertson, et al., 1994). A statistical interpretation also exists for the constrained/non-linear case, which suggests the choice of the weighting matrices (Robertson, 1996). Michalska and Mayne (1992) establishes the stability of a very restrictive form of moving hori-

zon estimator. Rao and Rawlings (1998) introduces a concept called 'arrival cost' which is dual to the 'cost-to-go' in the dynamic programming solution of the LQ problem and shows that the stability can be guaranteed by lower-bounding the arrival cost.

However, maintaining both optimality and stability seems to be a difficult task; it appears that additional somewhat artificial assumptions need to be made to guarantee stability (Tyler & Morari, 1996c).

#### 4.6. Improved optimization

A demanding feature of most model predictive controllers is that an optimization must be solved online. Depending on the nature of the model and the performance specification, this may be an LP, QP or NLP. Though LPs and QPs are thought to be easy to solve, they can still be computationally demanding for large-scale problems (problems with a large number of variables and/or those with large horizons). Often the NLP is solved by sequential quadratic programming (SQP), which is computationally very expensive and comes with no guarantee of convergence to a global optimum. For efficiency, many vendors currently solve the QP and LP in a heuristic manner, for example, by using dynamic weighting matrices.

Recently, the so called interior-point (IP) methods for solving LPs have been drawing much attention. Originally developed about 15 years ago, reliable public-domain and commercial codes are becoming available nowadays. A remarkable, though not proven, feature of these methods is that they seem to converge within 5–50 iterations regardless of the problem size (Boyd, 1997), an attractive feature for on-line use. Moreover, these methods are readily extendible to QPs and SQPs (Wright, 1996; Biegler, 1997). In IP methods each iteration involves solving a system of linear equations. For MPC problems, it should be possible to achieve a substantial speed gain through the use of a sparse solver. These developments are expected to have major bearings on the future practice of MPC since they will enable the user to solve large-scale problems very efficiently and reliably (without resorting to heuristics and fudge factors which may or may not work).

Another way to increase the efficiency and reliability is to exploit the structure of the problem. The Hessian and constraint matrices of the QPs are highly structured and exploiting this fact has been shown to speed up the computation by orders of magnitude (Biegler, 1997). This may be the key to solving NLPs and large-scale QPs reliably and efficiently. Similar efforts are also under way for highly structured, large-scale LPs (Doyle, Pekny, Dave & Bose, 1997).

#### 4.7. New opportunities by including integer decision variables in model predictive control

Integer variables and linear constraints can be used to represent heuristic process knowledge. Any relationship which can be expressed as propositional logic can be translated into this framework (Raman & Grossman, 1992). Apparently, it was not recognized that many possible applications of this approach exist in the area of control and detection (Tyler & Morari, 1996b; Bemporad & Morari, 1999).

The introduction of integer variables allows the extension of MPC/MHE techniques to hybrid systems, i.e. systems described by interdependent physical laws and logic rules. Hybrid systems include finite state machines driven by conditions on continuous dynamics, discrete event systems, and piece-wise linear systems. A framework for modeling such a class of systems through integer variables has been introduced by Bemporad & Morari (1999). The authors propose an MPC controller which is able to stabilize linear hybrid systems on desired reference trajectories while fulfilling operating constraints, and possibly take into account previous qualitative knowledge in the form of heuristic rules. The controller is also capable of prioritizing constraints as well as altering the control objective depending upon the positions of control inputs. Because of the presence of integer variables the resulting optimization problems are *mixed integer quadratic programs* (MIQP), for which efficient solvers have been developed recently.

Similar ideas can be applied in a moving horizon estimation framework. Integer variables can be used in detection problems to represent the occurrence of symptoms which are indicative of classes of failures. In applications where uncertain models must be used, false alarms due to uncertainty can be reduced by combining quantitative fault estimation with symptom based fault estimation. When residuals are primarily due to modeling uncertainty, the use of logic variables corresponding to symptoms will prevent erroneous fault alarms.

## 5. Conclusions

Over the last decade a mathematically clean formulation of MPC emerged which allows researchers to address problems like feasibility, stability and performance in a rigorous manner. In the non-linear area a variety of issues remain which are technically complex but have potentially significant practical implications for stability and performance and the computational complexity necessary to achieve them. The new software tools, however, which are becoming available for developing first-principle models efficiently have led to a steady increase in the use of non-linear MPC in

industry. There have been several innovative proposals how to achieve robustness guarantees but no procedure suitable for an industrial implementation has emerged. While a resolution of the aforementioned issues will undoubtedly change our understanding of MPC and be of high scientific and educational value, it may never have more than a minor effect on the practice of MPC. Seemingly peripheral issues like model identification and monitoring and diagnostics will continue to be decisive factors if MPC will or will not be used for a certain application. By generalizing the on-line MPC problem to include integer variables it will be possible to address a number of practical engineering problems directly which may lead to a qualitative change in the type of problems for which MPC is used in industry.

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