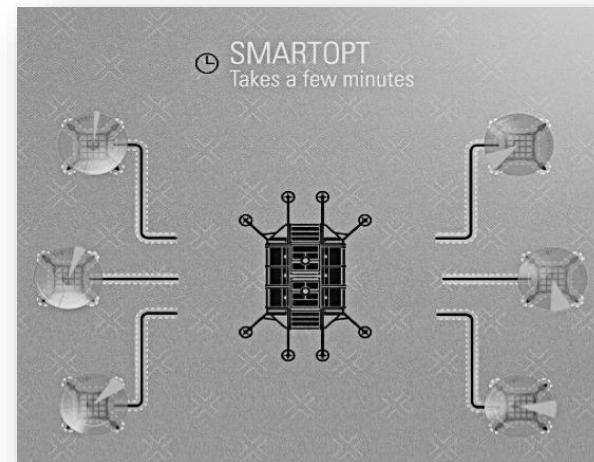
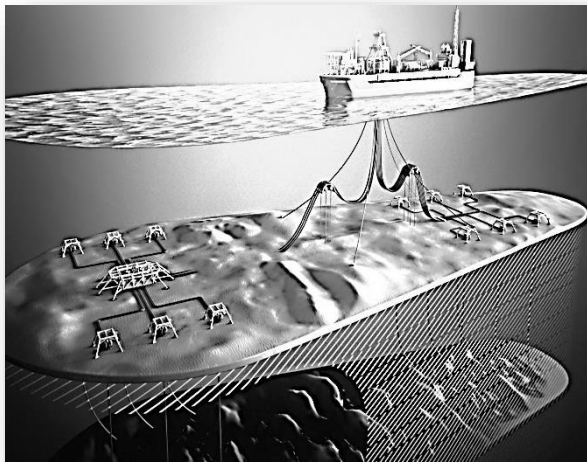


# Course module

## *Daily production optimization methods*



Fall 2014

**Stine Ursin-Holm, Vidar Gunnerud & Bjarne Foss NTNU**

**IO Center**

# Outline

- 1 Introduction
- 2 Mathematical optimization
- 3 SmartOpt approaches
- 4 Case
- 5 Results
- 6 Conclusion

# Outline

1 Introduction

2 Mathematical optimization

3 SmartOpt approaches

4 Case

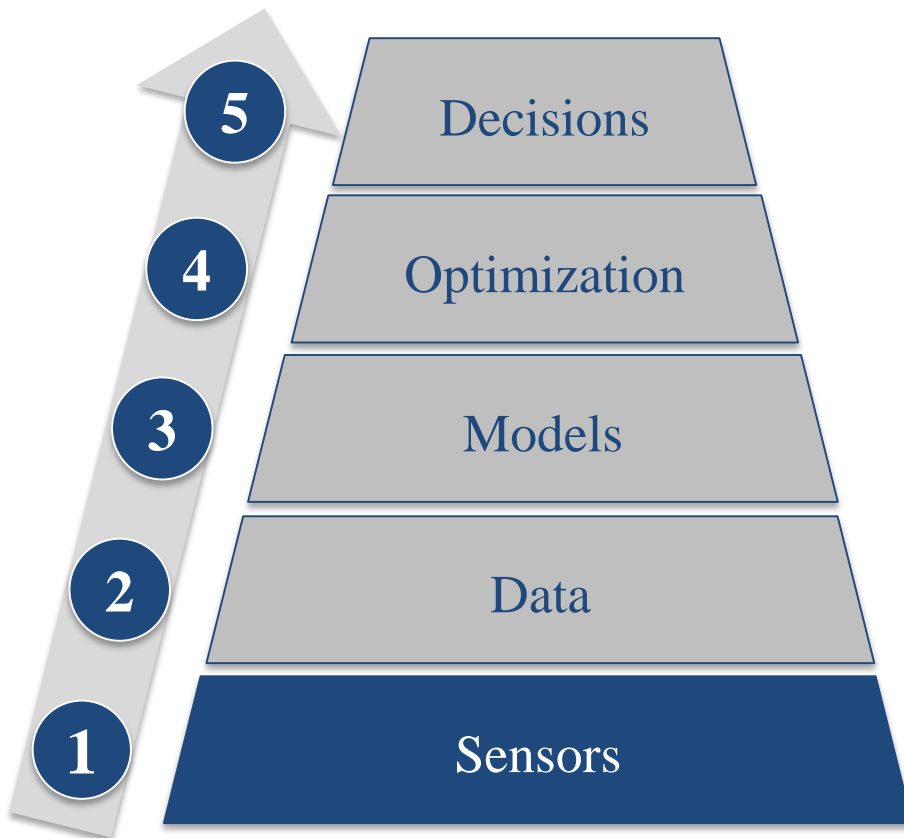
5 Results

6 Conclusion

# Decision support systems for operational decisions can be divided into a sequence of components

## From sensors to decisions

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### Automation level within process industry

- ❑ Arriving at a decision is still the task of an **experienced engineer**
- ❑ Computer models help the engineers **analyse and reason** about the systems behaviour
- ❑ However, the methods often **fail to provide satisfactory results**
- ❑ Increased level of automation can **improve the efficiency and quality** of the decisions made

Simulators have entered many engineering disciplines, due to their contribution in modelling complex systems

## **Simulation-based optimization**

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### **The industry uses simulators in a variety of ways:**

- ❑ “What-if” analyses of different solutions or alternative courses of action
- ❑ Sensitivity analyses, if derivatives are available
- ❑ Optimization approaches that can build directly on simulators, as the models usually scale up well

### **The IO Center approach:**

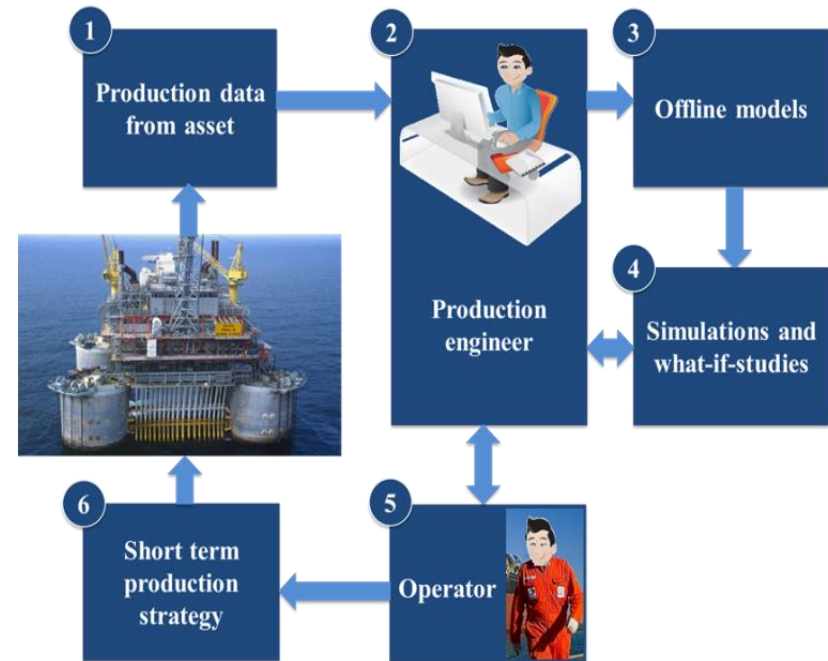
- Combine simulators with state-of-the-art optimization techniques
- Add optimization functionality onto simulators with certain structural properties

A key challenge in offshore petroleum field operations is to decide on the optimal day-to-day production strategy

## Daily production optimization process

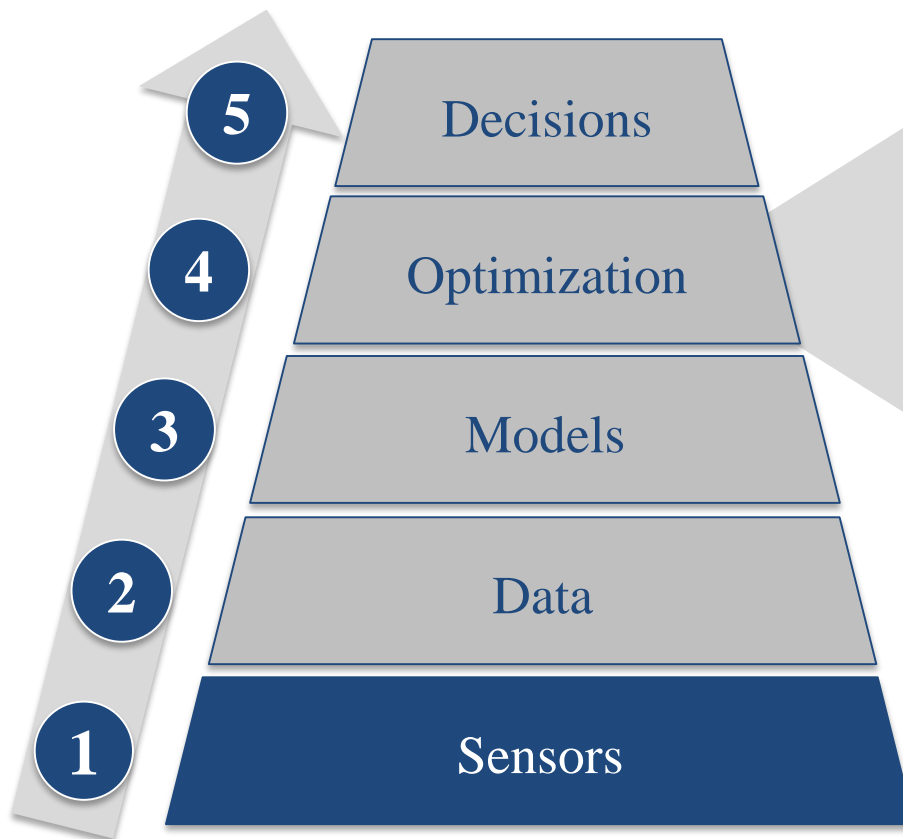
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- ❑ Several **tools and individuals** are involved in the process
- ❑ The time horizon are equal to a period where **changes in the reservoir are negligible (days up to a week)**
- ❑ State-of-the-art decision support tools are **frequently imprecise and slow** at computing recommendations
- ❑ We present efficient simulation-based optimization strategies that provide **reliable suggestions to optimal production strategies**



A key challenge in offshore petroleum field operations is to decide on the optimal day-to-day production strategy

*“The decision pyramid for short-term production optimization”*



### State of the art

- ❑ A significant amount of manual work
- ❑ Software tools are imprecise and slow

### New IO Center techniques:

- ❑ Enable optimization of larger and complex systems
- ❑ Substantial reduction in solution time
- ❑ On the fly recommendations

# Outline

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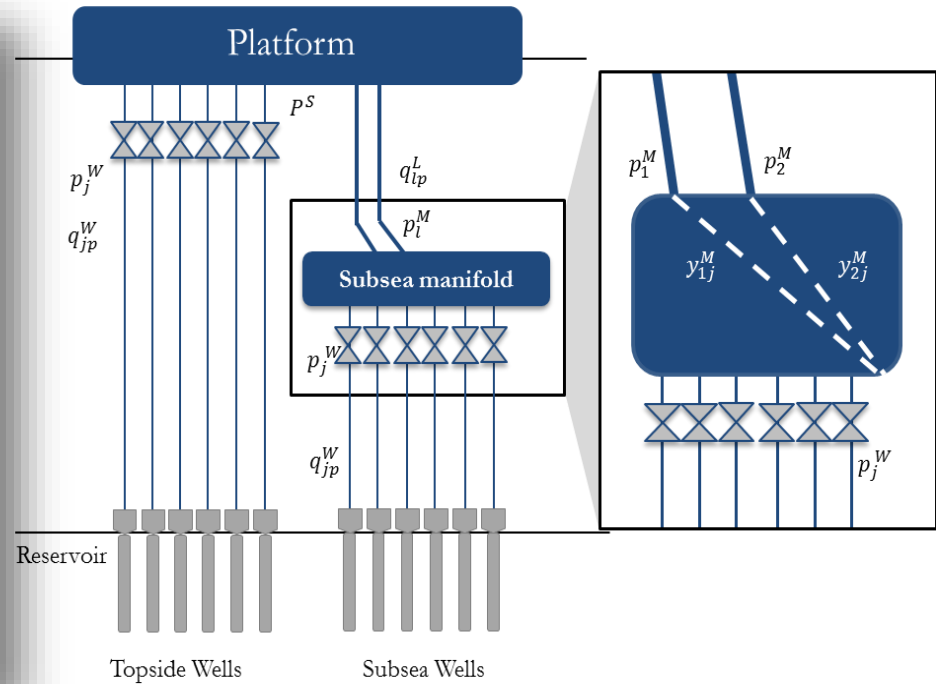
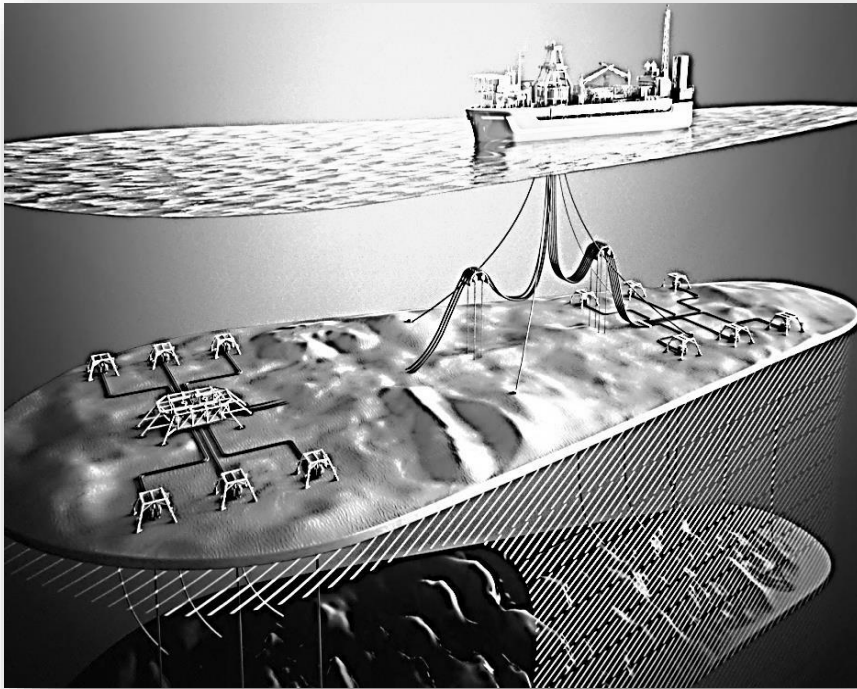


# 12 well production system with artificial gas lift

## 6 subsea wells and 6 satellite wells

### Case example

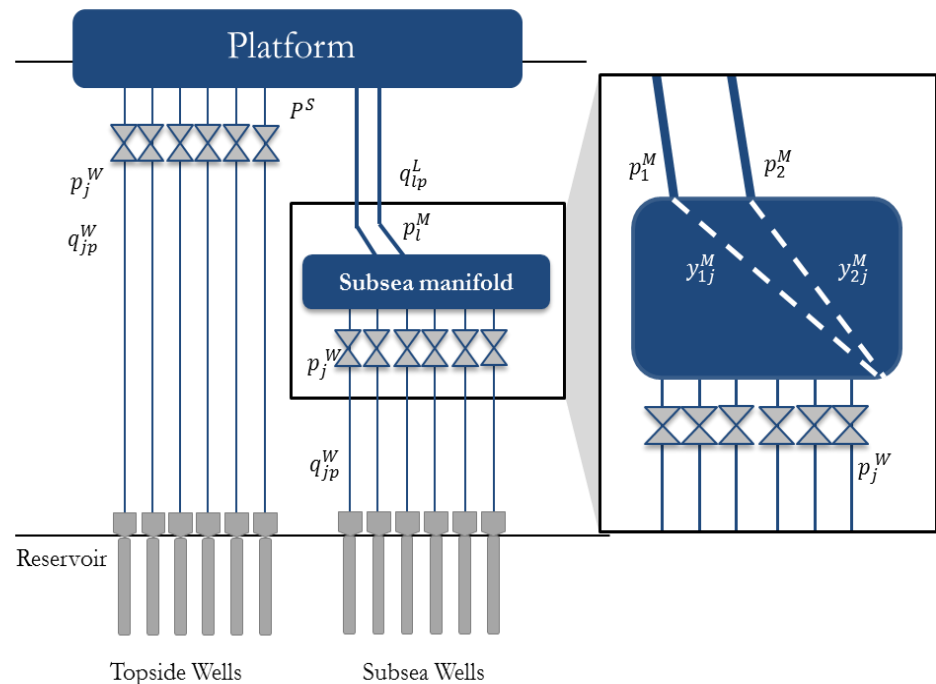
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# The production optimization problem aim to find the optimal operating plan for a production asset

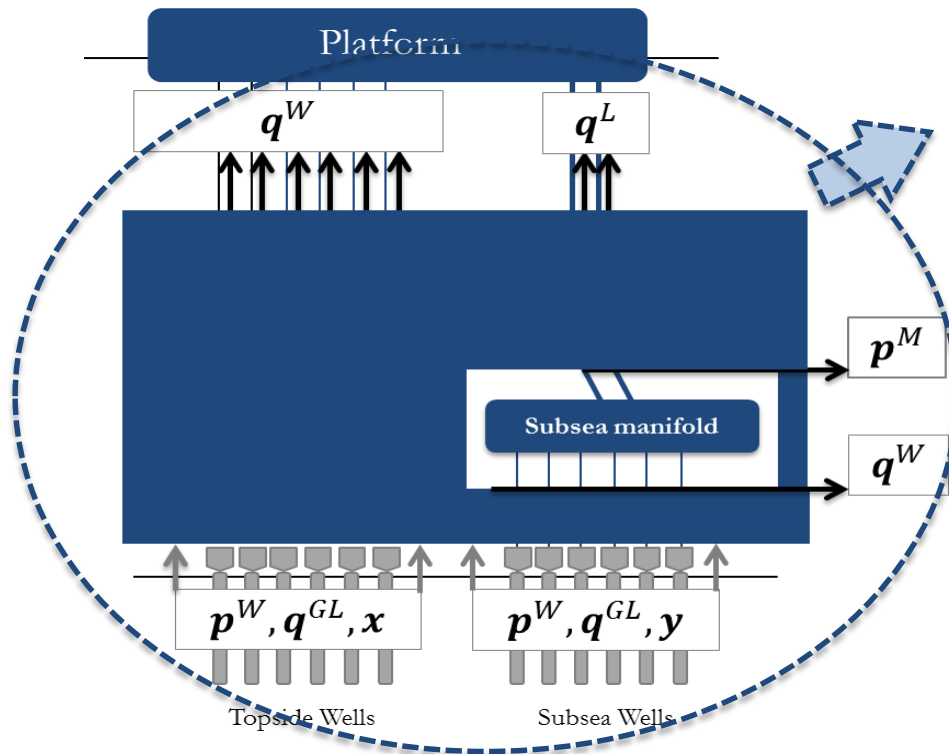
## About the optimization problem

- ❑ The production optimization problem aims to **find the best short term production strategy**
- ❑ Thus, it will provide optimal values on **the relevant variables**
- ❑ While at the same time fulfilling the requirements of **mass balances, capacities and pressure constraints**
- ❑ The time horizon should be equal to a period where **changes in the reservoir are negligible, and pipeline dynamics can be assumed steady-state**



The Black box simulator is a complex calculator giving output values based on some input values

## The Black box simulator in details



The whole network simulated as one simulator:

$$(q^W, q^L, p^M) = f^{BB}(p^W, q^{GL}, x, y)$$

**Inputs:**

- ❑ Wellhead pressures,  $p^W$
- ❑ Gas lift  $q^{GL}$
- ❑ Routing of all wells,  $x$  and on/off  $y$

**Outputs:**

- ❑ Production rates from all wells,  $q^W$
- ❑ Pipeline pressures at the subsea manifold,  $p^M$
- ❑ Pipeline flow rates,  $q^L$

Production optimization problem, objective and relevant variables and constraints depends on the case

### Alternative participants of an optimization problem

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#### *Objectives*

Maximize oil production  
Maximize NPV (Net Present Value)  
Minimize operational cost

#### *Variables*

Production choke opening  
Gas lift flow rate  
Well routing  
ESP speed

#### *Constraints*

Pressure constraints  
Water, gas, liquid and gas lift capacities  
Erosion or velocity constraints  
Pressure-Temperature envelope - hydrates

Mass balances and discrete decisions handled inside the box,  
pressure balances and capacity constraints treated outside

*“The Black box optimization problem”*

**Objective function**

$$\max \sum_{l \in L} q_{lo}^L + \sum_{j \in J_t} q_{jo}^W$$

**Black box simulator**

$$(q^W, q^L, p^M) = f^{BB}(p^W, q^{GL}, x, y)$$

**Capacity constraints**

$$\sum_{l \in L} q_{lp}^L + \sum_{j \in J_t} q_{jp}^W \leq C_p$$

**Pressure balances**

$$p_l^M \leq p_j^W \quad j \in J_s$$

**Routing**

$$\sum_{l \in L} y_{jl} \leq 1 \quad j \in J_s$$

$$\sum_{j \in J_s} \sum_{l \in L} q_j^{GL} y_{jl} + \sum_{j \in J_t} q_j^{GL} x_j \leq C^{GL}$$

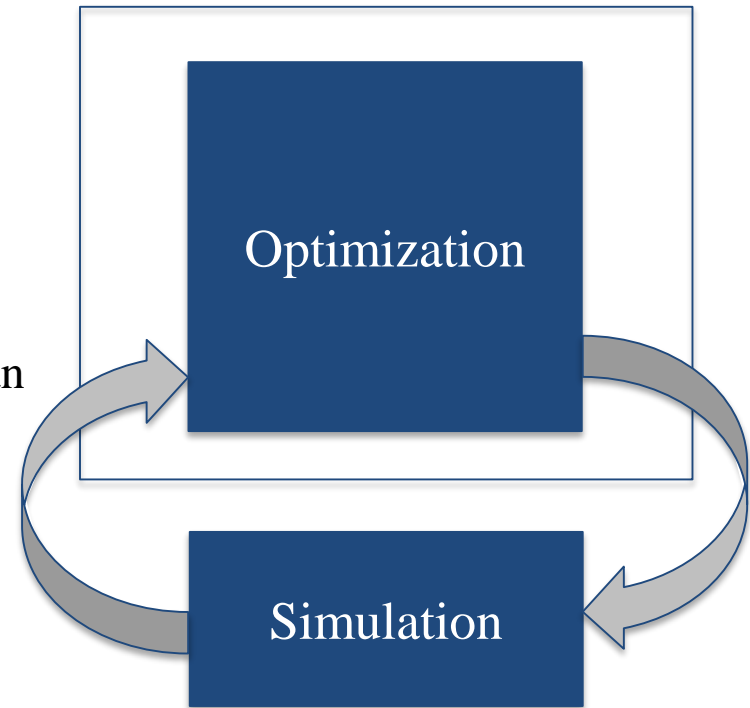
$$P^S \leq p_j^W \quad j \in J_t$$

# The traditional method optimize by simulate the network as one large black box

## An overview of the Black box approach

---

- ❑ A black box simulator represent the whole network
- ❑ Routing decisions are handled inside the black box simulator
- ❑ Additional constraints represent the boundary conditions
- ❑ Black box simulators usually only allow an optimization algorithm to make queries with discrete values on routing and on/off decisions
- ❑ Therefore it is only possible to get gradients for continuous variables
- ❑ The problem must then be solved with derivative-free algorithms



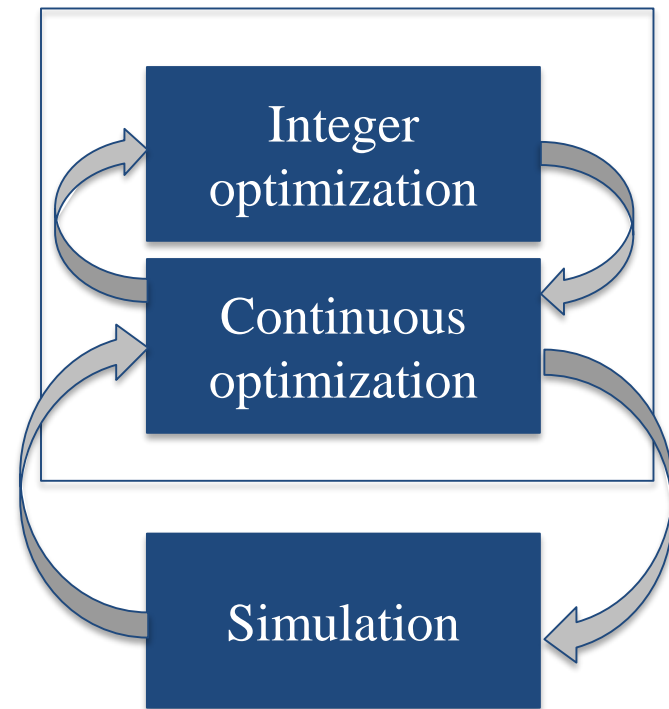
2-layer approach enables production optimization for complex systems by decomposing the problem and simulator

## Splitting the decision space

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### 2-layer approach

- ❑ The decision space is split in one integer subspace and one continuous subspace
- ❑ Still the mathematical formulation stays similar to the traditional approach
- ❑ However, it facilitates the use of optimization algorithms in two layers
- ❑ A derivative-free algorithm decides upon the routing and on/off decisions
- ❑ For each iteration of the derivative-free algorithm, a gradient-based algorithm communicates with the network simulator to find the optimal solution of the continuous subspace



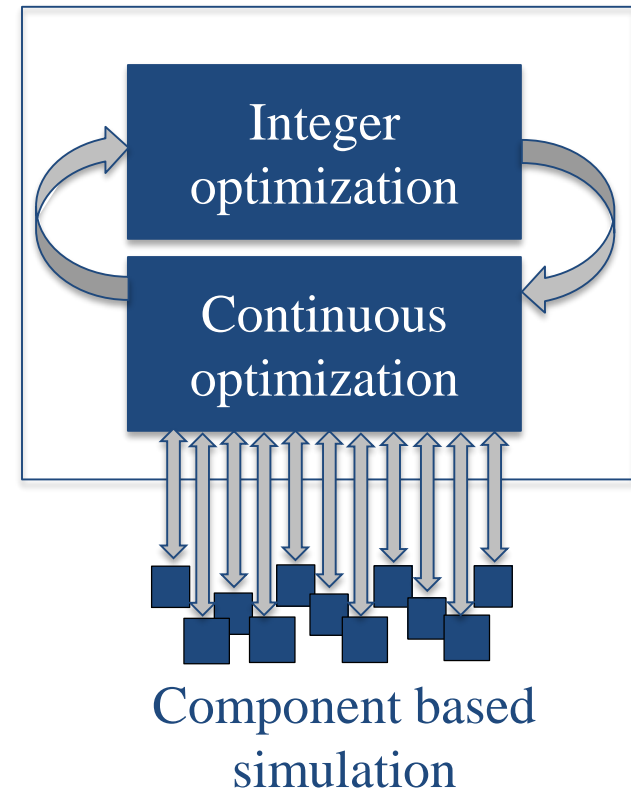
The SmartOpt approach enables production optimization for complex systems by decomposing the problem and simulator

## Splitting up the decision space and the network simulator

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### SmartOpt

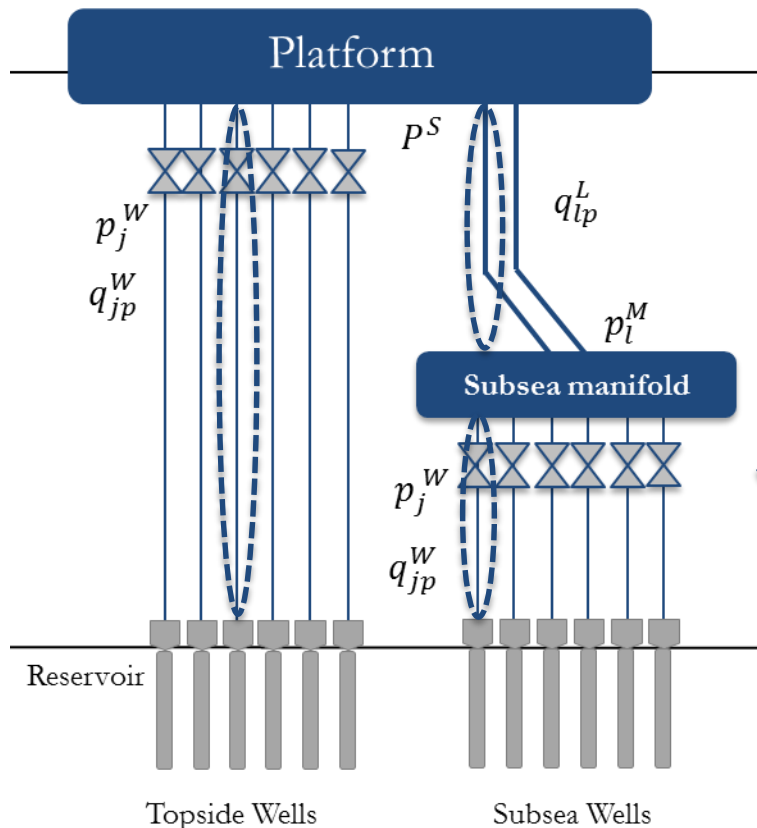
- ❑ The decision space is first split into integer and continuous subspaces
- ❑ It utilizes the information in the network structure and the simulator is decomposed into component simulators for wells and pipelines
- ❑ Mass and pressure balances can be treated inside the optimization algorithm
- ❑ The decomposition enables relaxation of all integer variables
- ❑ And facilitate the utilization of Branch & Bound and gradient based search algorithms





SmartOpt utilizes the production network structure and separates the simulations into smaller parts, one for each well and pipeline

## SmartOpt simulation strategy



**Splits network into simulators for each well and pipeline...**

- Well  $q^W = f^W(p^W, q^{GL})$
- Pipeline  $p^M = f^L(q^L)$

*...while mass balance and pressure relations are treated as explicit algebraic constraints*

By splitting up the simulator, discrete decisions are treated in analytical constraints, enabling Brach & Bound

*“The SmartOpt problem”*

### Objective function

$$\max \sum_{l \in L} q_{lo}^L + \sum_{j \in J_t} q_{jo}^W x_j$$

### Capacity constraints

$$\sum_{l \in L} q_{lp}^L + \sum_{j \in J_t} q_{jp}^W x_j \leq C_p$$

$$\sum_{j \in J_s} \sum_{l \in L} q_j^{GL} y_{jl} + \sum_{j \in J_t} q_j^{GL} x_j \leq C^{GL}$$

### Pressure and mass balance

$$\sum_{j \in J_s} q_{jp}^W y_{jl} = q_{lp}^L \quad \sum_{l \in L} y_{jl} \leq 1 \quad j \in J_s$$

$$p_l^M y_{jl} \leq p_j^W \quad j \in J_s$$

$$x_j P^S \leq p_j^W \quad j \in J_t$$

### Well and pipeline

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$

$$p_l^M - P^S = f_l^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$

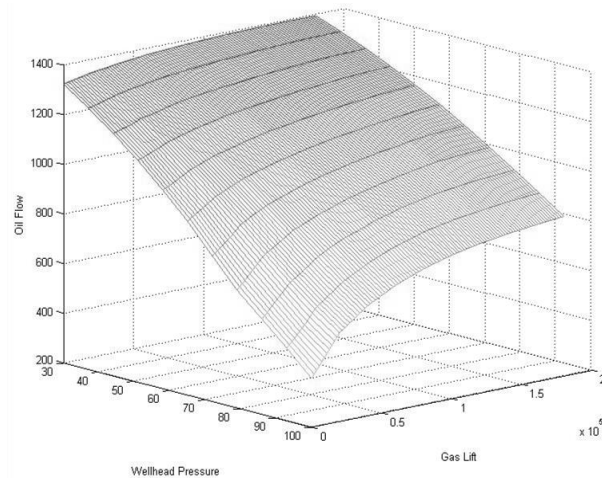
The simulators for each well and pipeline is of much lower dimension than the complete network simulator

## Well and pipeline simulator data

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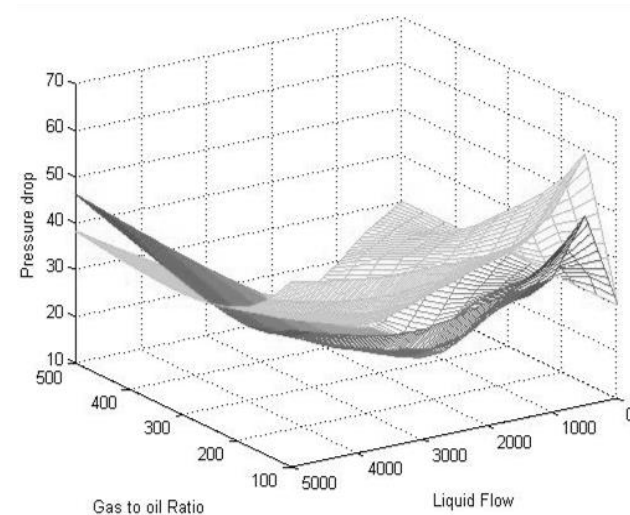
### Oil production from well:

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$



### Pressure drop over pipeline:

$$p_l^M - P^S = f_l^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$



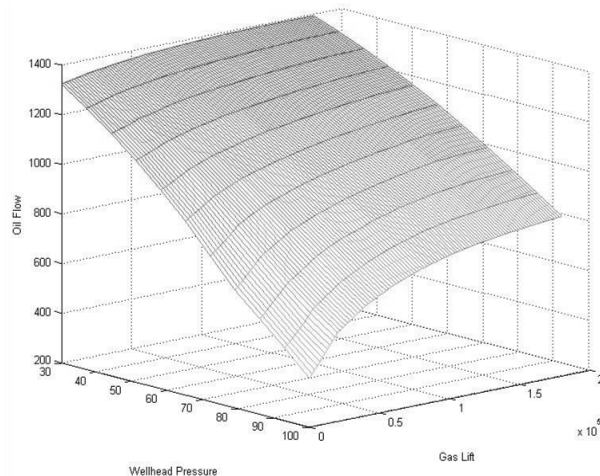
Gas and water production rates from each well are calculated using the GOR, WC and oil production rate

## Gas and water production equations

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### Oil production from well:

$$q_{jo}^W = f_{jo}^W(p_j^W, q_j^{GL})$$



### Gas and water production:

$$q_{jg}^W = GOR_j^W q_{jo}^W + q_j^{GL}$$

$$q_{jw}^W = q_{jo}^W \left( \frac{WC_j^W}{1 - WC_j^W} \right)$$

*The gas-to-oil ratio (GOR) and the water cut (WC) are assumed constant for each well...*

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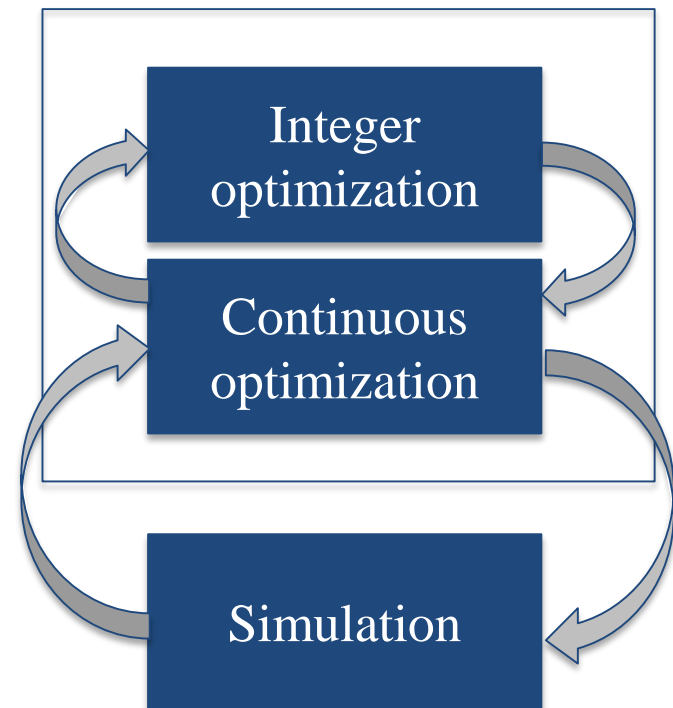
6 Conclusion

The simplest 2-layer approach divides the search space in a derivative free master problem and a continuous sub-problem

## Recap 2-layer approach

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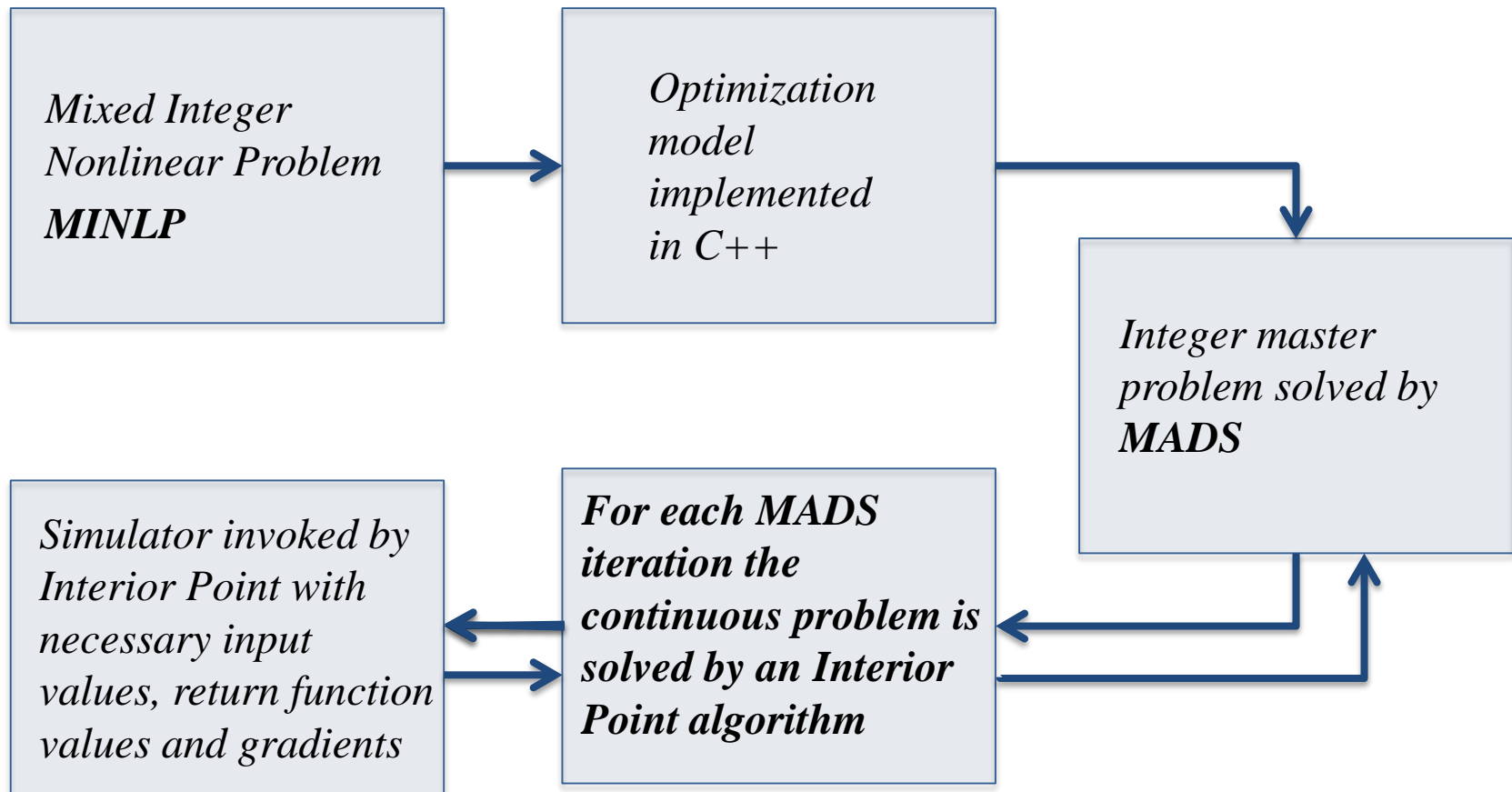
- ❑ One integer subspace and one continuous subspace
- ❑ Mathematical formulation similar to the traditional approach
- ❑ Optimization algorithms in two layers
- ❑ A **derivative-free algorithm** solves the integer master-problem
- ❑ For each iteration of the derivative-free algorithm, a **gradient-based algorithm** solves the continuous sub-problem



SmartOpt 2-layer method includes the network simulator, the algorithm does not see what happens in the simulator

## How wells and pipes are simulated, modeled and optimized

---

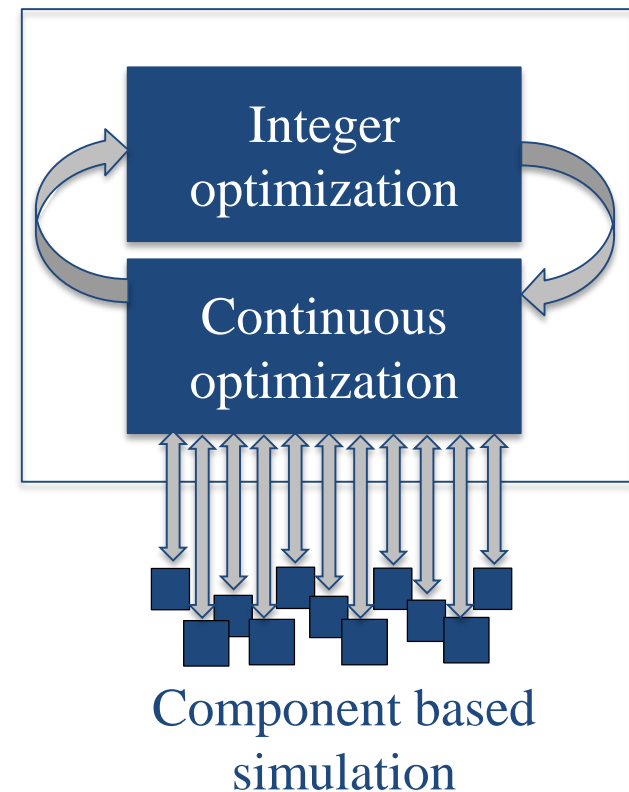


The more “sophisticated” SmartOpt approach further utilize the network structure to decompose the simulator

## Recap SmartOpt

---

- ❑ The decision space is split into integer and continuous subspaces
- ❑ The network simulator is decomposed into component simulators for wells and pipelines
- ❑ Mass and pressure balances are treated inside the optimization algorithm
- ❑ This facilitate the utilization of Branch & Bound and gradient based search algorithms



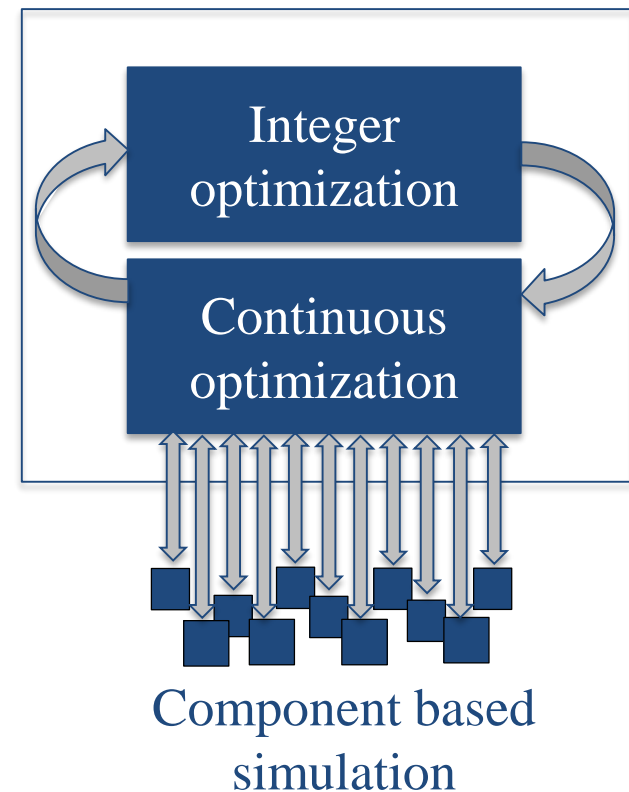


The more “sophisticated” SmartOpt approach further utilize the network structure to decompose the simulator

## Recap SmartOpt

---

- ❑ It is more efficient to call the specific component simulator as one needs information of that part of the system, than to call the entire network simulator each time
- ❑ This also opens up to the possibility of sampling the simulators prior to running the optimization algorithm, and use data tables and/or proxy models during the optimization
- ❑ Several representations of the component simulators will be presented

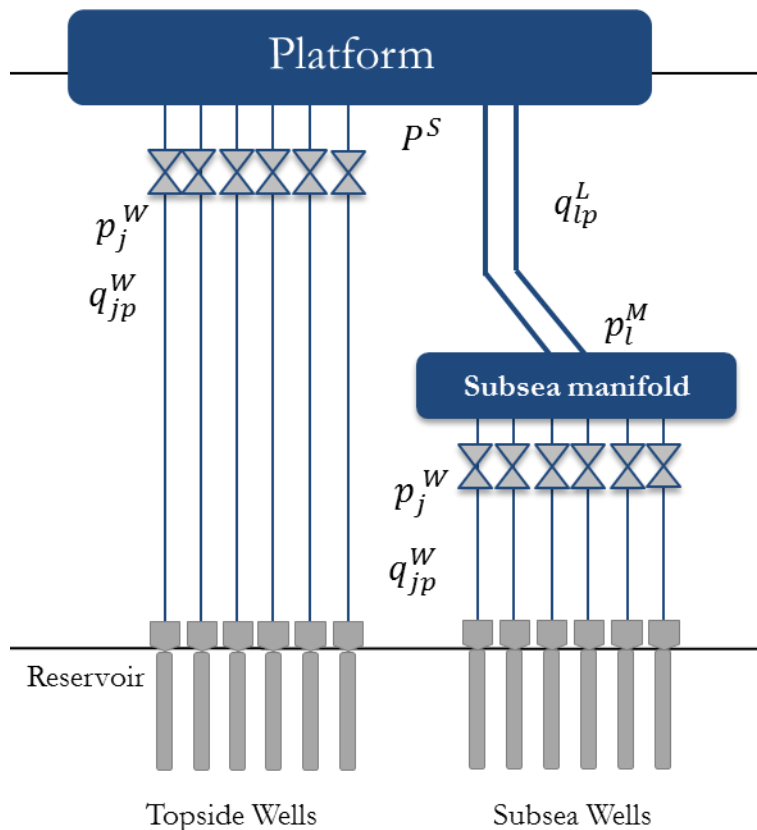


*Play SmartOpt movie..*

Well and pipeline simulators can be represented in several ways, either simulated directly or through approximations

## SmartOpt simulation strategies

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**SmartOpt**  
Simulation

**SmartOpt**  
Interpolation

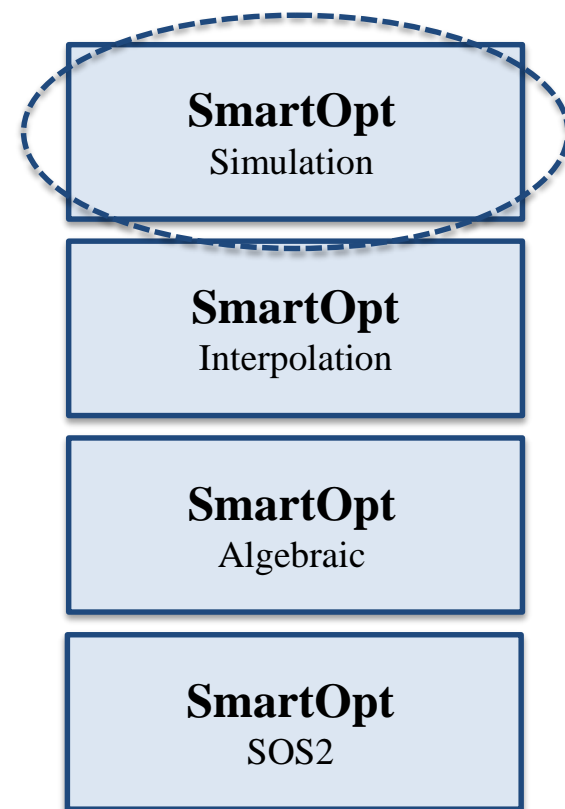
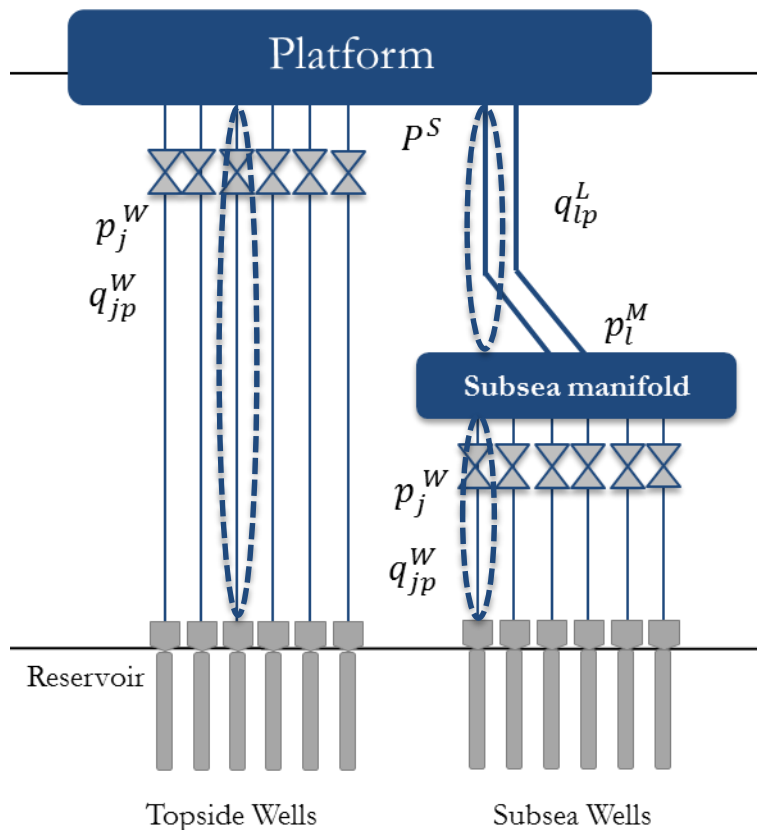
**SmartOpt**  
Algebraic

**SmartOpt**  
SOS2

Well and pipeline simulators can be represented in several ways, either simulated directly or through approximations

## SmartOpt simulation strategies

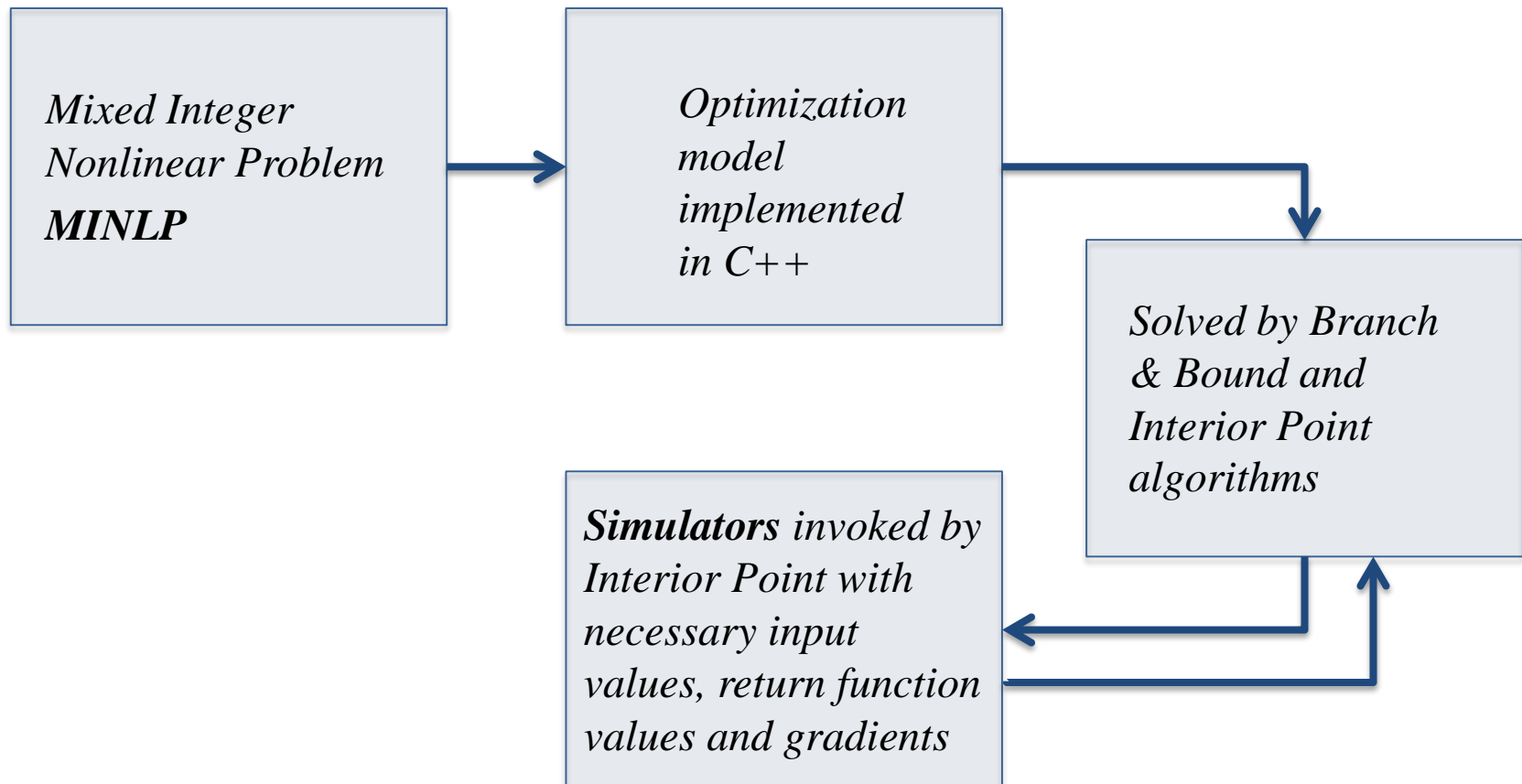
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SmartOpt simulation method includes simulators - the algorithm does not see what happens inside the simulator

**How wells and pipes are simulated, modeled and optimized**

---



By splitting up the simulator, discrete decisions are treated in analytical constraints, enabling Brach & Bound

*“The SmartOpt problem”*

### Objective function

$$\max \sum_{l \in L} q_{lo}^L + \sum_{j \in J_t} q_{jo}^W x_j$$

### Capacity constraints

$$\sum_{l \in L} q_{lp}^L + \sum_{j \in J_t} q_{jp}^W x_j \leq C_p$$

$$\sum_{j \in J_s} \sum_{l \in L} q_j^{GL} y_{jl} + \sum_{j \in J_t} q_j^{GL} x_j \leq C^{GL}$$

### Pressure and mass balance

$$\sum_{j \in J_s} q_{jp}^W y_{jl} = q_{lp}^L \quad \sum_{l \in L} y_{jl} \leq 1 \quad j \in J_s$$

$$p_l^M y_{jl} \leq p_j^W \quad j \in J_s$$

$$x_j P^S \leq p_j^W \quad j \in J_t$$

### Well and pipeline

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$

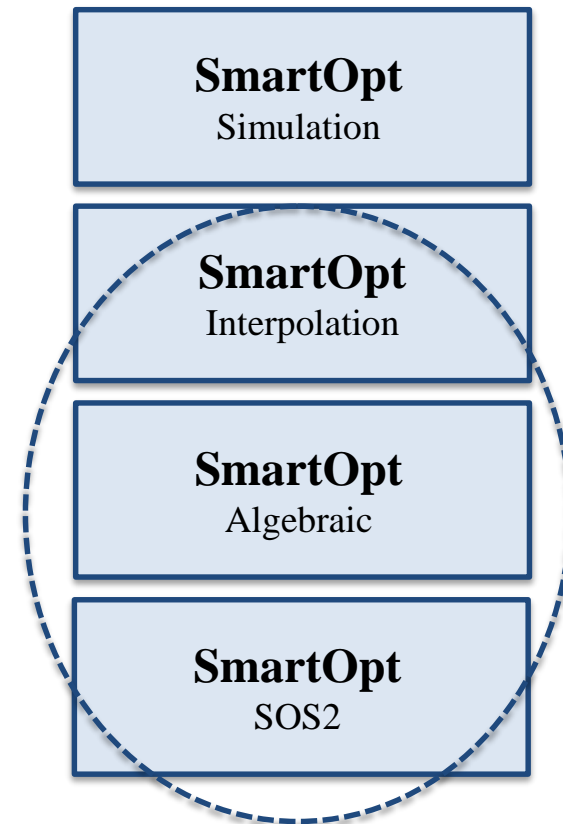
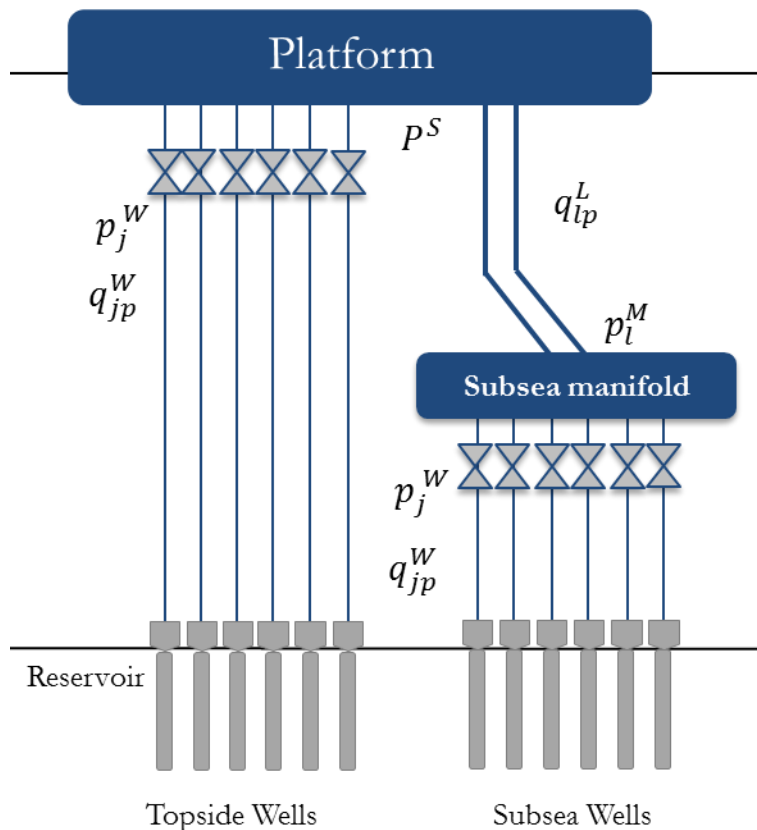
$$p_l^M - P^S = f_l^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$

Simulators are called directly; function value and gradients are returned

Well and pipeline simulators can be represented in several ways, either simulated directly or through approximations

## SmartOpt simulation strategies

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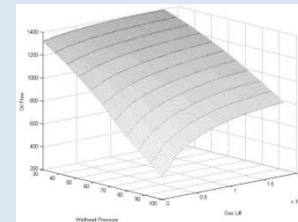
# The SmartOpt enables use of pre-generated data tables and interpolation techniques to reduce evaluation time

## How wells and pipes simulators can be represented

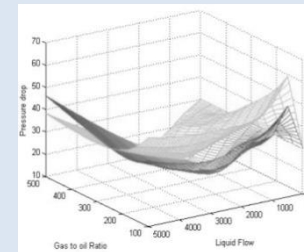
- ❑ The SmartOpt formulation decomposes the network simulator into smaller component simulators
- ❑ With the model parameters fixed, the well and pipeline simulator only have 2 or 3 degrees of freedom left
- ❑ Due to few variables the component simulators can be sampled and represented as data tables
- ❑ The data tables can be used for interpolation or pre-generation of proxy models

### Wells and pipelines

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$



$$p_i^M - P^S = f_i^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$



Including real simulators will affect the evaluation times, an alternative is to approximate the simulator

## How wells and pipes are simulated, modeled and optimized

---

### Simulators

$$q_{jp}^W = F_{jp}^W(p_j^W, q_j^{GL}, m_1, \dots, m_n)$$

- The parameters  $m_1, \dots, m_n$  are describing the plant.
- They introduce complexity in  $F_{jo}$ .
- $m_1, \dots, m_n$  are known and constant during optimization.

A numerical solver will only be faced with

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$

- Desirable to create an analytical replacement  $\hat{f}_{jp}^W$ , that is easily integrated in an optimization software and has short evaluation times.

### Approximation

$$\hat{q}_{jp}^W = \text{approx } \hat{f}_{jp}^W(p_j^W, q_j^{GL}) \cong f_{jp}^W(p_j^W, q_j^{GL})$$



By splitting up the simulator, discrete decisions are treated in analytical constraints, enabling Brach & Bound

*“The SmartOpt problem”*

**Objective function**

$$\max \sum_{l \in L} q_{lo}^L + \sum_{j \in J_t} q_{jo}^W x_j$$

**Capacity constraints**

$$\sum_{l \in L} q_{lp}^L + \sum_{j \in J_t} q_{jp}^W x_j \leq C_p$$

$$\sum_{j \in J_s} \sum_{l \in L} q_j^{GL} y_{jl} + \sum_{j \in J_t} q_j^{GL} x_j \leq C^{GL}$$

**Pressure and mass balance**

$$\sum_{j \in J_s} q_{jp}^W y_{jl} = q_{lp}^L \quad \sum_{l \in L} y_{jl} \leq 1 \quad j \in J_s$$

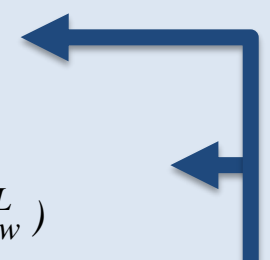
$$p_l^M y_{jl} \leq p_j^W \quad j \in J_s$$

$$x_j P^S \leq p_j^W \quad j \in J_t$$

**Well and pipeline**

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$

$$p_l^M - P^S = f_l^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$

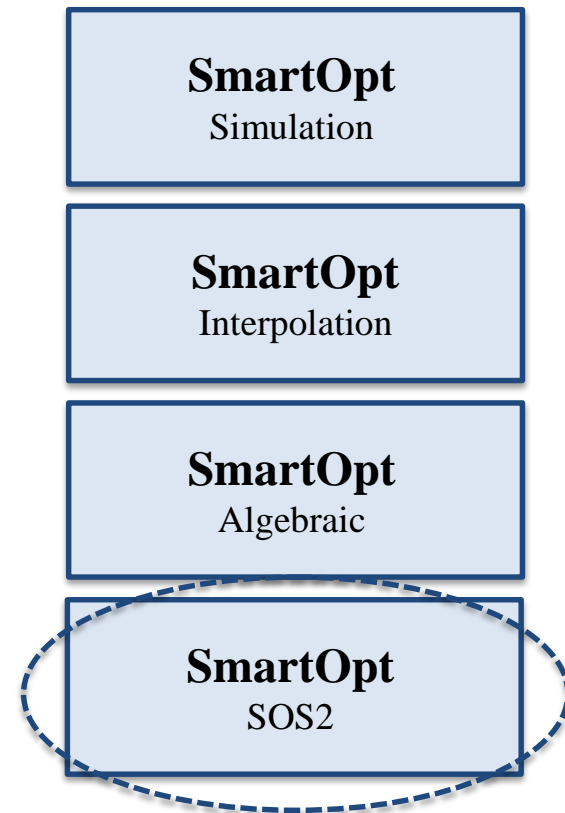
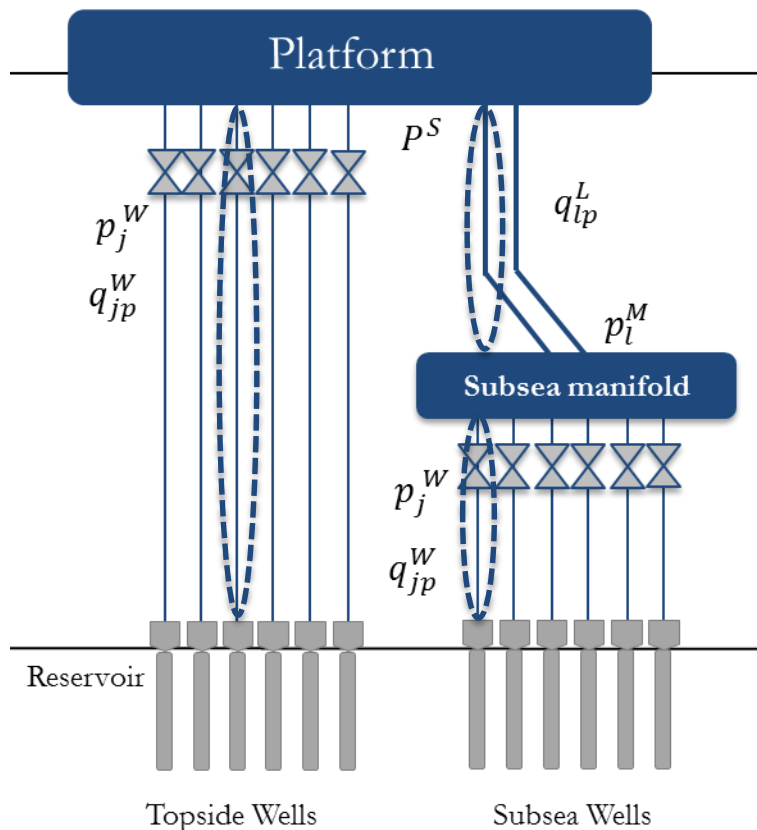


Simulators are approximated, and a surrogate model returns function value and gradients

Well and pipeline simulators can be represented in several ways, either simulated directly or through approximations

## SmartOpt simulation strategies

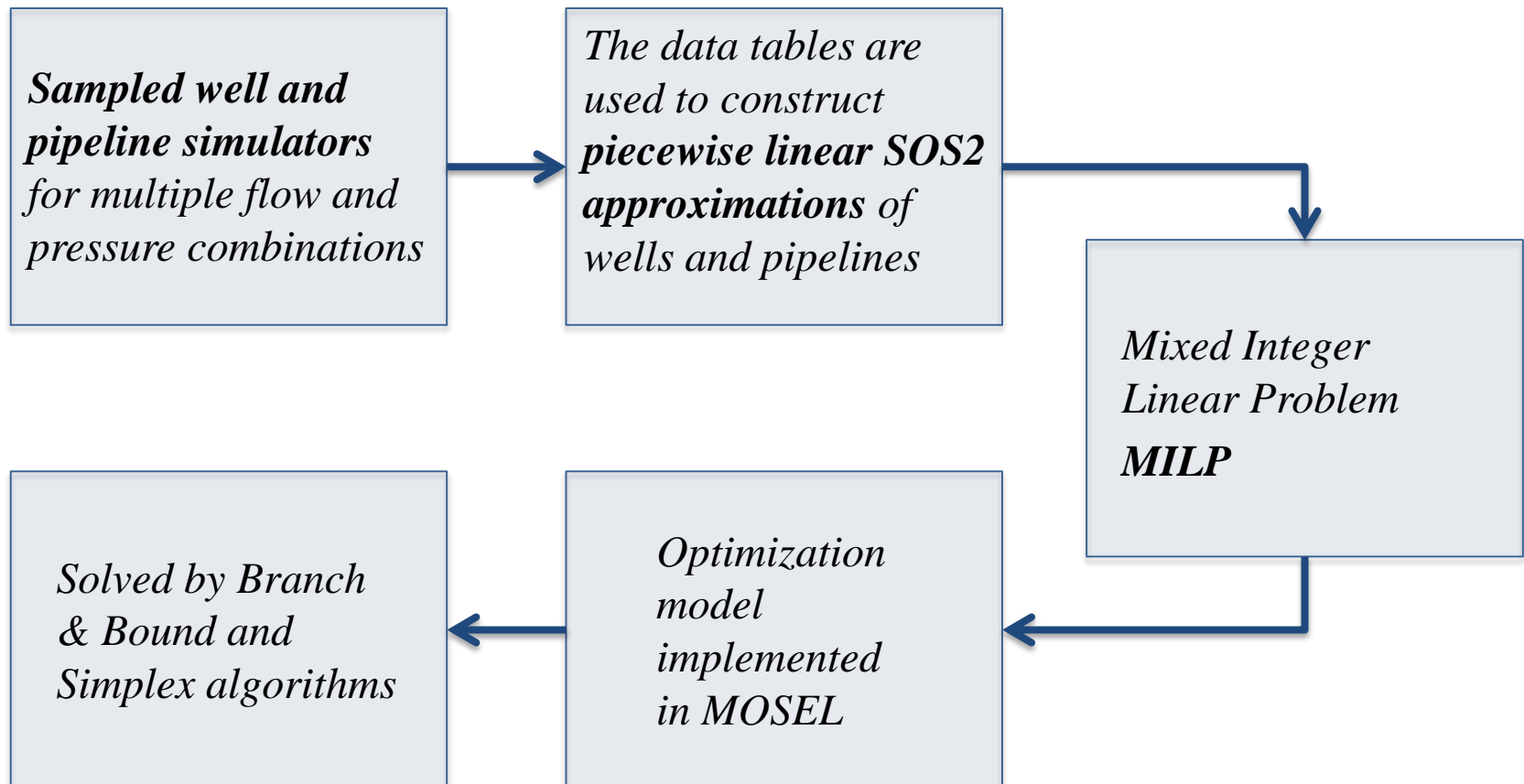
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# SmartOpt SOS2 becomes a linear and convex formulation of the production optimization problem

## How wells and pipes are simulated, modeled and optimized

---



To obtain a linear model all nonlinear constraints must be reformulated; by “Big M” and SOS2

***“The SmartOpt nonlinear problem”***

**Objective function**

$$\max \sum_{l \in L} q_{lo}^L + \sum_{j \in J_t} q_{jo}^W x_j$$

**Capacity constraints**

$$\sum_{l \in L} q_{lp}^L + \sum_{j \in J_t} q_{jp}^W x_j \leq C_p$$

$$\sum_{j \in J_s} \sum_{l \in L} q_j^{GL} y_{jl} + \sum_{j \in J_t} q_j^{GL} x_j \leq C^{GL}$$

**Pressure and mass balance**

$$\sum_{j \in J_s} q_{jp}^W y_{jl} = q_{lp}^L \quad \sum_{l \in L} y_{jl} \leq 1 \quad j \in J_s$$

$$p_l^M y_{jl} \leq p_j^W \quad j \in J_s$$

$$x_j P^S \leq p_j^W \quad j \in J_t$$

**Well and pipeline**

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$

$$p_l^M - P^S = f_l^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$

To obtain a linear model all nonlinear constraints must be reformulated; by “Big M” and SOS2

*“The SmartOpt SOS2 problem”*

**Objective function**

$$\max \sum_{l \in L} q_{lo}^L + \sum_{j \in J_t} q_{jo}^W$$

**Capacity constraints**

$$\sum_{l \in L} q_{lp}^L + \sum_{j \in J_t} q_{jp}^W \leq C_p$$

$$\sum_{j \in J} q_j^{GL} \leq C^{GL}$$

**Pressure balance**

$$P_l^M \leq P_j^W + P_l^{MAX} (1 - y_{jl}) \quad j \in J_s$$

$$P^S \leq P_j^W + P^S (1 - x_j) \quad j \in J_t$$

**Pressure and mass balance**

$$\sum_{l \in L} q_{jlp}^{WL} = q_{jp}^W \quad j \in J_s \quad \sum_{j \in J_s} q_{jlp}^{WL} = q_{lp}^L$$

$$\sum_{l \in L} y_{jl} \leq 1 \quad j \in J_s \quad q_{jlp}^W \leq Q_{jp}^W y_{jl} \quad j \in J_s$$

$$q_{jp}^W \leq Q_{jp}^W x_j \quad j \in J_t$$

**Well and pipeline**

$$q_{jp}^W = f_{jp}^W (P_j^W, q_j^{GL})$$

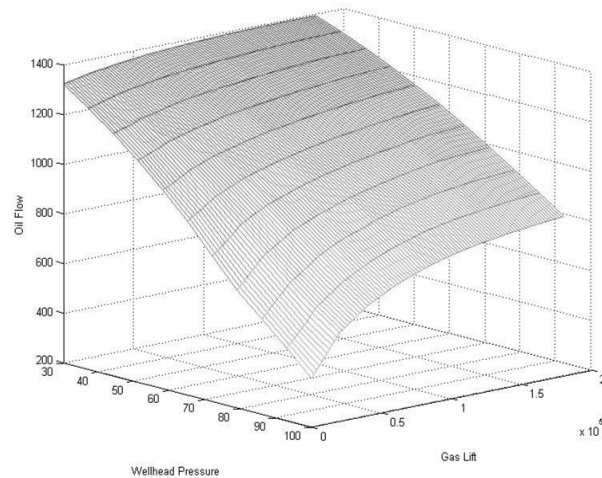
$$P_l^M - P^S = f_l^L (q_{lo}^L, q_{lg}^L, q_{lw}^L)$$

The component simulators are represented through data tables and piecewise linear SOS2 approximations

These plots is piecewise linearized

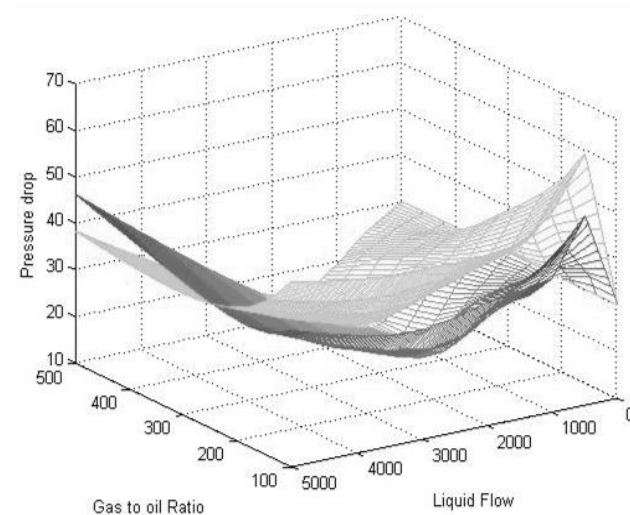
**Well:**

$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$



**Pipeline:**

$$P_l^M - P^S = f_l^L(q_{lo}^L, q_{lg}^L, q_{lw}^L)$$

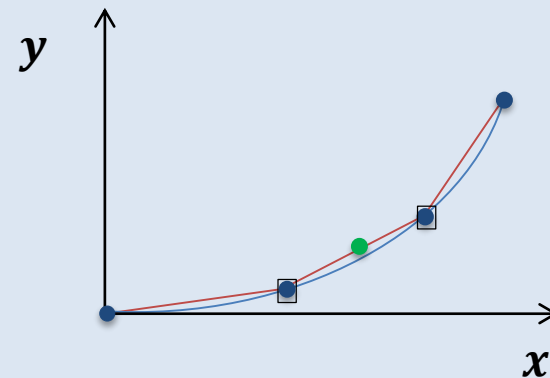


# SOS 2 formulation example for a fixed number of breakpoint values for $x$ and $y$ given by the curve $y=x^2$

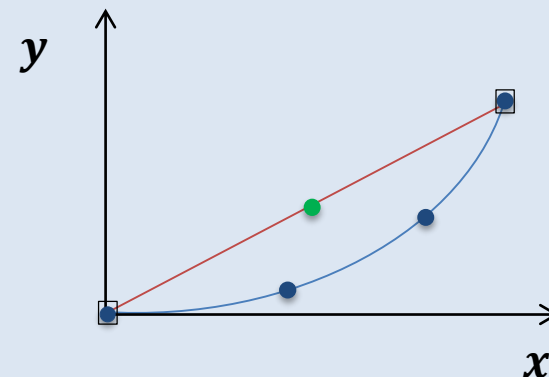
## Special ordered sets of type 2

- Defining the weighting variables as SOS2 means that at most two points can be non-zero, and they have to be adjacent
- This allows for interpolation between the two associated breakpoints
- Relaxing the neighboring requirements might lead to poor function approximations as seen in the figure
- SOS 2 can also be used when linearizing multidimensional functions

*With neighbor requirements...*



*Without neighbor requirements...*



# Introduction to piecewise linearization through SOS2; a simple “one to one dimension” example $y=x^2$

## Special ordered sets of type 2

### Piecewise linearization of function $y=x^2$

$$x = 0\lambda_0 + 1\lambda_1 + 2\lambda_2 + 2.5\lambda_3$$

$$y = 0\lambda_0 + 1\lambda_1 + 4\lambda_2 + 6.25\lambda_3$$

$$\lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 = 1$$

$$\lambda_0 \leq \alpha_0$$

$$\lambda_1 \leq \alpha_0 + \alpha_1$$

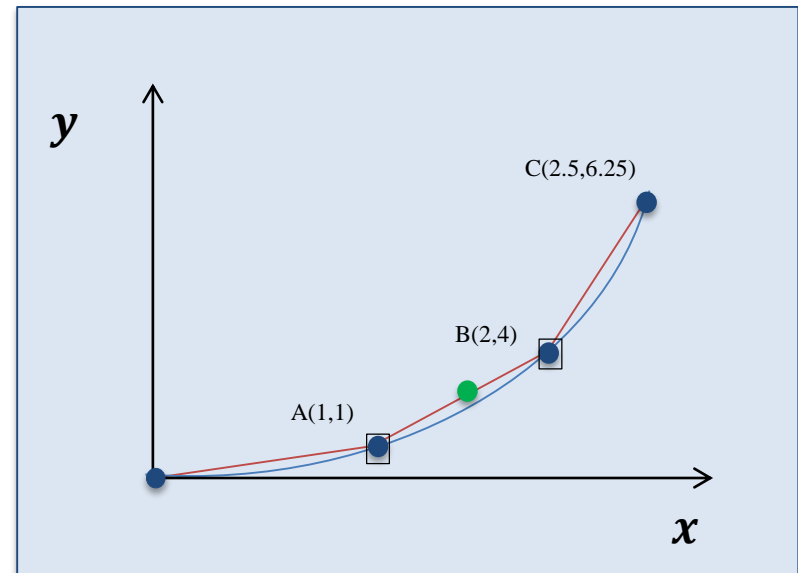
$$\lambda_2 \leq \alpha_1 + \alpha_2$$

$$\lambda_3 \leq \alpha_2$$

$$\lambda_i \in [0,1], i \in \{0, \dots, 3\}$$

$$\alpha_i \in [0,1], i \in \{0, \dots, 2\}$$

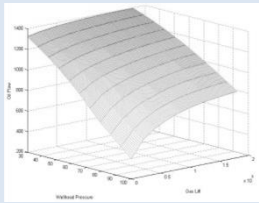
- A fixed number of breakpoint values for  $x$  and  $y$  can be defined and nonnegative weighting variables  $\lambda_i$  are assigned to each breakpoint  $i$
- Linear segments can be drawn between the data points





Well simulators are approximated through pre-generated data tables and linear pieces between the data points

*“The SOS 2 formulation for the well simulator”*



$$q_{jp}^W = f_{jp}^W(p_j^W, q_j^{GL})$$

$$q_{jp}^W = \sum_{k \in K} \sum_{n \in N} Q_{(jp)kn}^W \lambda_{(j)kn}$$

$$\eta_{(j)k}^K = \sum_{n \in N} \lambda_{(j)kn}$$

$\eta_{(j)k}^K$  is SOS 2 for  $k$

$$p_j^W = \sum_{k \in K} \sum_{n \in N} P_{(j)k}^W \lambda_{(j)kn}$$

$$\eta_{(j)n}^N = \sum_{k \in K} \lambda_{(j)kn}$$

$\eta_{(j)n}^N$  is SOS 2 for  $n$

$$q_j^{GL} = \sum_{k \in K} \sum_{n \in N} Q_{(j)n}^{GL} \lambda_{(j)kn}$$

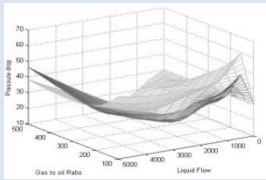
$$\eta_{(j)n}^N, \eta_{(j)k}^K \geq 0$$

$$\sum_{k \in K} \sum_{n \in N} \lambda_{(j)kn} = 1$$

$$\lambda_{(j)kn} \geq 0$$

Pipeline simulators are approximated through pre-generated data tables and linear pieces between the data points

**“The SOS 2 formulation for the pipeline simulator”**



$$p_l^M - P^S = f_l^L ( q_{lo}^L, q_{lg}^L, q_{lw}^L )$$

$$\sum_{k \in K} \sum_{n \in N} \sum_{r \in R} F_{(l)knr}^L \delta_{(l)knr} = P^S - p_l^M$$

$$q_{lo}^L = \sum_{k \in K} \sum_{n \in N} \sum_{r \in R} Q_{(l)k}^O \delta_{(l)knr}$$

$$\gamma_{(l)r}^R = \sum_{k \in K} \sum_{n \in N} \delta_{(l)knr}$$

$\gamma_{(l)r}^R$  is SOS 2 for  $r$

$$q_{lg}^L = \sum_{k \in K} \sum_{n \in N} \sum_{r \in R} Q_{(l)n}^G \delta_{(l)knr}$$

$$\gamma_{(l)n}^N = \sum_{k \in K} \sum_{r \in R} \delta_{(l)knr}$$

$\gamma_{(l)n}^N$  is SOS 2 for  $n$

$$q_{lw}^L = \sum_{k \in K} \sum_{n \in N} \sum_{r \in R} Q_{(l)r}^W \delta_{(l)knr}$$

$$\gamma_{(l)k}^K = \sum_{n \in N} \sum_{r \in R} \delta_{(l)knr}$$

$\gamma_{(l)k}^K$  is SOS 2 for  $k$

$$\sum_{k \in K} \sum_{n \in N} \sum_{r \in R} \delta_{(l)knr} = 1$$

$\gamma_{(l)r}^R, \gamma_{(l)n}^N, \gamma_{(l)k}^K \geq 0$

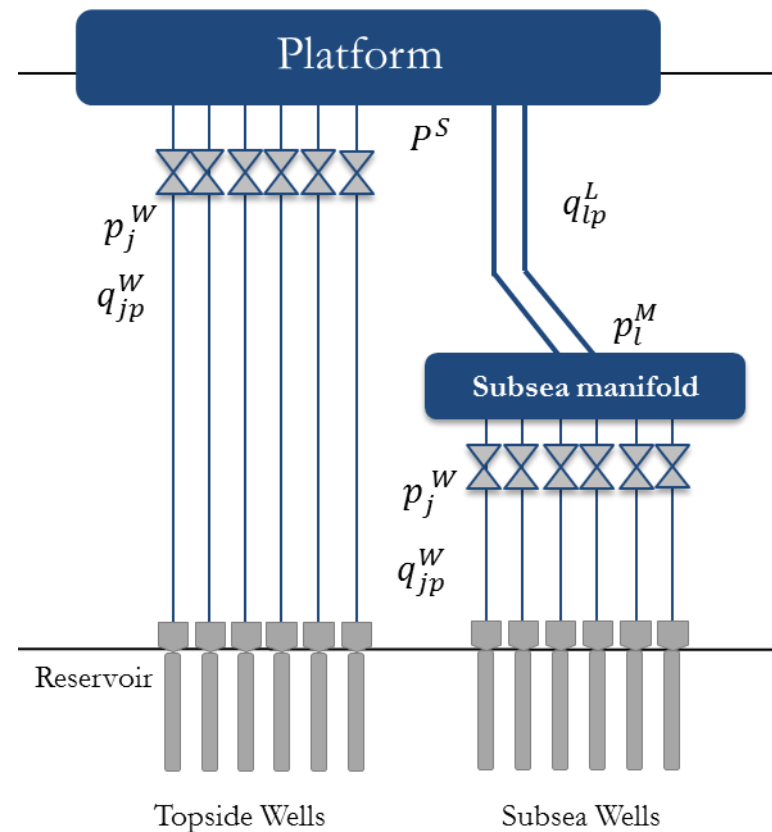
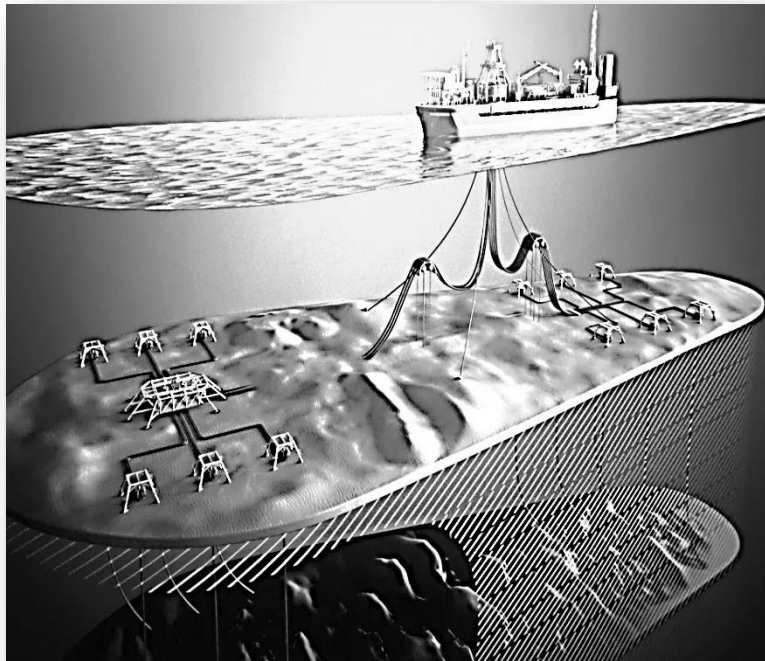
$$\delta_{(l)knr} \geq 0$$

# Outline

- 1 Introduction
- 2 Mathematical optimization
- 3 SmartOpt approaches
- 4 Case
- 5 Results
- 6 Conclusion

# 12 well production system, 6 subsea wells and 6 satellite wells, with artificial gas lift

## TEST CASE



1 base case and 5 similar cases are defined for available gas lift, gas and water handling capacities and separator pressure

## OPTIMIZATION PROBLEM

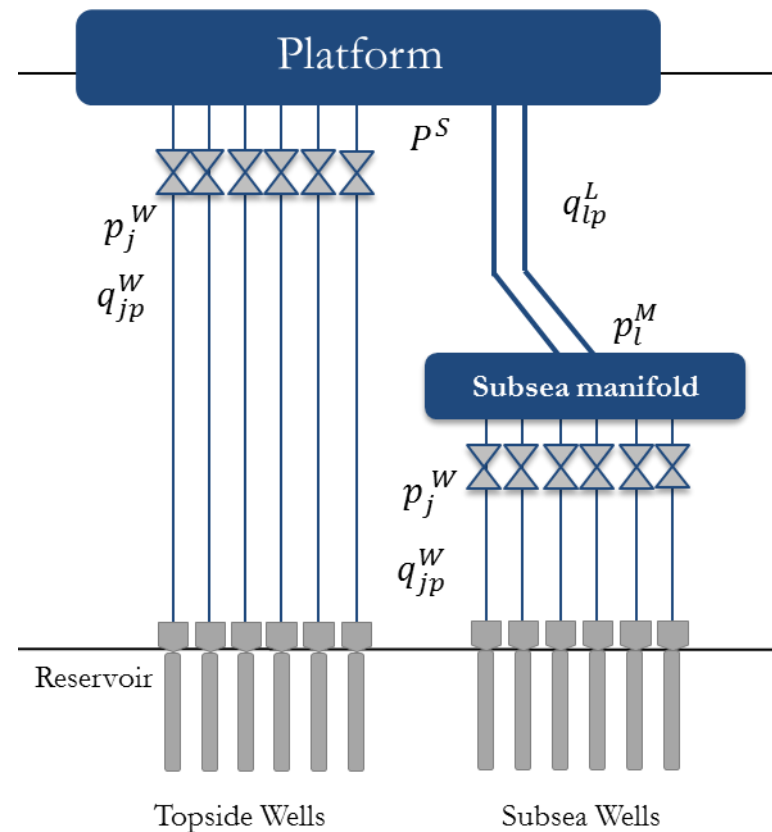
Maximize total oil production

*By adjusting:*

- Well head pressures
- Gas lift
- Subsea well routing

*And obeying:*

- Total handling capacity on water and gas production
- Total gas lift compression capacity
- Minimum separator pressure



# Outline

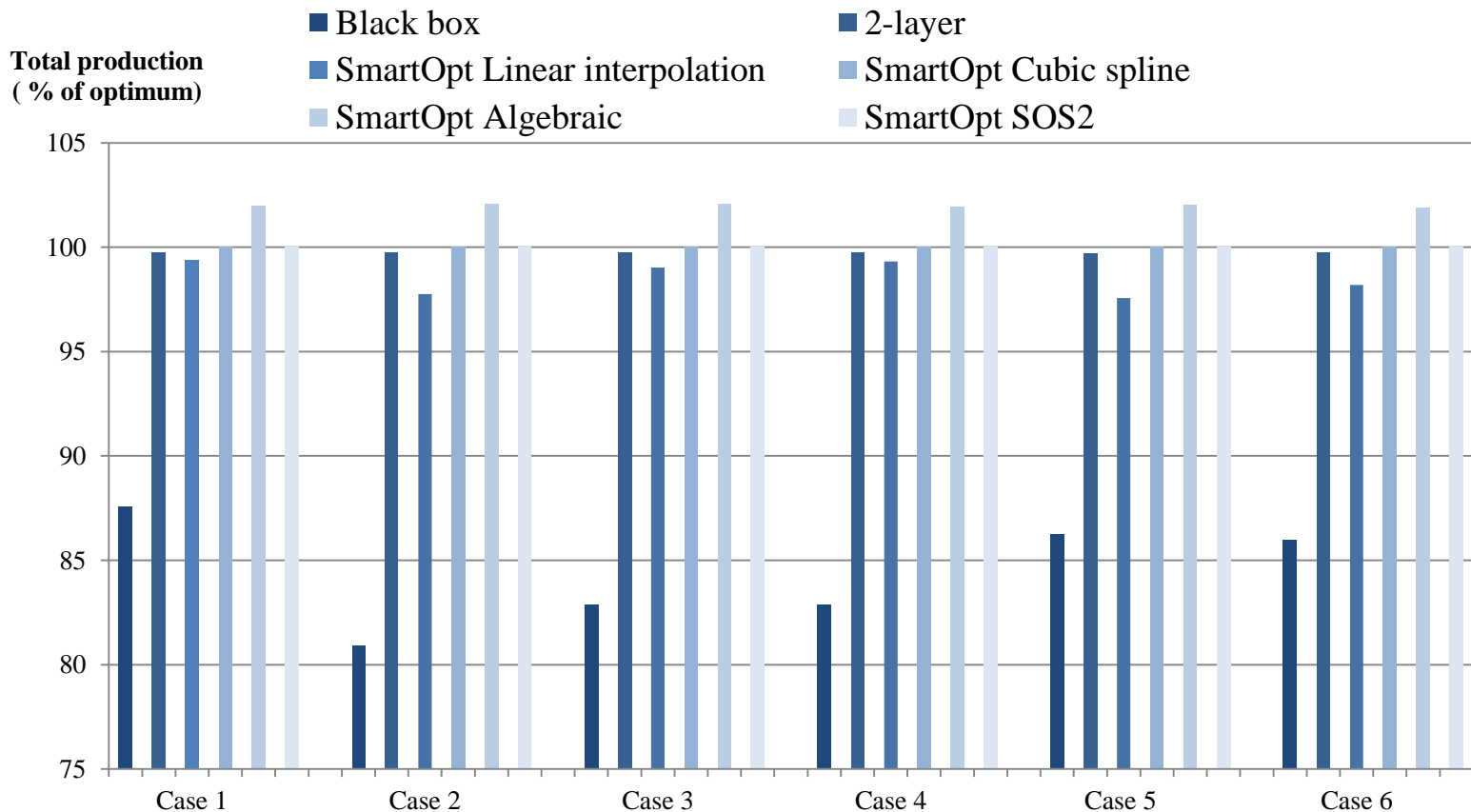
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# Six of the strategies presented are demonstrated on a real petroleum production problem

	Solution approach	Description	Simulator approximation	Implementation language	Problem class	Solution algorithm
Simulation	Black- Box	Traditional method Black box simulator	Simulation	C++	MINLP "simulation"	MADS
	2-layer	One discrete and one continuous problem				MADS + Interior point
Approximation	SmartOpt Interpolation (Linear and cubic spline)	Discrete and continuous subspaces  Component simulators	Interpolation algorithm Returns function value and derivatives	AMPL	MINLP	Branch & Bound + Interior point
	SmartOpt Algebraic	Well and pipeline models sampled upfront	Algebraic piecewise nonlinear approximations			
	SmartOpt SOS2	Component simulators represented by approximations	SOS2 formulations Convex problem Solutions taken as global optimum	MOSEL	MILP	Branch & Bound + Simplex

The graph present oil production from each well and indicate small differences between the methods

## Base case optimal solutions in % of SOS2 solution





The SmartOpt simulation approaches are very fast, but can be sensitive to starting point if not solved globally

	<b>Approaches</b>	<b>Solution time</b> Base case	<b>Performance</b> Base case	<b>Variance</b> Base case six initial points
Simulation	Black- Box	<b>463 seconds</b>	<b>84.7 %</b>	<b>± 9.1 %</b>
	2-layer	<b>151 seconds</b>	<b>99.7%</b>	<b>± 18.4 %</b>
Approximations	<b>SmartOpt</b> Linear interpolation	<b>1 second</b>	<b>99.4 %</b>	<b>± 6.4 %</b>
	<b>SmartOpt</b> Cubic spline	<b>107 seconds</b>	<b>100 %</b>	<b>± 0.0 %</b>
	<b>SmartOpt</b> Algebraic	<b>14 seconds</b>	<b>102 %</b>	<b>± 0.3 %</b>
	<b>SmartOpt</b> SOS2	<b>237 seconds</b>	<b>100 %</b>	<b>± 0.0%</b>

The SmartOpt simulation approaches are very fast, but can be sensitive to starting point if not solved globally

**Best objective functions and variations given in % of SOS2 solution**

	Black- Box	2-layer	SmartOpt Linear int.	SmartOpt Cubic spline	SmartOpt Algebraic	SmartOpt SOS2
Base case	84.7± 9.1 %	99.7±18.4 %	99.4± 6.4 %	100.0±0.0 %	102.0±0.3 %	100 %
Case 1	95.0±21.5 %	99.7±18.7 %	97.8±13.1 %	100.0±0.0 %	102.1±0.2 %	100 %
Case 2	89.5±15.3 %	99.7±18.9 %	99.0± 7.3 %	100.1±0.0 %	102.1±0.0 %	100 %
Case 3	92.5±18.3 %	99.8±18.3 %	99.3± 7.8 %	100.0±0.0 %	101.9±0.0 %	100 %
Case 4	88.0±10.9 %	99.7±18.5 %	97.6± 3.2 %	100.1±0.0 %	102.0±0.1 %	100 %
Case 5	92.3±17.8 %	99.7±18.0%	98.2± 9.6 %	100.1±0.0 %	101.9±0.0 %	100 %

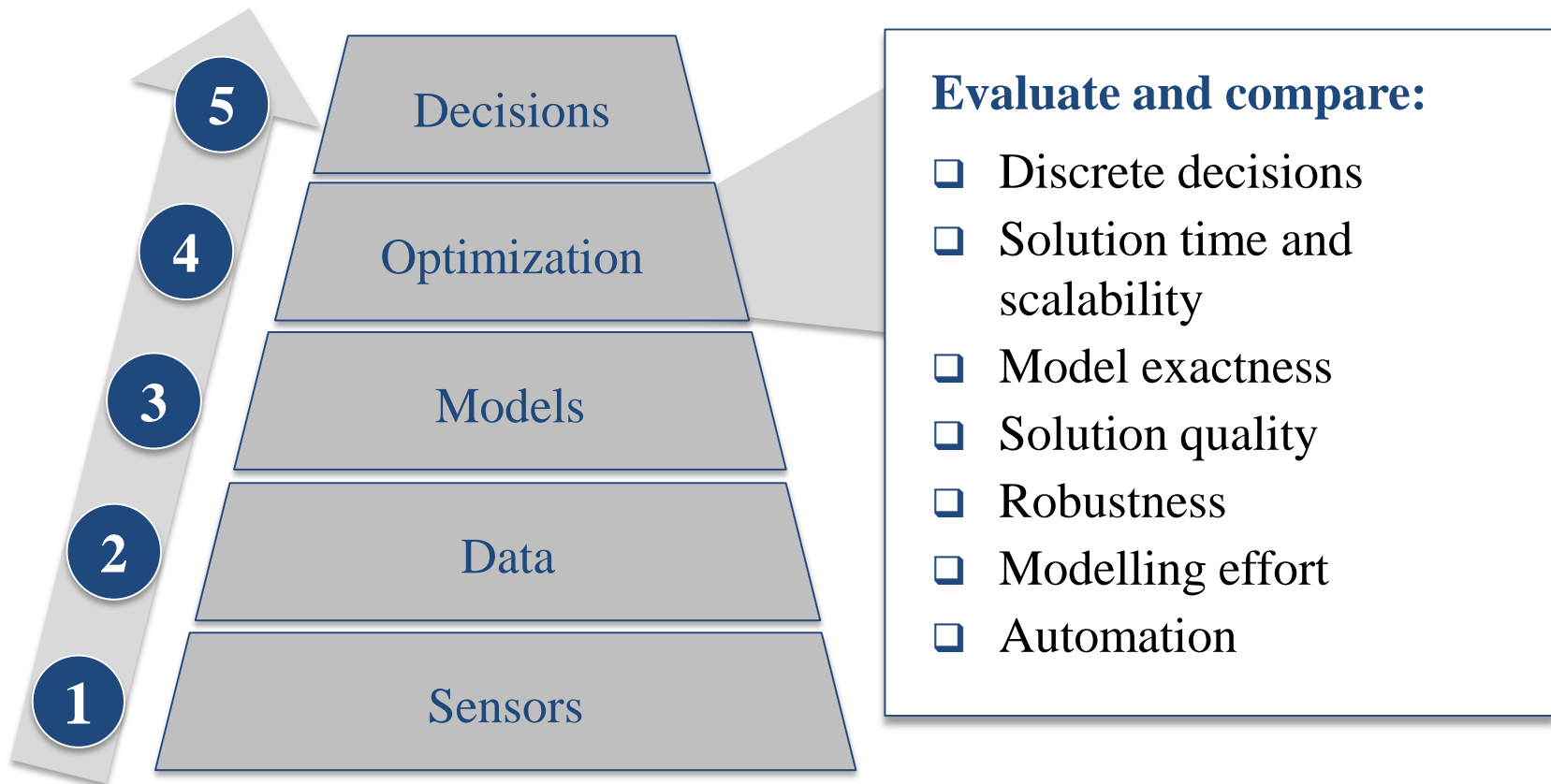
# Outline

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Six of the strategies presented are demonstrated on a real petroleum production problem

## EVALUATION CRITERIA

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There are pros and cons to all SmartOpt approaches, with one to choose depends on problem and preferences

## DISCRETE DECISIONS

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### *SmartOpt*

- ❑ Enables the use of Branch & Bound for efficient treatment of discrete decisions
- ❑ Optimal solution two orders of magnitude faster than the Black box approach
- ❑ Facilitates instant re-optimization
- ❑ Enable production engineers to focus on other important questions

### *Traditional approach*

- ❑ Simulators only allow the optimization algorithm to make queries with discrete values on the routing and on/off decisions
- ❑ Excludes the use of Branch & Bound based algorithms that rely on relaxing the integer requirements
- ❑ Must solve using derivative-free solution algorithms
- ❑ Leads to slow convergence and increased solution times

There are pros and cons to all SmartOpt approaches, with one to choose depends on problem and preferences

## **SOLUTION TIME & SCALABILITY**

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### *SmartOpt*

- ❑ The **SmartOpt** approaches sees a speed up compared to the traditional approach, thus much is gained by including explicit structural constraints
- ❑ **SmartOpt SOS2** is highly sensitive to an increase in data points and network components due to the subsequent increase in SOS2 variables
- ❑ **SmartOpt interpolation** methods scale better as the interpolation is quick
- ❑ Evaluations of more proxy models will have a noticeable effect on the solution speed of **SmartOpt algebraic**

### *Traditional approach*

- ❑ The **Black box approach** solves the petroleum production problem with reasonable solution times
- ❑ An increase in the number of decision variables that follows a larger production network is likely to slow down the derivative-free black box approach

There are pros and cons to all SmartOpt approaches, witch one to choose depends on problem and preferences

## MODEL EXACTNESS

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### *Simulator & interpolation based:*

- ❑ **Black box, 2-layer and SmartOpt interpolation** approaches provide realistic portrayals of the production network
- ❑ The number of data point and the interpolation scheme will affect the accuracy of **SmartOpt interpolation**

### *Algebraic based:*

- ❑ More detailed discretization will result in a **better match with the reality**
- ❑ SmartOpt SOS2 experience a trade off between **solution time and accuracy**
- ❑ **Algebraic proxy models** have been created based on visual inspections combined with least square fits. In this case the solutions are proven slightly infeasible, undermining the accuracy of the approximations used

(Some well approximations overestimating the oil production for certain wellhead pressures and gas lift allocations, while pipeline approximations at the same time underestimate the pressure losses over the pipelines for the resulting flows)

There are pros and cons to all SmartOpt approaches, witch one to choose depends on problem and preferences

## SOLUTION QUALITY

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### *SmartOpt*

- ❑ All **SmartOpt** approaches provide better solutions than the traditional approach
- ❑ **SmartOpt SOS2** provide global optimal solutions
- ❑ The **cubic spline interpolation** scheme proves very good and finds the same solutions as the SOS2 approximations, i.e. the optimal solution
- ❑ Clearly sensitive to the exactness of the proxy models the **SmartOpt algebraic** solutions surpasses the optimal values

### *Traditional approach*

- ❑ The **Black box approach** provide inferior solutions
- ❑ The **2-layer** approach lead to a significant improvement in objective function values



There are pros and cons to all SmartOpt approaches, witch one to choose depends on problem and preferences

## ROBUSTNESS

---

### *Simulator & interpolation based:*

- ❑ The **SmartOpt Cubic spline** interpolation can be efficiently convexified, proves very robust
- ❑ The **SmartOpt linear interpolation** method proves dependent on starting point, this can be solved by multi start
- ❑ The **Black box** and **2-layer** methods are also highly dependent of starting point, multi start is not so attractive due to solution time

### *Algebraic based:*

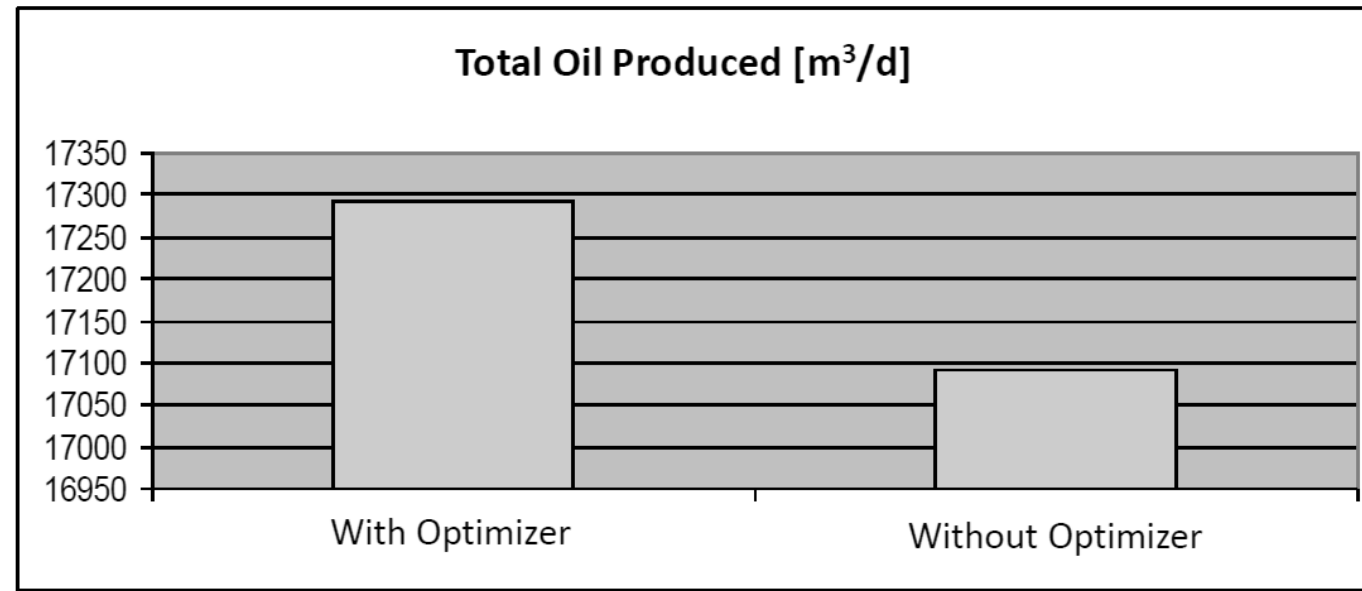
- ❑ Linearization and convexification of the optimization problem makes **SmartOpt SOS2** the most robust approach
- ❑ Almost convex proxy models makes **SmartOpt algebraic** method essentially independent of initial starting point

# Computational study and results indicate that SmartOpt performs well compared to traditional approach

Solution approach	Discrete decisions	Component simulators	Model exactness	Solution time/ scalability	Solution quality	Robustness	Modelling effort	Automation
Black- Box	÷	÷	--	÷	+	÷	+	++
2-layer	+	÷	--	÷	+	÷	+	++
SmartOpt	Linear interpolation	++	+	++	++	÷	+	++
	Cubic spline interpolation	++	++	++	++	++	+	++
	Algebraic	++	++	÷	+	+	++	+
	SOS2	++	++	+	÷	++	++	++

Petrobras decides to implement a SmartOpt SOS2 model to optimize gas lift allocation among 13 wells

## Petrobras field test of SmartOpt MILP model



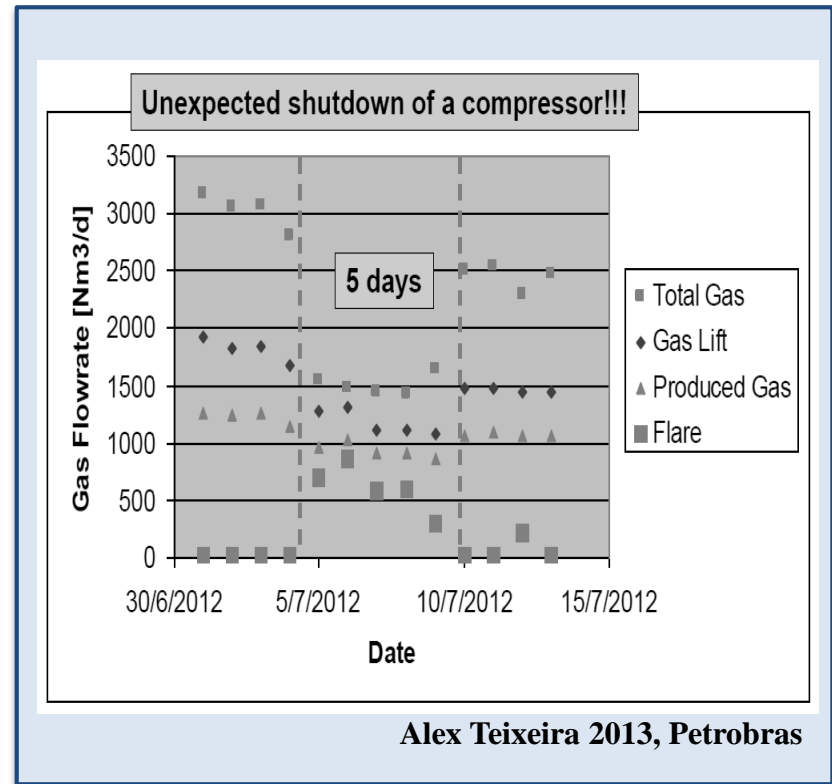
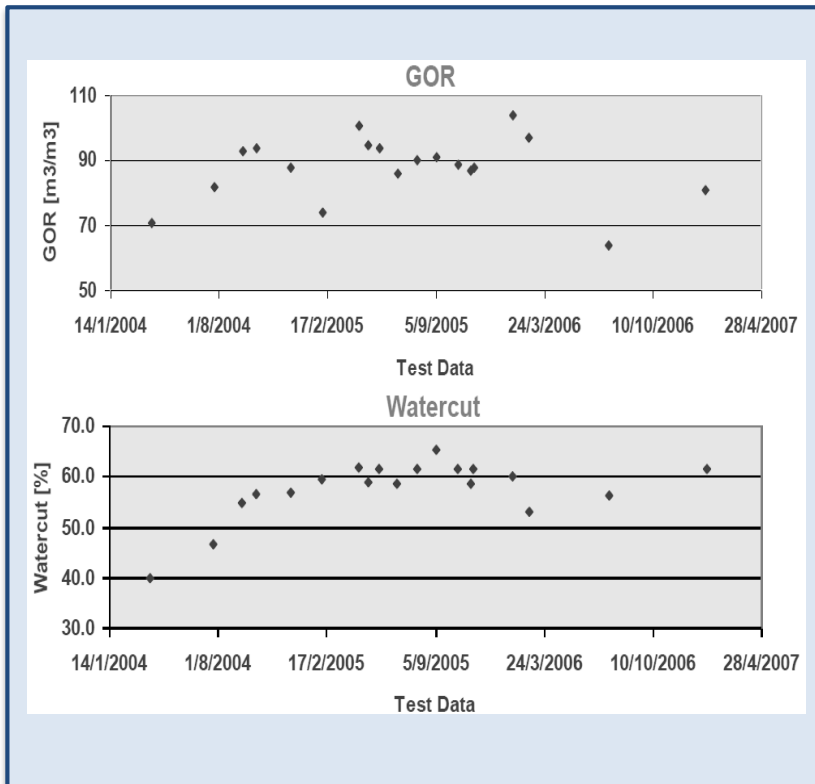
Alex Teixeira 2013, Petrobras

*Campos basin - 1.2 % increased production*

# Petrobras decides to implement a SmartOpt SOS2 model to optimize gas lift allocation among 13 wells

## GOR and water cut evolution, and unexpected compressor shut downs

*Why is it difficult to find the best production settings manually?*



Alex Teixeira 2013, Petrobras

*...because wells and operational conditions changes all the time.*