



Oil production optimization—A piecewise linear model, solved with two decomposition strategies

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ABSTRACT

This paper presents a new method for real-time optimization of process systems with a decentralized structure where the idea is to improve computational efficiency and transparency of a solution. The contribution lies in the application and assessment of the Lagrange relaxation and the Dantzig–Wolfe methods, which allows us to efficiently decompose a real-time optimization problem. Furthermore, all nonlinearities are modeled by piecewise linear models, resulting in a mixed integer linear program, with the added benefit that error bounds on the solution can be computed.

The merits of the method are studied by applying it to a semi-realistic model of the Troll west oil rim, a petroleum asset with severe production optimization challenges due to rate dependent gas-coning wells. This study indicates that both the Lagrange relaxation and in particular the Dantzig–Wolfe approach offers an interesting option for complex production systems. Moreover, the method compares favorably with the non-decomposed method.

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1. Introduction

Development of a petroleum field asset requires planning on several horizons. On a long-term horizon, typically from one year and up to the field's lifetime, strategic reservoir planning is based on market conditions, field properties and strategic considerations of the developing company. Decisions related to technology for an offshore field will include; how to develop the subsea solution, whether to process the fluid onshore or offshore, and how to export the different products produced. It is also possible to include extra flexibility. By for instance accepting a higher investment cost, it may be possible to allow future development such as tie-ins from possible neighboring assets. The analyses and subsequent development plan seek to maximize the net present value of the asset by maximizing oil and gas recovery while honoring safety and environmental constraints. Nygreen, Christiansen, Haugen, Bjorkvoll, and Kristiansend (1998) discuss some of these issues.

On a medium time horizon, often referred to as tactical reservoir management, the planner will seek to extract as much oil and gas from the reservoir as possible, within the bounds of the earlier strategic decisions. In the Troll field which will be discussed later, the extraction of gas was severely limited to ensure higher pressure in the reservoir for an easier extraction of oil. Usually during the green field stage it is important to plan, drill and commission

new wells to reach some pre-defined plateau rate as soon as possible. During the plateau production there may be an in-field drilling program for production and/or injection wells. This involves decisions on the location and completion of wells. Later, during the decline phase of a field, artificial lift technology may be applied to boost production.

Operational production planning considers shorter time horizons, typically days and weeks, and is usually denoted real-time production optimization (RTPO) problem. Production may be constrained by reservoir conditions such as coning effects and/or the production equipment like pipeline capacity or downstream water handling capacity. Hence, this requires modeling of both the subsurface part (reservoir and wells) and the surface part (pipelines and downstream production equipment) of the value chain. Decision variables in RTPO include production and possibly injection rates, artificial lift inputs like lift gas rates and electric submersible pump (ESP) rates, and routing of well streams. The goal will be to maximize daily production rates. Overviews on RTPO can be found in Wang (2003), Saputelli, Nikolaou, and Economides (2005) and Bieker, Slupphaug, and Johansen (2006). RTPO is a widely used expression in the industry. In this paper, however, by RTPO we mean planning which involves the solution of a mathematical optimization problem.

This paper centers on the RTPO problem. The main contribution is a mixed integer linear program (MILP) formulation of the production network combined with a decomposition strategy. Two decomposition methods, Lagrange decomposition (LD) and Dantzig–Wolfe decomposition (DWD), are explored and tested on

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a realistic example. A key feature of our approach is the use of a divide-and-conquer strategy to decompose a RTPO problem into tractable sub-problems. Related references are Foss, Gunnerud, and Dueñas Díez (2009) where LD was introduced on a rather conceptual level and Gunnerud, Foss, Nygreen, Vestbø, and Walberg (2009) where DWD was initially proposed as a decomposition method for RTPO.

There has been some interest in applying decomposition techniques within the process systems literature. Alabi and Castro (2009) applies DWD for refinery planning while Cheng, Forbes, and Yip (2008) uses DWD to coordinate decentralized model predictive controllers for plant-wide control. In particular decentralized target calculations are coordinated by applying DWD.

There are many similarities between developing and operating a petroleum asset and a downstream process system, like a refinery. Hence, it makes sense to frame RTPO within the well established process control hierarchy which includes regulatory control, supervisory control, local dynamic or static optimization, site-wide optimization, and scheduling and planning; see e.g. Backx, Bosgra, and Marquardt (2000). As mentioned earlier RTPO means short term production planning. This is comparable to RTO in the process industries. One obvious difference, however, is the fact that a petroleum field in a life-cycle perspective is a depletable asset which can and should be viewed as a batch process as opposed to a plant like a refinery. Hence, conditions will vary significantly during the lifetime of a petroleum asset.

The remainder of this paper is organized as follows. To begin with, the complete nonlinear RTPO model will be presented. Then the techniques used for handling the nonlinearities, i.e. how to create piecewise linear representations and formulate the MILP model, is discussed. Further, we will look into why the problem is suitable for decomposition and present two decomposition schemes. Subsequently, results from a field case will be presented and a discussion on challenges related to the alternative solution methods is included before some conclusions end the paper.

2. The real-time production optimization problem

2.1. Methods and technology

RTPO applications exist in limited numbers. Two commercial products are GAP from Petroleum Experts and MaxPro from FMC Technologies. Both model the wells and pipeline systems, and solve the optimization problem by combining linear and nonlinear techniques. Wang (2003) provides a comprehensive overview of models and solution algorithms for different problems in the industry, again including both linear and nonlinear formulations with appropriate techniques for solving them. Bieker et al. (2006) presents an overview of the oil production problem which includes a description of production planning, processing facilities and well model updating. Another recommendable reference is Saputelli et al. (2003) since it ties RTPO to application challenges such as the availability of appropriate technologies. The value chain may be divided into an upstream part, which includes reservoirs, wells, pipelines and a downstream part which includes a process system for separating oil, water and gas as well as some export facility. The literature mentioned above takes a silo approach in the sense that the upstream part is optimized without including a model of the downstream system. Usually the downstream boundary is a fixed pressure on the inlet separator of the downstream process. Optimizing across this boundary by including the upstream and downstream system is rarely seen. One exception is Foss and Halvorsen (2009) which shows that a significant gain can be made by bridging the gap.

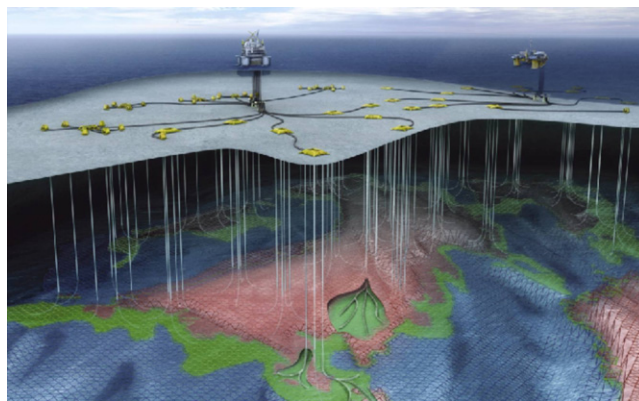


Fig. 1. Troll west structure (StatoilHydro 2008).

Some work discuss the consistency between production optimization and the medium term horizon decisions involving a full field reservoir simulator. In Awasthi, Sankaran, Nikolaou, Saputelli, and Mijares (2007) model consistency is emphasized while Awasthi, Sankaran, Nikolaou, Saputelli, and Mijares (2008) focus on decomposition between time scales and a moving-horizon approach is used for operational planning. As in all model-based applications model maintenance is important. It is particularly important to update well models since these models may change significantly over time. Cramer, Goh, Dolan, and Moncur (2009) present a data driven monitoring approach towards this end.

The literature is fairly limited on optimization models and solution algorithms for upstream petroleum production systems. Bieker (2007) solve the problem by piecewise linearization of nonlinearities and end up with a MILP formulation. Network topology is kept quite simple and in particular no routing issues are included. Kosmidis, Perkins, and Pistikopoulos (2005) uses a similar approach on a richer network topology. They allow for routing of the fluids from wells between different pipelines and to different separators. Kosmidis et al. (2005) piecewise linearizes the well models. They do, however, end up with a MINLP model, which results in completely different solution algorithm compared to MILP solvers.

2.2. Problem structure

Fig. 1 illustrates the Troll B and C platforms including the subsea production network. The structure is typical for a large scale offshore oil and gas production system. Such systems can be divided into clusters where one cluster contains a collection of wells which are connected to a platform through common pipelines. An illustration of a cluster is shown in Fig. 2. Each cluster will consist of a number of wells, manifolds and importantly several production lines. Troll C, which we will revisit later, contains eight clusters, each with two parallel production lines, two manifolds, and up to eight wells per cluster. The fluids from these eight clusters feed into a common platform-based process section.

Inflow from the reservoir into the wellbore, i.e. in the bottom part of the well, is known as the inflow performance relationship (IPR). It depends on reservoir pressure, pressure in the wellbore as well as the condition of the well itself. Further, a vertical lift performance (VLP) curve is commonly used to relate downhole conditions to wellhead (surface) conditions. This relationship depends on well geometry, and fluid rates and composition. Reservoir conditions will change over time due to the drainage effect. Since we are interested in short term optimization it is fair to assume constant conditions on the optimization horizon of interest. The well stream entering a manifold is routed to one of the production lines, see Fig. 2. The fluid is then transported through these pipelines to

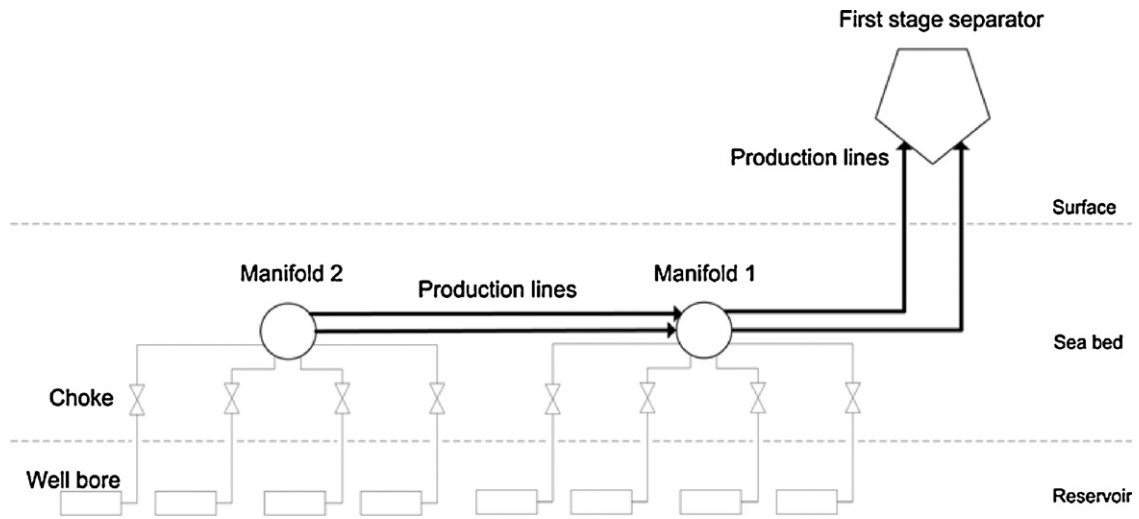


Fig. 2. Typical structure of one cluster.

the platform, and the pressure drop along the lines is modeled by multiphase flow models.

There will be constraints related to each cluster. They arise from reservoir analyses and capacity limits in the production equipment itself. As an example short term production boosting may harm long term drainage efficiency. The reason is that increased pressure gradients may damage the formation close to wells and hence reduce long term productivity. Further, there will as always be capacity limits on production equipment like wells, valves and pipelines, as well as constraints which originate from the downstream part of the value chain, for instance gas and water handling capacities. All appropriate constraints will be detailed in the next section.

A complete formulation is complex, thus the RTPO problem is both challenging and hard to solve. It may be noted, however, that most of the constraints are local to each cluster. This observation is essential for the decomposition approach applied later.

3. Model formulation

In the following we present a system model which encompasses a substantial class of upstream production systems. It is based on a relatively general network topology including fields like Troll. We will start by stating the complete model and from there, derive the MILP model which is used as a foundation for the two decomposition strategies.

3.1. Nonlinear model

In the interest of clarity, we will formulate the problem for clusters containing only one manifold. An extension to several manifolds per cluster, as in Fig. 2, is quite straightforward and is actually implemented in the case example discussed later.

At Troll C, there are two parallel pipelines transporting the produced fluid from a cluster. The number of pipelines can vary from one asset to another, from simple applications with only one line to network structures with even more than two lines, and with other possible structures than the tree structure in our model.

The following indexing conventions are used throughout the paper. Each cluster is identified by a single index i . Each well is identified by two indices ij , the index of the cluster in which it lies and its own index within the cluster. Each pipeline is also identified by two indices il , the index of the cluster where it lies and its own pipeline index.

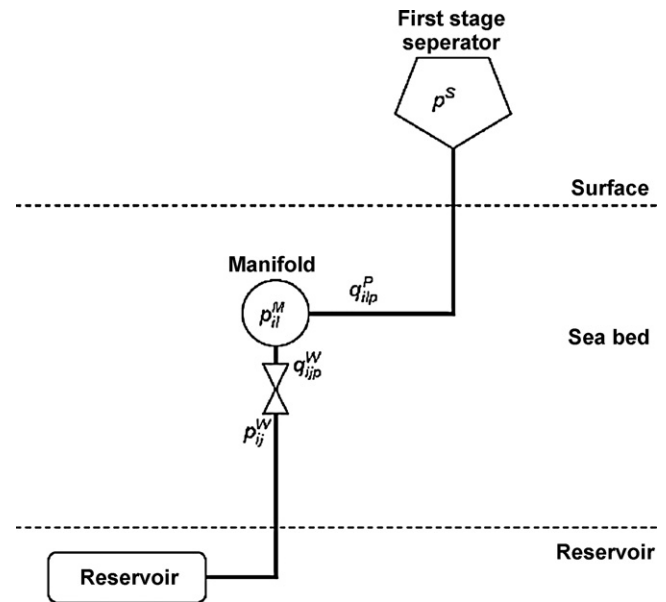


Fig. 3. Cluster topology with key variables.

Table 1
Indices.

i	Cluster
j	Well
l	Pipeline
p	Phase

The most common variables are illustrated in Fig. 3. The choke valve is found between the reservoir and the manifold, and is used to control the flow from the well. Some definitions are needed. In Table 1 all the indices are given, while the sets are given in Table 2. Further, Table 3 contains the data, while Table 4 contains the variables we will use.

Table 2
Sets.

\mathcal{I}	Set of clusters
\mathcal{J}_i	Set of wells in cluster i
\mathcal{L}_i	Set of pipelines in cluster i
\mathcal{P}	Set of phases (g for gas, o for oil, and w for water)

Table 3
Data.

C_p^T	Capacity of phase p in the first stage separator at platform
$f_{ij}^p(\cdot)$	Function for pressure drop in pipeline l
$f_{ij}^W(\cdot)$	Function for pressure drop in pipeline from wellbore to manifold
$f_{ijp}^{IN}(\cdot)$	Well model for each phase p
p_{ij}^R	Reservoir block pressure near well j
p^S	Pressure at first-stage separator on platform
q_{ij}^M	Maximum liquid production from well ij

In the following we present the objective function and constraints. The constraints are divided into two groups: common constraints and local constraints for each cluster.

3.1.1. Objective function

The objective function sums the oil flowrates from all clusters

$$\max Z = \sum_{i \in \mathcal{I}} q_{ip}^C, \quad p \in \{o\} \quad (1)$$

It should be noted that the objective function is additive on a cluster level. Further, the use of oil flowrate is a common production measure since it reflects short term revenue in most cases.

3.1.2. Common capacity constraints

The inequality below defines the common constraints. It states that the sum of gas and water rates from all clusters must be less than the gas handling and water handling capacities of the downstream processing equipment. Hence, these are the only constraints which connect the clusters, all other constraints apply for each cluster separately

$$\sum_{i \in \mathcal{I}} q_{ip}^C \leq C_p^T, \quad p \in \{g, w\} \quad (2)$$

3.1.3. Cluster constraints

All constraints, except for the common capacity constraints, are defined for all cluster $i \in \mathcal{I}$. In this section this statement will be omitted for the sake of simplicity.

The well model consists of two parts. First, the multiphase flow from the reservoir into the wellbore of each well is defined. It depends on reservoir pressure, geological properties of the formation close to the well and the wellbore itself, and it is usually represented by the well's IPR curve

$$q_{ijp}^W = f_{ijp}^{IN}(p_{ij}^R, p_{ij}^{WF}), \quad \forall j \in \mathcal{J}_i, \quad p \in \mathcal{P} \quad (3)$$

Further, the pressure upstream the choke has to be equal to the bottomhole flowing pressure p_{ij}^{WF} subtracted the pressure loss

between the wellbore and the wellhead:

$$p_{ij}^W = p_{ij}^{WF} - f_{ij}^W(q_{ijg}^W, q_{ijw}^W, q_{ijw}^W), \quad \forall j \in \mathcal{J}_i \quad (4)$$

This represents the VLP curve of a well.

The two nonlinear functions representing IPR and VLP respectively can be combined to create a nonlinear well performance curve (WPC). This equation then links wellhead pressure to the flowrate of each phase from one particular well

$$q_{ijp}^W = f_{ijp}^{WPC}(p_{ij}^W, p_{ij}^W), \quad \forall j \in \mathcal{J}_i, \quad p \in \mathcal{P} \quad (5)$$

Long term recovery may impose limits on liquid production from a well

$$q_{ijw}^W + q_{ijw}^M \leq q_{ij}^M, \quad \forall j \in \mathcal{J}_i \quad (6)$$

The well routing constraint (7) assures that flow from a well j either is closed or routed to one of the pipelines leaving the manifold

$$x_{ij} + \sum_{l \in \mathcal{L}_i} y_{ijl} = 1, \quad \forall j \in \mathcal{J}_i \quad (7)$$

The mass balance constraints (8) sum the flow from all wells that are routed to a particular pipeline

$$q_{ilp}^P = \sum_{j \in \mathcal{J}_i} q_{ijp}^W y_{ijl}, \quad \forall l \in \mathcal{L}_i, \quad p \in \mathcal{P} \quad (8)$$

Similarly (9) aggregates the flow from all pipelines in cluster i to one variable q_{ip}^C , i.e. the total production of phase p from cluster i

$$\sum_{l \in \mathcal{L}_i} q_{ilp}^P = q_{ip}^C, \quad \forall p \in \mathcal{P} \quad (9)$$

Pressure relations in manifolds and separator: Constraint (10) assures that the pressure for line l at the manifold in cluster i must be lower than the pressure upstream the choke in all wells j connected to the manifold and routed to line l . The reason for the inequality, instead of an equality, is to allow a pressure drop across the choke, i.e. when the choke is partly closed

$$y_{ijl} p_{il}^M \leq p_{ij}^W, \quad \forall j \in \mathcal{J}_i, \quad l \in \mathcal{L}_i \quad (10)$$

The pressure drop in pipelines between the manifold and the separator is formulated in (11). As the separator pressure is fixed the absolute pressure will not be included in the pressure drop model. For a problem containing more than one manifold in a cluster, this will not be the case and absolute pressure should be considered

$$p^S = p_{il}^M - f_{il}^P(q_{ilg}^P, q_{ilo}^P, q_{ilw}^P), \quad \forall l \in \mathcal{L}_i \quad (11)$$

3.2. MILP model

There are especially two factors that make the above optimization problem challenging. First, the nonlinearities related to well models (5) and pressure losses in pipelines (11), and second, well routing, which forces integer properties to be taken into consideration, cf. (7). These aspects have been key issues when choosing the solution approach for this optimization problem.

We piecewise linearize all nonlinearities which transform the MINLP into a MILP, to take advantage of the features that come with this formulation. This includes the possibility to use algorithms like Simplex, and Branch and Bound, see e.g. Rardin (2000), and the ability to compute performance bounds on a global solution.

Compared to Bieker (2007) we treat several clusters, which makes decomposition interesting. Further, we include the ability to route well flows to different pipelines. Kosmidis et al. (2005)

Table 4
Variables.

p_{il}^M	Pressure at the manifold level in pipeline l
p_{il}^D	Pressure drop across pipeline l
p_{ij}^W	Pressure at manifold level upstream choke
p_{ij}^{WF}	Wellbore flowing pressure, well j
q_{ip}^C	Flowrate of phase p from cluster i
q_{ilp}^P	Flowrate of phase p in line l
q_{ijp}^W	Flowrate of phase p from well j
x_{ij}	Binary variable. 1 if well j is closed
y_{ijl}	Binary variable. 1 if well j is routed to line l

Table 5
Indices.

k	Interpolation coordinate for the piecewise linearization of the WPC
n_p	Interpolation coordinate for the piecewise linearization of the pressure drop in pipelines, dependent on phase p (n_g for gas, n_o for oil, n_w for water)

Table 6
Sets.

\mathcal{K}_{ij}	Set of interpolation coordinates for the piecewise linearization of the well performance curve (WPC)
\mathcal{N}_{ip}	Set of interpolation coordinates for the piecewise linearization of the pressure drop in the pipelines

ends up with a MINLP model since he keeps the nonlinear models for the pipeline pressure drops as opposed to our MILP formulation.

Table 5–8 define the new indices, sets, data and variables needed for linearizing the problem.

3.2.1. Gas and water capacity constraints

The capacity constraints (2) are already linear, however we would like to restate them here to have a complete MILP model in this section

$$\sum_{i \in \mathcal{I}} q_{ip}^C \leq C_p^T, \quad \forall p \in \{g, w\} \quad (12)$$

3.2.2. Linearization of well performance curves

We apply a modal formulation to piecewise linearize the WPC (5). By this we replace the nonlinear constraints with SOS2 sets Williams (2005). The link between wellhead pressure p_{ij}^W and flow q_{ijp}^W is given by (13)–(17). γ_{ijk} are the weighting variables introduced by the SOS2 set which decides on the weighting of the brake points ($p_{ijk}^{WPC}, q_{ijk}^{WPC}$)

$$p_{ij}^W = \sum_{k \in \mathcal{K}_{ij}} p_{ijk}^{WPC} \gamma_{ijk}, \quad j \in \mathcal{J}_i \quad (13)$$

Table 7
Data.

$p_{iln_g n_o n_w}^D$	Breakpoints for pressure drop in pipeline l
p_{ij}^{MAX}	Maximum well head pressure
p_{ijk}^{WPC}	Pressure at manifold level before choke, for breakpoint k
$q_{ilpn_p}^D$	Flow rate in pipeline l of phase p corresponding to breakpoint n_p
q_{ijp}^{MAX}	Maximum flow of phase p from well j
q_{ijk}^{WPC}	Flow rate of phase p from well j corresponding breakpoint k

Table 8
Variables.

q_{ijlp}^S	Flow from well ij to line l of phase p into line l
γ_{ijk}	Weighting variable associated with each breakpoint k
$\lambda_{iln_g n_o n_w}$	Weighting variable associated with the breakpoint n_g, n_o, n_w
η_{ilpn_p}	Weighting variable associated with the SOS2 sets for the piecewise linearization pipeline models

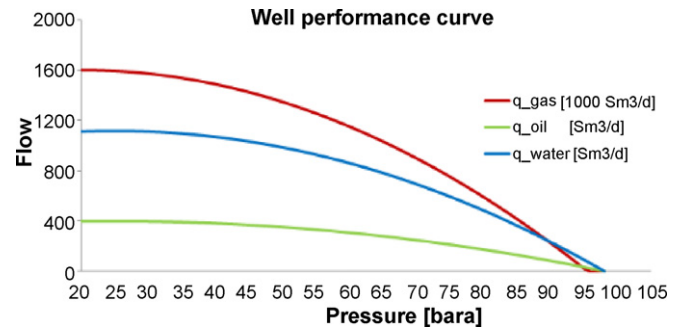


Fig. 4. WPC based on typical data from a gas-coning well.

$$q_{ijp}^W = \sum_{k \in \mathcal{K}_{ij}} q_{ijk}^{WPC} \gamma_{ijk}, \quad j \in \mathcal{J}_i, \quad p \in \mathcal{P} \quad (14)$$

$$\sum_{k \in \mathcal{K}_{ij}} \gamma_{ijk} = 1, \quad j \in \mathcal{J}_i \quad (15)$$

$$\gamma_{ijk} \geq 0, \quad j \in \mathcal{J}_i, \quad k \in \mathcal{K}_{ij} \quad (16)$$

$$\gamma_{ijk} \text{ is SOS2 for } k, \quad j \in \mathcal{J}_i \quad (17)$$

Note that p_{ij}^W is mapped to three flow variables q_{ijp}^W , cf. Fig. 4.

3.2.3. Linearization of well routing constraints

To handle well routing, several steps are made. The linear routing constraints (7) are restated

$$x_{ij} + \sum_{l \in \mathcal{L}_i} y_{ijl} = 1, \quad \forall j \in \mathcal{J}_i \quad (18)$$

Further, we have changed the nonlinear constraints (8) and (10) into a set of linear constraints. A new set of variables, q_{ijlp}^S , which are the flowrates of phase p from well ij to pipelines il leaving the manifold, is defined. Further, two new sets of constants, q_{ijp}^{MAX} and p_{ij}^{MAX} are introduced. q_{ijp}^{MAX} defines the maximum possible flowrate for each phase from a given well, and p_{ij}^{MAX} defines the maximum possible pressure drop over the manifold. (19)–(22) then describe new mass and pressure balances for wells and manifolds

$$\sum_{l \in \mathcal{L}_i} q_{ijlp}^S = q_{ijp}^W, \quad \forall j \in \mathcal{J}_i, \quad p \in \mathcal{P} \quad (19)$$

$$q_{ijlp}^S \leq q_{ijp}^{MAX} y_{ijl}, \quad \forall j \in \mathcal{J}_i, \quad l \in \mathcal{L}_i, \quad p \in \mathcal{P} \quad (20)$$

$$q_{ilp}^P = \sum_{l \in \mathcal{L}_i} q_{ijlp}^S, \quad \forall l \in \mathcal{L}_i, \quad p \in \mathcal{P} \quad (21)$$

$$p_{il}^M \leq p_{ij}^W + p_{ij}^{MAX} (1 - y_{ijl}), \quad \forall j \in \mathcal{J}_i, \quad l \in \mathcal{L}_i \quad (22)$$

To elaborate (22) limits the pressure in the manifold when the connection between well ij and pipeline il is open. When the connection is closed, however, then $y_{ijl} = 0$ and (20) ensures that there is no flow from the well into that pipeline, and (22) is relaxed so that there is no link between the wellhead and pipeline pressures.

3.2.4. Linearization of pressure drop in pipelines

The pressure drop in the pipelines depends on gas, oil and water flowrates as shown in (11). Hence, it is necessary to select break points in three dimensions, i.e. the gas, oil and water flowrates which results a more complicated remodeling procedure. Bieker (2007) used a similar procedure for a pipeline model with four inputs since he also includes the absolute pressure at the inlet of the pipeline in addition.

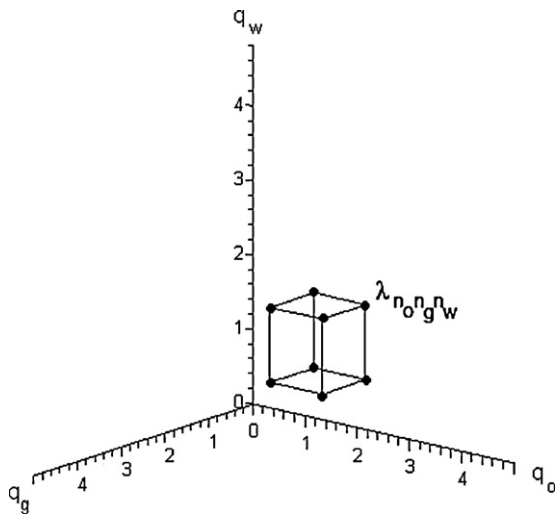


Fig. 5. Illustration of brakepoint coordinates.

To explain this procedure, we start by defining a grid for each pipeline il ($q_{ilg}^p, q_{ilo}^p, q_{ilw}^p$), not necessarily equidistant, in three dimensions. q_{ilg}^p will be the flowrate of gas in pipeline l , and similar for oil and water. Associated non-negative weighting variables $\lambda_{iln_g n_o n_w}$ with each point in the grid are also defined, where $n_g \in \mathcal{N}_{ig}$, $n_o \in \mathcal{N}_{io}$, $n_w \in \mathcal{N}_{iw}$. Further, \mathcal{N}_{ig} , \mathcal{N}_{io} , and \mathcal{N}_{iw} is the set of flowrate brake points along the gas, oil and water axes, respectively, see Fig. 5.

If the values of ($q_{ilg}^p, q_{ilo}^p, q_{ilw}^p$) at the brake points are denoted ($q_{ilng}^D, q_{ilno}^D, q_{ilnw}^D$) and the associated pressure drop ($p_{ilpn_g n_o n_w}^D$), it is possible to approximate function (11) by means of the following relations:

$$p_{il}^D = \sum_{n_g \in \mathcal{N}_{ig}} \sum_{n_o \in \mathcal{N}_{io}} \sum_{n_w \in \mathcal{N}_{iw}} p_{ilpn_g n_o n_w}^D \lambda_{iln_g n_o n_w}, \quad \forall l \in \mathcal{L}_i \quad (23)$$

$$q_{ilp}^p = \sum_{n_g \in \mathcal{N}_{ig}} \sum_{n_o \in \mathcal{N}_{io}} \sum_{n_w \in \mathcal{N}_{iw}} q_{ilpn_g n_o n_w}^D \lambda_{iln_g n_o n_w}, \quad \forall l \in \mathcal{L}_i, \quad p \in \mathcal{P} \quad (24)$$

$$\sum_{n_g \in \mathcal{N}_{ig}} \sum_{n_o \in \mathcal{N}_{io}} \sum_{n_w \in \mathcal{N}_{iw}} \lambda_{iln_g n_o n_w} = 1, \quad \forall l \in \mathcal{L}_i \quad (25)$$

$$\lambda_{iln_g n_o n_w} \geq 0, \quad \forall l \in \mathcal{L}_i, \quad n_g \in \mathcal{N}_{ig}, \quad n_o \in \mathcal{N}_{io}, \quad n_w \in \mathcal{N}_{iw} \quad (26)$$

In addition at most eight neighboring $\lambda_{iln_g n_o n_w}$ can be non-zero as illustrated by Fig. 5. This condition is a generalization of a SOS2 set, and can be imposed as below, where η_{ilpn_g} , η_{ilpn_o} and η_{ilpn_w} are auxiliary weighting variables of \mathcal{N}_{ig} , \mathcal{N}_{io} and \mathcal{N}_{iw} elements defined as SOS2 sets

$$\eta_{ilpn_p} = \sum_{n_o \in \mathcal{N}_{io}} \sum_{n_w \in \mathcal{N}_{iw}} \lambda_{iln_g n_o n_w}, \quad \forall l \in \mathcal{L}_i, \quad p \in \{q\}, \quad n_g \in \mathcal{N}_{ig} \quad (27)$$

$$\eta_{ilpn_p} = \sum_{n_g \in \mathcal{N}_{ig}} \sum_{n_w \in \mathcal{N}_{iw}} \lambda_{iln_g n_o n_w}, \quad \forall l \in \mathcal{L}_i, \quad p \in \{o\}, \quad n_o \in \mathcal{N}_{io} \quad (28)$$

$$\eta_{ilpn_p} = \sum_{n_g \in \mathcal{N}_{ig}} \sum_{n_o \in \mathcal{N}_{io}} \lambda_{iln_g n_o n_w}, \quad \forall l \in \mathcal{L}_i, \quad p \in \{w\}, \quad n_w \in \mathcal{N}_{iw} \quad (29)$$

$$\eta_{ilpn_p} \geq 0, \quad \forall l \in \mathcal{L}_i, \quad p \in \mathcal{P}, \quad n_p \in \mathcal{N}_{ip} \quad (30)$$

$$\eta_{ilpn_p} \text{ is SOS2 for } n_p, \quad \forall l \in \mathcal{L}_i, \quad p \in \mathcal{P} \quad (31)$$

Finally, the relationship between the pressure in the manifold and the flowrates q_{ilp}^p in the pipelines (11) can be formulated

$$p^S = p_{il}^M - p_{il}^D, \quad \forall l \in \mathcal{L}_i \quad (32)$$

3.2.5. Aggregated flow variable

To obtain a complete model formulation the linear constraints (9) are restated

$$\sum_{l \in \mathcal{L}_i} q_{ilp}^p = q_{ip}^C, \quad \forall p \in \mathcal{P} \quad (33)$$

Constraints (13)–(33), together with the common capacity constraints (12) and the objective function (1), defines the complete MILP problem. This problem will be the basis for the two decomposition strategies investigated in the next section.

4. Decomposition strategies

Decomposition approaches for optimization problems is a mature field in operations research. In this section we argue that decomposition is a suitable strategy for the RTPO problem. Alternative decomposition approaches exist and their applicability depends on the structure of the underlying problem. We present two alternative and related strategies, Lagrange decomposition (LD) and Dantzig–Wolfe decomposition (DWD), as a means to decompose the RTPO problem. A third strategy, Bender’s decomposition (Dantzig & Thapa, 2003), is excluded since it cannot exploit the structure in our problem efficiently.

The general idea is to relax global constraints, i.e. constraints which span across large parts of a problem. There are alternative ways of doing this. The basic mechanism in all decomposition principles, however, is to decompose the original problem into smaller sub-problems which are coordinated by a “master” problem. An iterative procedure is then used to achieve convergence towards a global solution.

There are two aspects to consider when choosing which constraints to place in the sub-problem. It should be easy to solve since it is re-optimized several times, and it should provide good quality bounds on the intermediate solutions. As mentioned earlier, the presented RTPO problem has a decentralized topology, which is a suitable structure for some decomposition strategies. An oil and gas producing cluster has many internal couplings such as mass and pressure balances, and routing of wells. Further, there are only a few common constraints, in this case total gas and water production. By relaxing these constraints a decomposition strategy may result in one sub-problem for each cluster i . Each sub-problem can be assigned its own part of the objective function if this is additive on a cluster basis as in (1), and if a second term takes common capacity constraints into account.

4.1. Lagrange decomposition

Lagrange relaxation is a well known technique for finding upper bounds on maximization problems. This involves a technique that attaches Lagrange multipliers to some of the constraints of the original problem, and relaxes them into the objective function. The resulting problem is then solved, and the value provides an upper bound of the solution of the original maximization problem. If the solution also is feasible with respect to the original problem, it provides a lower bound as well. Further, if common constraints are subject to relaxation, the resulting problem will fall apart into smaller optimization problems, one for each sub-problem. A general description can be found in Beasley (1993).

When building a solution method using LD, two issues have to be decided. First, which constraints to relax to define the sub-problems, and second, the strategy for how to update the Lagrange multipliers for the relaxed constraints. Both are presented below.

4.1.1. Relaxed constraints/sub-problems

The Lagrangian objective function for the RTPO is presented below. In this case the gas and water handling capacity constraints (12) are relaxed. When including all other constraints in the MILP model (13)–(33) this will represent the Lagrangian upper bound problem for the RTPO problem. π_g^{CAP} and π_w^{CAP} are the Lagrange multipliers associated with the gas and water handling capacity constraints

$$\max Z = \sum_{i \in \mathcal{I}} q_{io}^C + \pi_g^{CAP} (C_g^T - \sum_{i \in \mathcal{I}} q_{ig}^C) + \pi_w^{CAP} (C_w^T - \sum_{i \in \mathcal{I}} q_{iw}^C) \quad (34)$$

This objective function, together with the MILP constraints (13)–(33), define the Lagrangian upper bound problem (LUBP). By relaxing these constraints the problem can be separated into one sub-problem for each cluster. Hence, we obtain the following objective function (35) for each cluster i

$$\max Z_{(i)} = q_{io}^C - \pi_g^{CAP} q_{ig}^C - \pi_w^{CAP} q_{iw}^C \quad (35)$$

This objective function will be solved subject to local constraints only (13)–(33).

4.1.2. Lagrange multiplier update

When the sub-problems are defined the next step involves updating the Lagrange multipliers. There are several alternatives in the literature, herein, the subgradient methods described in Beasley (1993) has been implemented.

The procedure is iterative, and generates multipliers in a systematic fashion from an initial set of multipliers. Upper and lower bounds on the solution have to be calculated after each iteration. These bounds can also be used to quantify the quality of the solutions obtained. The upper bound Z_{UB} will equal (34) while the lower bound is usually found by some heuristics. In this case we only update this bound when the sum of production of gas and water from all the sub-problems is less than the handling capacities. It can then be defined to be equal to the original objective function

$$Z_{LB} = \sum_{i \in \mathcal{I}} q_{io}^C \quad (36)$$

Algorithm structure

1. Select ν , satisfying $0 < \nu \leq 2$. Then initialize a lower bound, Z_{LB} , from some heuristics of the problem. We choose $Z_{LB} = 0$. Then select an initial set of Lagrange multipliers, i.e. π_g^{CAP}, π_w^{CAP} .
2. Solve \mathcal{I} local optimization problems by using the Lagrange multipliers and get an upper bound of the solution, i.e. Z_{UB} . If the solution also is feasible with respect to the original problem, update the Z_{LB} as well.
3. Compute the subgradients G_g^{CAP}, G_w^{CAP} for the relaxed constraints

$$G_g^{CAP} = \sum_{i \in \mathcal{I}} q_{ig}^C - C_g^T \quad (37)$$

$$G_w^{CAP} = \sum_{i \in \mathcal{I}} q_{iw}^C - C_w^T \quad (38)$$

4. Define a step size T when gas and water handling capacity is relaxed

$$T = \frac{\nu(Z_{UB} - Z_{LB})}{\sum_{p \in \{g, w\}} (G_p^{CAP})^2} \quad (39)$$

The step size depends on the gap between the lower and the upper bound and the user defined parameter ν . The denominator acts as a scaling factor.

5. Update π_p^{CAP} using the rule below and then go to step 2 to resolve the LUBP with this new set of multipliers

$$\pi_g^{CAP} = \max(0, \pi_g^{CAP} + T G_g^{CAP}) \quad (40)$$

$$\pi_w^{CAP} = \max(0, \pi_w^{CAP} + T G_w^{CAP}) \quad (41)$$

4.1.3. Convergence and integer handling

The iterative procedure needs a termination criterion. The procedure may terminate after a certain number of iterations or when $Z_{UB} - Z_{LB}$ is below some value, e.g. 1% of Z_{UB} .

LD and the subgradient method is developed for LP problems (Beasley, 1993). Since the sub-problems are MILP the method will not necessarily converge to a 0% gap due to the integer variables, as would be the case for LP problems. The reason is that marginal changes in the Lagrange multipliers may lead to different integer values and hence shifts in the objective functions of the sub-problems.

4.2. Dantzig–Wolfe decomposition

When applying DWD to the RTPO problem the sub-problems will be identical to LD if the same common constraints are subject to relaxation. However, while the Lagrange multipliers are updated by a simple heuristics in the LD case, the update is now done by solving a LP-master problem.

4.2.1. DWD principle

We start by assuming linear constraints and continuous variables, i.e. a LP-problem instead of a MILP problem. The master problem is a reformulation of the integrated problem. By taking advantage of the fact that a convex combination of basic feasible points, i.e. corner points of the feasible set defined by the linear constraints of a problem, also is a feasible solution, an alternative formulation can be achieved. Each basic feasible point in each sub-problem is then represented as a variable in the master problem. Note that each basic feasible point in a sub-problem represents a specific production setup for this sub-problem. The number of basic feasible points for any practical problem can clearly be prohibitively high, and in reality only a small number of these basic feasible points will ever enter the basis in the master problem. The idea is then to restrict the master problem by reducing the number of basic feasible points. This is called a restricted master problem (RMP). Hence, we start with a few basic feasible points and check if the solution of the integrated problem is within a convex combination of these points. If this is not the case, new basic feasible points are included in a structured way until the optimal solution has been found, Dantzig and Thapa (2003) and Williams (2005). Details of the algorithm are given below with some related comments specific to the RTPO problem above.

Algorithm structure

1. Choose two initial basic feasible points for each local optimization problem, i.e. two different modes of operating a cluster.
2. Specify the RMP for the given set of basic feasible points. Then solve it and compute values for the Lagrange multipliers for the global constraints, i.e. π_g^{CAP}, π_w^{CAP} .
3. Solve \mathcal{I} local optimization problems by using the Lagrange multipliers computed in 2.
- 4a. For $i \in \mathcal{I}$: If the solution of a local optimization problem i lies outside the convex set defined by the basic feasible points used in 2; add these basic feasible points to the RMP, and go to 2. Hence, the RMP is then resolved, also including these new basic feasible points generated in 3.
- 4b. If the solutions of all the local optimization problems are unchanged, the optimal solution has been found; and the algorithm terminates.

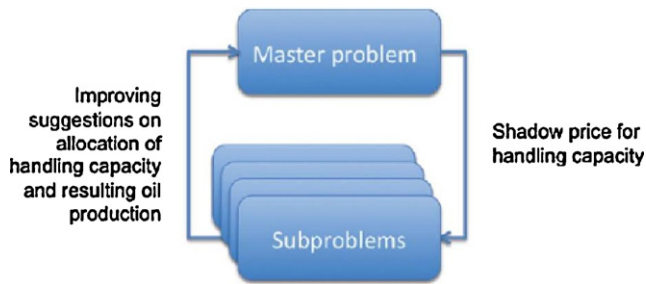


Fig. 6. Iteration structure for Dantzig–Wolfe decomposition (DWD) and Lagrangian decomposition (LD).

To illustrate, the main iteration loop is shown in Fig. 6. This figure is also applicable for LD if “Master problem” is replaced by “Subgradient updating”.

4.2.2. Sub-problem

The procedure is to update the Lagrange multipliers such that the consumption of the relaxed common constraints converges to their optimal values. Let q_{ig}^C , q_{io}^C and q_{iw}^C denote the output flow of the phases in a feasible solution to cluster i . The corresponding reduced cost $\bar{c}_{(i)}$, if these values are used to construct a column in the master problem, will be:

$$\bar{c}_{(i)} = q_{io}^C - \pi_g^{CAP} q_{ig}^C - \pi_w^{CAP} q_{iw}^C - \pi_i^C, \quad \forall i \in \mathcal{I} \quad (42)$$

π_g^{CAP} and π_w^{CAP} are the Lagrange multipliers in the master problem for the common gas and water handling capacity constraints (44) and (45) while π_i^C is the multiplier for the convexity constraint for cluster i (46). The objective in the sub-problem for this cluster is to find the maximum reduced cost among feasible solutions for the cluster. By maximizing $\bar{c}_{(i)}$, given the local constraints (13)–(31), this column will be found. If this value is positive then a new column, say s , is added to the master problem with Z_{isg} , Z_{iso} and Z_{isw} equal to the optimal values of q_{ig}^C , q_{io}^C and q_{iw}^C , respectively, from the sub-problem. s is added to set \mathcal{S}_i , which is used in (43).

As mentioned π_i^C are the Lagrange multipliers for the convexity constraints in the RMP defined below. Since no sub-problem variables are associated to it, it will only act as a constant in the sub-problem.

4.2.3. Restricted master problem

The RMP can now be formulated. The production rate corresponding to each Z_{isp} represents one basic feasible point s from sub-problem i , i.e. a possible production allocation for cluster i . Z_{isp} could in principle include the optimal value of all decision variables for sub-problem i after solving it given π_g^{CAP} and π_w^{CAP} . However, only the variables also present in the objective function and the common constraints will be relevant for the RMP. Hence, Z_{isp} will for this RTPO problem contain the flow variables (q_{ig}^C , q_{io}^C , q_{iw}^C), but no pressure and routing variables. μ_{is} , the optimization variables for the RMP, is the weight the RMP will give this basic feasible point after it is solved to optimality. The objective function of the restricted master problem is given in (43). Further, (44) and (45) represent the constrained common resources, while (46) is the convexity constraints

$$\max \sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}_i} Z_{iso} \mu_{is} \quad (43)$$

$$\sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}_i} Z_{isg} \mu_{is} \leq C_g^T \quad (44)$$

$$\sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}_i} Z_{isw} \mu_{is} \leq C_w^T \quad (45)$$

$$\sum_{s \in \mathcal{S}_i} \mu_{is} = 1, \quad \forall i \in \mathcal{I} \quad (46)$$

$$\mu_{is} \geq 0 \quad (47)$$

4.2.4. Integer handling

DWD will find exact optimal solutions for LP problems. If it is extended to a MILP problem, Branch and Price Desrosiers and Lubbecke (2006) or some heuristics have to be applied to handle the integer properties. When solving the master problem, we have not imposed integer restrictions on μ_{is} , i.e. the RMP is solved as a LP to achieve Lagrange multipliers for (44)–(46). The resulting solution may then be infeasible with respect to the original MILP problem since a convex combination of two basic feasible points may not be feasible. As an example a basic feasible point represents a specific production allocation for a particular cluster, while another basic feasible point will represent another allocation for the same cluster. A convex combination of these is impossible if the two allocations have different routings, i.e. different integer solutions.

As mentioned, integer handling could be handled in several ways. In this work we apply the following heuristic approach. If a sufficient number of basic feasible points is generated up front, a feasible solution can simply be found by demanding integer values for μ_{is} by solving the RMP as an IP problem.

4.2.5. Convergence

For both LP and MILP problems, upper and lower bounds on the objective function can be computed. The LP solution of the RMP plus the sum of the objective values of the sub-problems, i.e. the reduced cost, will act as an upper bound Z_{UB} , Karlof (2006, chap. 4). In the LP case, the solution of the RMP alone will give a feasible lower bound, while for the MILP problems, the master problem has to be solved as an IP or some heuristics has to be applied to create the feasible lower bound Z_{LB} . By using these bounds actively during the optimization process, it is possible to terminate the optimization problem when an acceptable gap is achieved.

5. Implementation

An RTPO has been investigated by two decomposition strategies, LD and DWD. These strategies are compared with the solution of the integrated problem before relaxation. In this way, we compare the effect of the two decomposition strategies against each other, and against a regular global method.

Piecewise linear well and pipeline models were generated as follows. Initially analytic functions approximating data from Troll C were created. These functions were then used to generate breakpoints for the piecewise representation of the models. This information was subsequently stored as tables. For the well models; this meant that for every wellhead pressure brake point, there is a related gas, oil and water flowrate. For the pipeline models on the other hand; every combination of gas, oil and water flowrates gives a pressure drop across it. If the solution algorithm requests a pressure between the brake points, it will simply linearly interpolate between the neighboring points. The reason for introducing the analytic functions was to obtain flexibility when choosing the interpolation points for the well models used in the optimization. The underlying data points vary in resolution since some operating regions are more heavily sampled than others.

Both optimization algorithms are implemented in Xpress-IVE which is a state of the art software for mixed integer linear problems. Brake point tables together with other topology information

Table 9
Results.

No. of clusters	2	4	6	8
Gas cap [Sm ³ /day]	3,000	12,000	18,000	24,000
Continuous variables	13,898	27,805	41,766	55,688
Discrete variables	1,029	2,134	3,725	4,819
Constraints	491	981	1,639	2,185
Strategy 1—global				
Solution time [min]	0.26	7.38	237.0	720.0
Oil prod [Sm ³ /day]	1,777	6,487	11,641	14,365
Optimality gap [%]	0.00	0.00	0.00	7.50
Strategy 2—LD				
Solution time [min]	1.42	8.54	18.2	19.2
Oil prod [Sm ³ /day]	1,774	6,467	11,640	14,440
Optimality gap [%]	0.17	0.30	0.21	0.38
LD iterations	12	32	16	13
Strategy 3—DWD				
Solution time [min]	1.86	1.43	6.16	11.3
Oil prod [Sm ³ /day]	1,777	6,458	11,629	14,473
Optimality gap [%]	0.02	0.46	0.36	0.15
DWD iterations	10	4	3	5

are supplied through data files. The user is able to choose between solving the data by a global solution algorithm, or with LD or DWD. In the case of the global method, all data will be loaded into the solver and solved directly.

6. Results

The Troll field is a huge oil and gas field on the continental shelf west of Norway. There are severe production optimization challenges due the size of the asset and because of rate dependent gas-coning wells (Hauge & Horn, 2005). We study a model of the Troll C production system shown in Fig. 1 where primarily oil is produced from an oil rim through more than 50 wells.

This system changes with time since new wells are drilled and commissioned continuously. Further, the reservoir conditions change significantly due to medium and long term depletion effects. Hence, well models are usually updated twice a year by running well tests to collect data for parameter estimation. Within a day or a week, however, changes are usually quite limited. Therefore RTPO as presented in this paper has a potential for increasing value creation.

For the moment, only one common constraint is active; the total handling capacity for gas production (12). Water handling will, however, become an issue in the future as the reservoir drains and therefore produces more water. The production system includes 8 clusters and each cluster contains 6–8 wells and two manifolds.

The purpose of the numerical study is to investigate three solution strategies.

1. A global strategy where all clusters are solved in one large MILP problem.
2. The LD strategy we propose in this paper.
3. The DWD strategy we propose in this paper.

The results shown in Table 9 are an extension of the results in Gunnerud et al. (2009). The values in the table are representative numbers for the different cases and the three alternative methodologies. The computations are performed on an IBM Thinkpad T60P with a 2.33 GHz processor.

The solution strategies were solved for two, four, six and eight clusters as shown in Table 9, and the average gas capacity per cluster was chosen at about 3000 Sm³/day per cluster. The large number of continuous and discrete variables per cluster comes from the piecewise linearization approach. A typical well model is linearized

by 20–100 brake points. The total number of brake points in the pressure drop models is much higher since it is necessary to interpolate in three dimension: the gas, oil and water flowrates.

7. Discussion

There are two main contributions in this paper. The first is the formulation of a full field RTPO model only containing linear and integer variables while the second is a detailed description of two methods for decomposing the RTPO problem into smaller and hence easier problems.

As mentioned earlier, by piecewise linearizing all nonlinearities and thereby creating a MILP formulation it is possible to apply known and well developed algorithms such as the Simplex algorithm and the Branch and Bound algorithm to solve the RTPO problem. An additional effect of this formulation is the fact that the underlying wells and pipeline models are completely disconnected from the MILP formulation. Hence, they could be replaced on a later occasion without affecting the structure and solution procedure of the MILP formulation. Hence data points can be created from any reservoir, well and pipeline simulator as an alternative to real production data.

Flexible accuracy is an attribute of the formulation since the break points can be placed arbitrarily. The price to pay for higher accuracy, however, is a longer calculation time.

There are still challenges using the piecewise linearization approach for pipeline models. Since the pressure drop depends on the gas, oil and water flowrates the number of weighting variables increases with the power of three. 20 brake points for each phase will, e.g. result in 8000 interpolation elements (λ_{ilng, n_o, n_w}). In addition there will be $20 \times 3 = 60$ integer variables (η_{ilp, n_p}) in this case. Since the number of weighting variables increases so rapidly it is essential to minimize the number of brake points. This has however not been studied in any detail in the present paper.

The second contribution in this paper, the decomposition methods, is tested in a realistic numerical study. The decomposition strategies, DWD and LD, perform better than the global method for the combined rate allocation and routing problem for all sizable problems. Moreover, it may be observed that the global method does not converge for the eight cluster problem and it has a hard time solving problems consisting of more than 6 clusters.

To elaborate on the above the global method finds the optimal solution for all except the full field problem with 8 clusters. In this case it terminates after 12 h with more than a 7.5% duality gap. DWD and LD terminate with less than 0.5% duality gap for all problems.

The global method works fine for the 2 and 4 cluster problems. This is not a surprising result since the benefit of a decomposition strategy is limited in these cases. For larger problems, however, we observe that the two decomposition methods are much faster than the global method. Further, DWD performs better than LD. The reason is the mechanism for updating of Lagrange multipliers. The DWD master problem finds good multipliers with fewer iterations than for the LD case, and usually converges after fewer iterations. This is observed in the table where DWD converges in 3 and 5 iterations for the 6 and 8 cluster cases while LD requires more than 10 iterations. The run-time for solving the DWD master problem, i.e. the LP-problem, is negligible compared to solving the local MILP-problems even though it is more time-consuming than solving the LD master problem. Hence, this is no issue when comparing DWD and LD.

DWD is more stable with respect to solution time than LD. DWD has fewer tuning parameters and works well for different data sets. On the other hand LD is quite sensitive to perturbations of the data set. A small change may for instance double the solution time.

Decomposition algorithms using DWD or LD have some interesting properties. First, the sub-problems may be solved by

different algorithms or even different software packages. This feature has a potential for value chain optimization applications which may encompass reservoir, wells, pipelines and downstream processing facilities. The duality gap, however, can only be computed if upper and lower bounds on the solution can be found. This is in general not possible if the sub-problems are nonlinear programs as opposed to MILPs. Second, it is quite obvious that decomposition methods are suitable for parallel computing since each sub-problem is self-contained and has no direct dependence on the other sub-problems.

The routing and well allocation problem is usually solved by re-optimizing the stationary optimization problem, typically once a day. Hence, it is treated in a quasi-dynamic way. A couple of wells are usually selected for more frequent production changes to compensate for variations in for instance gas processing capacity. The use of dynamic models is an issue, in particular during start-up of wells. Start-up occurs quite often since wells are shut-in from time-to-time due to maintenance or operational problems. Further, applications with long pipelines may benefit from dynamic pipeline models provided the dynamics are important for optimal performance.

There are at least two trends which can aid the implementation of the methodology proposed in this paper. First, the possibility to include third party applications into commonly used software systems for monitoring and control purposes has increased significantly in the latter years. Second, there is an initiative to develop a new standard for production optimization, PRODML, which is supported by many leading vendors and end users.

8. Conclusions

This paper presents a complete MILP model for the RTPO problem in the petroleum industries. Further, the paper argues that decomposition is well suited for this problem. There are many reasons for this. Decomposition methods clearly outperform a global method for full field problems in terms of computational efficiency. Further, DWD gives better performance than LD in all the relevant cases tested in this paper and is therefore the preferred decomposition method. This is because of a more efficient updating algorithm of the Lagrange variables. The MILP formulation allows the computation of a duality gap on the solution of the production optimization problem. This is clear information of interest to any user.

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