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Production Optimization – Facilitated by Divide and Conquer Strategies

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Abstract: This paper explores the use of divide and conquer algorithms to solve production optimization problems. Exploitation of inherent structure facilitates efficient formulations and solutions to production optimization problems that frequently appear in the upstream oil and gas sector. We present the daily production optimization, which gives rise to network optimization problems, and also add some comments to the closely linked reservoir optimization problem. Subsequently, a common non-invasive optimization approach is presented prior to three concepts that apply decomposition and structure exploitation. The first approach splits a network simulator into interconnected parts and exploits the local components and their couplings in devising efficient formulations and algorithms. This opens for a range of opportunities in terms of formulations and algorithms, several of which will be discussed. This part is accompanied by a realistic numerical study. The second approach exploits a decomposable network structure, which frequently appears in large production systems, by applying a Lagrangian type decomposition. The last approach deals with reservoir optimization where there is no apparent structure to exploit. However, by recognizing this as a dynamic optimization problem, it may be split in the time domain. The paper ends with some conclusions.

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Keywords: production optimization; reservoir optimization; formulations; decomposition; structure exploitation; optimization.

1. INTRODUCTION

Oil and gas assets are found in subsurface reservoirs in which hydrocarbons have been trapped in some suitable geological structure. Having discovered an asset, and assessed its economical viability, it can only be drained through wells that connect it to the surface. Thus, during production reservoir fluid enters the wells and subsequently invades a collection system comprised of manifolds and pipelines before the fluids are processed and exported through some suitable means. It is useful to divide production planning of an asset through different planning horizons ranging from a life-cycle perspective to daily production planning. Decisions on various horizons may be portrayed in a multilevel control hierarchy as shown in Fig. 1. The topmost level includes life-cycle related decisions such as the choice of technologies and the investment strategy.

On a shorter time frame, normally from one to five years, decisions are made on production strategies such as the location of new wells, injection rates and target production rates. Such decisions are typically supported by a simulator based on high fidelity reservoir models. Reservoir models are based on a (qualitative) geological model of the asset and are calibrated against appropriate data, a process denoted history matching. To account for this uncertainty an ensemble of reservoir models is customarily used in reservoir simulation studies. These studies concentrate on the subsurface domain, which includes wells, even though attempts are being made to include the collection system and processing facilities, thus using an integrated model that covers the value chain from source to export. Optimization based on reservoir models and with time frames of one to five years is denoted the *Reservoir Optimization problem.*

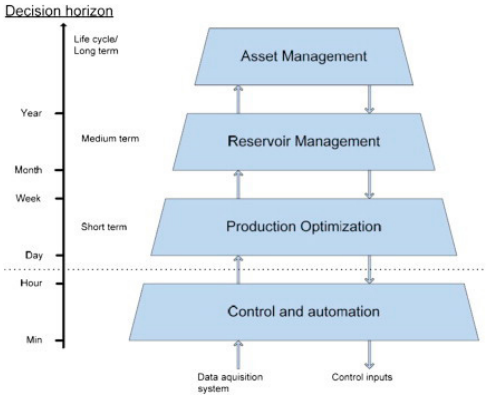

Fig. 1. A multilevel control hierarchy

On level three in the control hierarchy in Fig. 1, which we refer to as the *Daily Production Optimization (DPO)* problem, the planning horizon typically ranges from a few hours to a week. There are two key differences to the reservoir optimization problem. First, the shorter time horizon facilitates the use much simpler reservoir models since reservoir fluid movements within this time span is limited. There do exist exceptions to this rule, however, for instance in wells with severe gas coning (Hasan et al.,

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2013) and in mature shale gas wells (Knudsen and Foss, 2013). Second, it is critical to account for the production network, i.e. manifolds, pipelines and processing system, since production bottlenecks normally is found anywhere in the system.

The lowest level comprises fast decisions that are tackled by automatic control systems. This includes control logic and conventional control for flow, pressure and level control functions, and it is normally implemented in proprietary systems.

DPO will be the focus of this paper even though reservoir optimization also will be discussed. To elaborate on the former, there is both a research based and business-wise motivation for this paper. The DPO problem is hard to solve, particularly in mature fields due to lowered reservoir pressure, and higher gas rates (for oil fields) or water rates. From a business perspective it is challenging to assess the value of applying mathematical optimization compared a baseline with skilled production engineers and operators. There are, however, industry-based claims that a production increase in the range from 1-4% is obtainable (Stenhouse et al., 2010; Teixeira et al., 2013), which is an indication that optimization can indeed make a difference.

This paper contributes to *formulations* for DPO and reservoir optimization problems, i.e. how these problems can be formulated in ways that ensure efficient and predictable solutions. We are in particular interested in exploiting structure as a means to achieve this. Exploiting structure implies the use of a divide and conquer strategy, an approach that is used extensively to solve complex problems. The paper continues with a formulation of the DPO problem before entering into a discussion on how this problem normally is solved. Subsequently, some alternative divide and conquer strategies are explored. A synthetic, albeit realistic, offshore case is used to substantiate the arguments. Finally, benefits and limitations of the proposed formulations as well as other ways of introducing structure are discussed, before some conclusions end the paper.

The paper will have a bias towards offshore oil and gas fields, and to a lesser extent refer to onshore developments or oil sand exploration. Further, it will be biased towards production optimization research at the IO Center at NTNU, research that is performed in cooperation with six oil companies and three engineering companies. The paper will not deal with non-technical issues, such as work flows and incentive systems, which are important ingredients in order to spread the use of mathematical optimization.

2. DPO PROBLEM DESCRIPTION

Consider the subsea production system illustrated in Fig. 2, which consists of reservoirs, wells, manifolds, subsea processing, flowlines, risers and separators. Well geometries may be complicated with several branches connected to one well outlet, and the perforated parts may be highly deviated or even horizontal. On the seabed streams are commingled, and in some instances some processing is done. This is exemplified through a multiphase pump that energizes the commingled well stream from low pressured (weak) wells. Two pipelines leave the second manifold, and each of the wells that are directly connected to this manifold, as well as the flowline from the first manifold, will be connected to *one* of these flowlines. The second manifold connects to two inlet separators, which are placed on a floating unit, through a flowline including a riser. The downstream system boundary is given by the inlet separators in the figure. Downstream this boundary there will be further processing equipment before products are exported through pipelines or ships. In this paper we will mainly be concerned with production systems where the upstream and downstream boundaries are placed in the reservoir and the inlet separator, respectively.

Table 1. Utility sets

Set	Description
$\mathbf{N}^{\mathrm{d}}$	Nodes with discrete leaving edges, i.e. $\mathbf{N}^{\mathrm{d}} = \{i : i \in \mathbf{N}, \mathbf{E}_i^{\mathrm{out}} \subset \mathbf{E}^{\mathrm{d}}\} \subset \mathbf{N}$.
$\mathbf{E}_i^{\mathrm{in}}$	Edges entering node i, i.e. $\mathbf{E}_i^{\mathrm{in}} = \{e : e = (j,i) \in \mathbf{E}\}$.
$\mathbf{E}_i^{\mathrm{out}}$	Edges leaving node i, i.e. $\mathbf{E}_i^{\mathrm{out}} = \{e : e = (i,j) \in \mathbf{E}\}$.
$\mathbf{E}^{\mathrm{snk}}$	Edges entering a sink node, i.e. $\mathbf{E}^{\mathrm{snk}} = \{e : e = (i,j), \mathbf{E}_j^{\mathrm{out}} = \emptyset\}$.

We can now formulate a fairly general network optmization problem. First, the topology can be represented by a directed graph $G = (\mathbf{N}, \mathbf{E})$, with nodes $\mathbf{N}$ and edges $\mathbf{E}$ (Ahuja et al., 1993). In the sequel we adopt the notation in Grimstad et al. (2015). There are three mutually exclusive sets of nodes, $\mathbf{N}$, which all represent a junction: source nodes ($\mathbf{N}^{\mathrm{src}}$), sink nodes ($\mathbf{N}^{\mathrm{snk}}$) and intermediate nodes ($\mathbf{N}^{\mathrm{int}}$). An edge $\mathbf{E}$ connects two nodes and represents a pipe segment such as a well or a flowline, a valve, or an active element like a pump. A subset of edges, $\mathbf{E}^{\mathrm{d}}$, represents the on/off valves. These edges have two states: either open or closed. Thus, discrete edges are used to route the flow through the network by restricting the flow through the valve. All other edges ($\mathbf{E} \backslash \mathbf{E}^{\mathrm{d}}$) represent pipes. It is advangeous to define certain utility sets, and certain requirements need to be placed on the graph structure, cf. Grimstad et al. (2015). Some utility sets are defined in Table 1 in order to compactify notation.

Table 2. Variables

Variable	Description
p_i	Pressure at node $i \in \mathbf{N}$.
Δp_e	Pressure drop over edge $e = (i,j) \in \mathbf{E}$, e.g. $\Delta p_e = p_i - p_j$.
$q_{e,r}$	Flow rate of phase $r \in \mathbf{R}$ on edge $e \in \mathbf{E}$.
y_e	Binary variable associated with an edge $e \in \mathbf{E}^{\mathrm{d}}$. The edge may be open ($y_e = 1$) or closed ($y_e = 0$).

Second, variables are considered. For three-phase flow the phases are $\mathbf{R} = \{\mathrm{oil}, \mathrm{gas}, \mathrm{wat}\}$, denoting the hydrocarbon liquid phase, the hydrocarbon gas phase, and the water liquid phase, respectively. Alternative formulations like compositional models may, however, be necessary. The variables of the problem, listed in Table 2, are given as mass flow rates or as volumetric flow rates in standard conditions. For the sake of compactness, the phase flow rates on an edge $e \in \mathbf{E}$ are collectively denoted $\mathbf{q}_e$, that is, with an oil, gas, and water phase, $\mathbf{q}_e = [q_{e,\mathrm{oil}}, q_{e,\mathrm{gas}}, q_{e,\mathrm{wat}}]^{\mathsf{T}}$. Further, we denote all the flow rates, pressures, and pressure drops in the network with $\mathbf{q}$, $\mathbf{p}$, and $\mathbf{\Delta p}$, respectively.

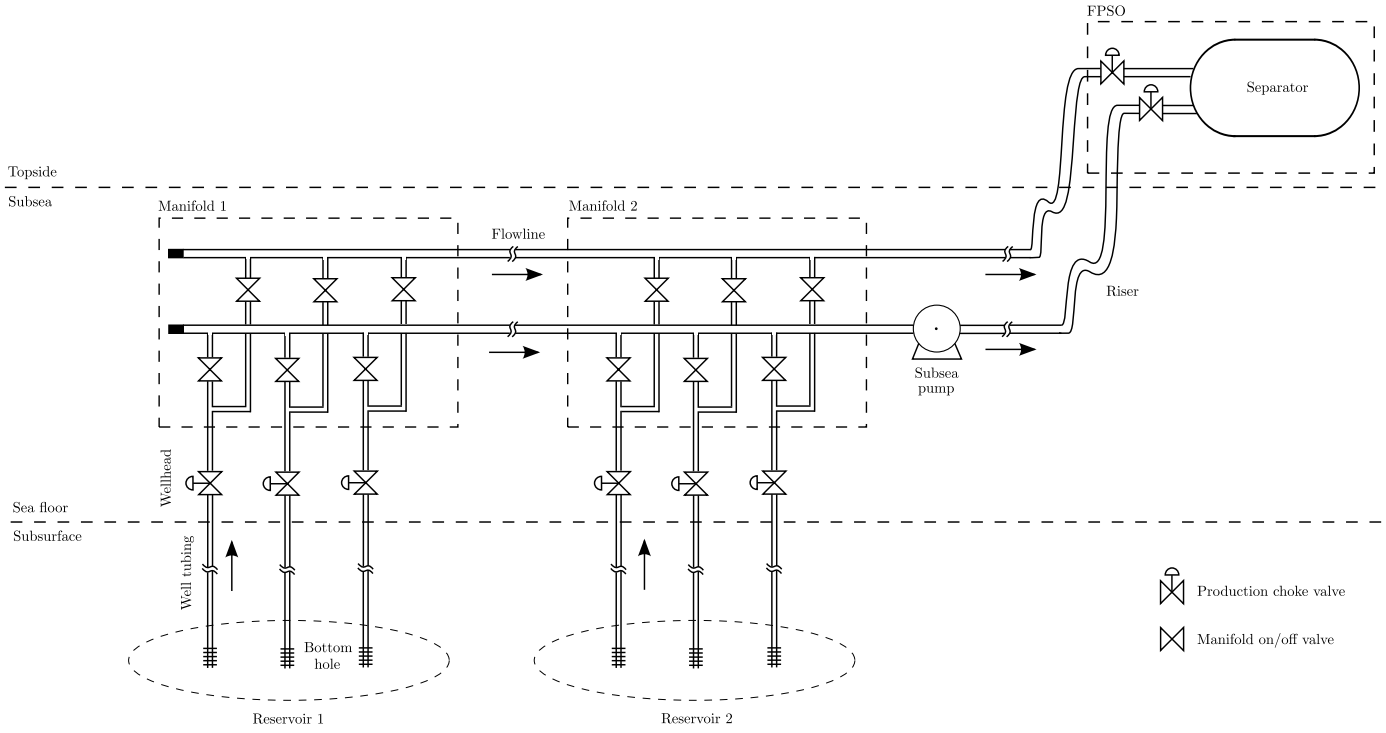

Fig. 2. A subsea production system

Third, the DPO problem is posed as the following mixed-integer nonlinear programming (MINLP) problem, denoted **P**, before commenting on its different parts:

$$\underset{\mathbf{y},\mathbf{q},\mathbf{p},\mathbf{\Delta p}}{\text{maximize}} \quad z = \sum_{e\in\mathbf{E}^{\text{snk}}} q_{e,\text{oil}} \tag{1}$$

subject to

$$\sum_{e\in\mathbf{E}_i^{\text{in}}} q_{e,r} - \sum_{e\in\mathbf{E}_i^{\text{out}}} q_{e,r} = 0, \qquad \forall r\in\mathbf{R}, i\in\mathbf{N}^{\text{int}} \tag{2}$$

$$\zeta_{i,r}(\mathbf{q}_e, p_i) = 0, \qquad \forall r\in\mathbf{R}, i\in\mathbf{N}^{\text{src}} \tag{3}$$

$$p_i = \text{const.}, \qquad \forall i\in\mathbf{N}^{\text{snk}} \tag{4}$$

$$\Delta p_e = f_e(\mathbf{q}_e, p_i), \qquad \forall e\in\mathbf{E}\setminus\mathbf{E}^{\text{d}} \tag{5}$$

$$\Delta p_e = p_i - p_j, \qquad \forall e\in\mathbf{E}\setminus\mathbf{E}^{\text{d}} \tag{6}$$

$$\begin{aligned} -M_e(1-y_e) &\le p_i - p_j - \Delta p_e, \\ M_e(1-y_e) &\ge p_i - p_j - \Delta p_e, \end{aligned} \qquad \forall e\in\mathbf{E}^{\text{d}} \tag{7}$$

$$\sum_{e\in\mathbf{E}_i^{\text{out}}} y_e \le 1, \qquad \forall i\in\mathbf{N}^{\text{d}} \tag{8}$$

$$y_e q_{e,r}^L \le q_{e,r} \le y_e q_{e,r}^U, \qquad \forall r\in\mathbf{R}, e\in\mathbf{E}^{\text{d}} \tag{9}$$

$$q_{e,r}^L \le q_{e,r} \le q_{e,r}^U, \qquad \forall r\in\mathbf{R}, e\in\mathbf{E}\setminus\mathbf{E}^{\text{d}} \tag{10}$$

$$p_i^L \le p_i \le p_i^U, \qquad \forall i\in\mathbf{N} \tag{11}$$

$$y_e \in \{0,1\}, \qquad \forall e\in\mathbf{E}^{\text{d}} \tag{12}$$

$$\sum_{e\in\mathbf{E}^{\text{snk}}} q_{e,\text{gas}} \le C_{\text{gas}}, \tag{13}$$

$$\sum_{e\in\mathbf{E}^{\text{snk}}} q_{e,\text{wat}} \le C_{\text{wat}}. \tag{14}$$

The objective function (1), total oil production, is a fairly general goal in operational settings even though other elements, in particular cost, could be included.

Constraint (2) is linked to mass balances for each phase, and included for all the internal nodes in the system. The upstream and downstream boundary conditions are covered by constraints (3) and (4). The former inflow condition may be linear or nonlinear depending on the actual well and describes the mass flowrate from the reservoir into the well. Common models are linear productivity index models and nonlinear Vogel curves. Downstream conditions may vary. However, a constant separator pressure is quite common and is adopted here.

Equations (5) and (6) defines the momentum balance, or pressure drop, across pipe segments. These are modelled as a function of inlet flow rates and inlet pressure. For a discrete edge the momentum balance must be deactivated when it is closed. This is modelled through a big-M constraint as shown in (7). Equation (8) simply states that no more than one discrete edge leaving a node can be open at a time, i.e. a well stream can at most be routed to one flowline.

Constraints (9) and (10) limit the flowrate in edges through upper and lower bounds. These bounds will, however, always be zero for closed discrete edges since $y_e = 0$ in that case. This implies that the domain of the nonlinear function $f_e(\cdot)$ should include $\mathbf{q}_e = \mathbf{0}$; otherwise, $y_e = 0 \implies \mathbf{q}_e = \mathbf{0}$ is infeasible. Constraint (11) limits the pressure in a node. A typical limitation will be a lower bound on the bottomhole pressure in a well, which is imposed to prevent damage to the well or near-well reservoir.

The two final constraints are linked to capacity constraints for handling gas and water.

To summarize, the DPO problem as defined here is about solving **P** in a robust and predictable way. The next three section discusses this.

3. A NON-INVASIVE OPTIMIZATION APPROACH

We will start to outline a solution method for DPO by describing a standard approach where the production network simulator is treated as one black box, i.e. only allowing the optimization algorithm to interrogate the simulator by requesting outputs based on a set of inputs as shown in the upper part of Fig. 3. This is referred to as a *black-box* method. Referring to the DPO problem this implies that equations (2)-(5) are embedded in a simulator. A simulator may in some cases provide gradients in addition to normal simulator outputs. This outset facilitates a range of options. First, the use of a proxy model for optimization could be conceivable in which the proxy models mimic the simulator's input output mapping. This is however practically infeasible for production optimization problems due to the number of decision variables, which typically runs into several tens and sometimes more. Second, a derivative free approach may be used. Nonetheless, limitations as to the dimension of the decision vector remain, a fact that limits the applicability of derivative free methods. If applying such an approach the Mesh-Adaptive Direct Search algorithm is a reasonable choice since it does not rely on building a local model at each new iteration point. It is designed for difficult black box problems such as very nonsmooth problems and hidden constraints. To elaborate on the latter, the optimizer may investigate a wide range of operating points in its search for an optimum. Some of these points may lie in a region where the simulator experiences convergence issues, or undefined and erroneous behaviour. The optimizer may venture to these regions when **P** lacks constraints that are hidden in the simulator. One immediate example is the case where the domain of the nonlinear function $f_e(\cdot)$ should include $\mathbf{q}_e = \mathbf{0}$, as discussed in the previous section. For more information on derivative free methods we refer to Conn et al. (2009).

4. STRUCTURE EXPLOITATION IN NETWORK OPTIMIZATION

The DPO problems clearly exhibits structure due to the topology of the layout of pipes and components. Scrutinizing **P** and recalling that it can be described by a directed graph with nodes and edges in which nodes relate to physical components, all nonlinearities are confined to component models while the couplings between them are given by linear relationships. Further, discrete routing decisions are confined to couplings only, i.e. there are no discrete decision variables in the components themselves.

We will now explore the option of decomposing the simulator into parts as a means to streamline optimization. Decomposition facilitates the use of surrogate models as an alternative to simulators, since the input dimension of the component model will be much lower than for the complete network model. As an example the input dimension to pressure drop models (5) is four. Thus, the use of surrogate models is indeed feasible and thereby enabling the use alternative model formats. We will in particular be interested in basis functions with compact support, i.e. they are zero outside of a compact set, such as piecewise linear models (PWL) and B-spline representations. The choice of component models rests on several considerations.

First, the accuracy and trust region of a model is important. High accuracy on a large input set requires more complex proxy models, for instance more break points for PWL models, than high accuracy on a smaller input set. Bearing in mind that proxies are generated from simulators there is a trade off since proxies may be updated or extended during an optimization run, i.e. we may start off with proxies that are accurate on limited sets and subsequently update them based on the progression of the optimization run. Alternatively, more complex proxies may be generated prior to an optimization run with no subsequent change in these.

Second, severe nonlinearities may occur in network optimization, a typical example is pressure drop multiphase flowlines when flow patterns change. Such nonlinearities, which even may be discontinuous, always hamper optimization. Thus, there exists a trade off between accuracy and smoothness when generating proxies. This is easier handled with small component models rather than for large network models.

Third, the network optimization problem is transformed into an MILP for PWL models. This is a distinct advantage since MILP solvers is a mature technology where solutions come with a quality certificate. The downside is that an accurate representation may require an overly high dimensional MILP. Returning to nonlinear component models leave us with an MINLP. However, special purpose algorithms may be exploited for specific classes of nonlinear functions as will be shown in a later example with B-spline functions.

Fourth, the component models may take on different forms within one application. Some parts may be represented by a simulator, while others are pre-sampled and available through tables, and yet other through surrogate models.

Fifth, communication between an optimizer and simulators, i.e. I/O operations, may be time consuming. This problem is not present for surrogate models. Thus, this perspective favors proxy models over simulators.

Proxy models also have the favourable property that they are analytic functions with gradients. As mentioned, this is not always the case for simulators.

Splitting the network into parts makes the coupling variables readily available and in particular it is straightforward to include bounds on these variables since this amounts to linear inequality constraints that hardly affects the run time of the algorithm. As a contrast, such variables will appear as nonlinear constraints in the standard approach. The penalty for this availability is of course a large increase in the number of optimization variables.

An important advantage of using the decomposed model is the fact that constraints can be relaxed during an optimization run, which is particularly important for the equality constraints on the boundary between nodes. This

means that an MINLP algorithm can relax all decision variables, in particular the discrete variables related to routing that appear on the boundaries between components.

Splitting the network into parts has two negative implications. First, as alluded earlier, the number of optimization variables increases substantially. It is important to note, however, that the decomposed approach does not introduce any extra integer variables. Second, the communication between the optimizer and models is more extensive than for the standard approach, since information about the structure must be communicated (once) – while traditionally, communication is limited to the request and response for evaluating the simulator at a point.

5. EXAMPLE

In this example we test four approaches to production optimization. This includes the black-box approach and three variations that exploit the network structure, see Table 3. To compare the approaches we consider the maximization of oil production from the system depicted in Fig. 2. The system is modelled with realistic parameters for the wells and flowlines, and the incidence matrix is shown in an Appendix.

Table 3. Optimization approaches

Approach	Structural information	Analytical derivatives	Routing
Black-box	No	No	No*
Grey-box	Yes	No**	Yes
White-box	Yes	Yes	Yes
Surrogate	Yes	Yes	Yes

* Exhaustive search is required
** Constraints related to structure may have derivatives

Here, the black-box method refers to the traditional use of a gradient-based solver, where gradients are computed by finite differences. The manipulated/input variables are the production choke pressure drops for each well and the desired pressure rise over the pump. With this approach, routing is not handled by the solver, that is, the well routings must be fixed. The output variables are related to the objective and production capacity constraints, which become nonlinear functions in the input variables. No structural information is used in this approach.

The *gray-box* and *white-box* approaches exploit structural information, allowing the solver to include routing variables and relax them during solving. In the white-box approach all information is available and analytical derivatives are supplied to the solver. This approach is usually not a realistic option since simulator models are implemented with advanced computer codes that occlude for analytic derivatives.

Finally, in the *surrogate* approach structural information is utilized and unknown functions are approximated with cubic B-spline surfaces, as illustrated in the lower part of Fig. 3. The B-spline surfaces are smooth functions with analytical derivatives readily available for the solver.

The nonconvex problems that result from the approaches in Table 3 are solved to local optimality. The black-box approach employs Ipopt (Wächter and Biegler, 2006) and an integer heuristic that exhaustively tries all routing-feasible routing combinations (there are 4096 routing combination, 729 being routing settings that are feasible according to (8)). In the other three approaches Bonmin (Bonami et al., 2008) is used as a heuristic, which may accelerate the search by (sometimes wrongfully) disregarding branches with poor routing combinations. In the black- and grey-box approach, finite differences are used to compute derivatives. The results are recorded in Table 4.

Table 4. Results

Approach	Precomp. time (sec)	Run time (sec)	Optimality gap (%)
Black-box	0	301	0
Grey-box	0	114	0
White-box	0	40	0
Surrogate	24	42	0

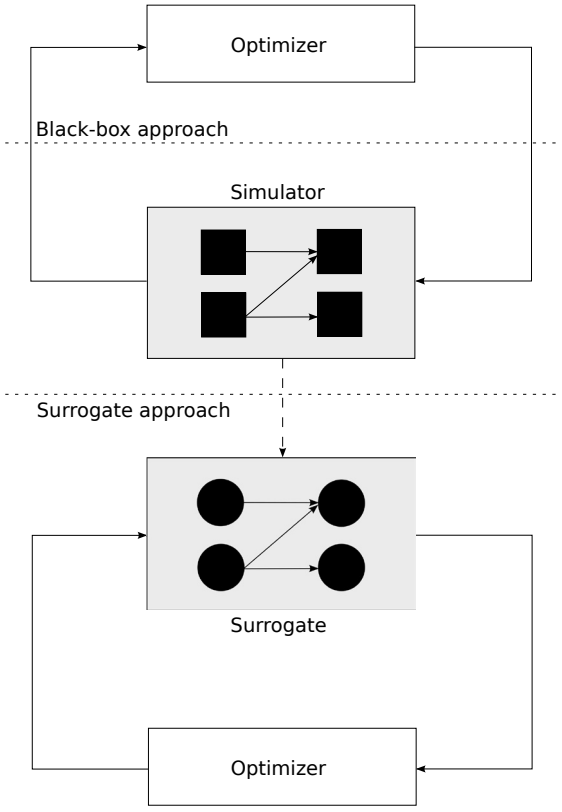

Fig. 3. Illustration of the black-box and the surrogate optimization approaches

All approaches are able to close the optimality gap in this particular case (the global optimum was certified by a global solver). The results are clearly in favor of exploiting structure. As to the three divide and conquer strategies we may first recall that the white-box approach is an unrealistic option in most cases. Left with the grey-box and surrogate methods, the surrogate approach is the preferred option in this example. The surrogate model requires an additional 24 seconds for sampling and building the surrogate models. This is however only done once.

Delays caused by I/O operations were not considered in Table 4. We believe the introduction of I/O time delays would favor the surrogate method. To substantiate this, in our experience with black-box optimization on real field cases the distribution of total solution time typically has been: simulation 90%, I/O operations 5%, and time spent in the optimizer 5%.

6. DISCUSSION

This discussion section will first address the issues investigated above before extending it to loosely coupled networks and subsequently to the reservoir optimization domain.

6.1 DPO problems

The rationale for applying a divide and conquer strategy is, not unexpectedly, substantiated through the test case. Thus, there are distinct advantages using such an approach. This is not surprising since a structure exploitation strategy embeds more system knowledge into the formulation of the optimization problem. Thus, if one is able to utilize this in a meaningful manner the expectation is improved performance. The non-invasive, or black-box, approach has the advantange that implementation is easier, in particular in cases where the network model is implemented in one simulator.

A key decision rests with the use of simulators or surrogate models in optimizers that exploit structure. There are essentially three options; use the component simulators as they are, create surrogate models offline, or a mix in which the surrogate models are updated when the optimizer moves outside the trust region of a component model as proposed in Gunnerud et al. (2013). It is worth noting that there are always hazards when running a simulator. This includes communication issues, numerical convergence problems and error messages. Thus, a reasonable strategy is indeed to avoid simulator evaluations altogether during an optimization run.

The surrogate approach necessitates the use of a flexible approximation scheme, in particular the ability to vary accuracy in different parts of the operating envelope for each component model, to reduce the need for offline evaluations. This calls for approximation schemes with basis functions with compact support, of which PWL and B-splines are two apparent choices. PWL allows for the use of powerful MILP solvers. The size of the optimization problem however may however limit the use of PWL. An alternative is the use of nonlinear basis functions with a dedicated solver. Our experience with B-splines coupled to a tailored, global MINLP solver are excellent (Grimstad and Sandnes, 2014). This solver, CENSO, accepts nonlinear constraints based on B-splines. The importance of applying a global solver is debatable since a local solution usually suffices given that the networks models are approximations of reality in the first place.

The network formulation allows for nonlinearities in the nodes only. This is no limitation since a nonlinear coupling, i.e. a nonlinear edge, between two nodes can be removed by splitting the edge into two (linear) edges and one node, and confine the nonlinearities to the node only.

The divide and conquer formulation assumes that discrete decisions are constrained to the edges only. This limits the use of discrete variables in the nodes, an option which can be of interest when modelling for instance discontinuities. However, as alluded earlier, smoothness in the (nonlinear) nodes is important to secure robust and reliable solvers. This is the same problem we obtain when trying to optimize discrete variables as inputs to black-box models; the model is then discontinuous and thus optimization becomes challenging. In the case of PWL models the binary variables are also part of the model. Since the model is analytical, however, it is possible to treat the binary variables as continuous variables while solving – thus removing the discontinuities.

Problem **P** is not without challenges and it cannot be applied to all situations. First, cases where different oil compositions are produced together present a definite challenge. In such cases it may be necessary to resort to a compositional fluid model. This increases the input dimension of (5) and the need to include specific thermodynamic models to allow for phenomena like flashing. These facts complicate the development of proxy models. Extensions of **P** are, however, practically feasible. As an example, Grimstad et al. (2015) extended **P** with energy models to model temperature to include gas velocity constraint in a riser, and applied surrogate models based on B-splines successfully, even with a global solver.

Second, the discussion so far has treated DPO as a static problem. Thus, the approach is to re-optimize as frequently as necessary and implement the solution in a control system operated by the production team. This strategy relies on well-tuned controllers such that the recommended operating point is swiftly reached. To elaborate, the control logic on the lowest level in Fig. 1 needs to be well tuned. The dynamics of the production system is typically much faster than the frequency with which the DPO is performed. Normally the dominant dynamics of the production systems is less than one hour while DPO is performed once or a couple of times a day. In this context optimization on a static formulation makes sense since it is much simpler and less error prone than a dynamic optimization strategy like Model Predictive Control. There are, however, exceptions. One case occurs when reservoir dynamics are fast, as in oil reservoir with severe gas coning or mature shale gas wells. By fast reservoir dynamics we mean dynamics with dominant time constants in the hours to one day region. In such cases dynamic optimization does indeed make sense. Two other situations where dynamic optimization is an option are when lengthy procedures like well startup occur and in production systems with very long flowlines. One example of the latter is offshore gas condensate production connected through a long flowline to an onshore LNG plant. In Statoil's Snøhvit field offhore northern Norway the dominant dynamics of the pipeline alone is 8 hours.

Problem **P** does encompass systems with subsea processing including equipment like subsea separators and pumps, i.e. active components in addition to valves and manifolds. Thus, the approach is applicable in these situations provided static models suffices and the input dimension to models like (5) is limited. The same hold for downstream extension to topside facility components.

6.2 Higher level decomposition

DPO problems can be large, and may be composed of several subsystems, as in Fig. 2, where the couplings between these subsystems are weak. A typical example is systems where the subsystems are connected only through downstream separator pressures and gas and water handling constraints as defined by (4), (13) and (14) in **P**. This structure paves the way for decomposition techniques.

As a means of illustration we inspect Fig. 3 where there are close links between manifold 1 and 2 while the coupling to manifold 3 is much weaker. If we assume pressure control on both inlet separators, which is the normal case, the only coupling from an optimization point of view are constraints in the processing facilities, i.e. gas handling and water handling constraints. Such optimization structures invite the use of some decomposition technique since there are only a few global constraints, in this case Lagrangian relaxation or Dantzig Wolfe decomposition are appropriate methods. The general idea is to relax global constraints, and thereby decompose the DPO problem into smaller sub-problems, which are coordinated by a "master" problem. An iterative procedure is then used to achieve convergence towards a solution. An efficient solution of the sub-problems, which may be carried out in parallel, is a key. Thus, it is important to apply structure exploitation as discussed in Section 4 to each sub-problem.

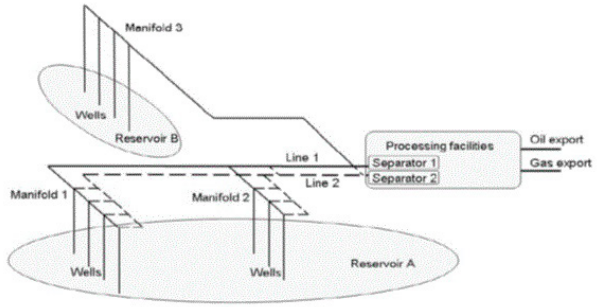

Fig. 3. Topology for a typical upstream production system.

Fig. 4. Production system composed of two weakly coupled subsystems

An application of the above approach is the DPO problem presented in Gunnerud and Foss (2010) for the Troll oil field on the Norwegian continental shelf. Each subsystem was modelled using PWL models and thus each sub-problem amounted to an MILP. Results indicated a substantial runtime reduction by applying Lagrangian relaxation or Dantzig Wolfe decomposition as compared to a centralized approach, i.e. solving the complete problem as one large MILP. Runtime was reduced by 10 to 100 times for the complete system comprised of more than 50 wells. These results were achieved without taking advantage of parallelizing the solution of the sub-problems. As a side comment it may be added that Dantzig Wolfe decomposition was deemed the preferred method because of a more efficient updating algorithm for the Lagrange variables.

6.3 Decomposition in the reservoir optimization domain

This paper has focussed on the DPO problem. However, it is appropriate to include some remarks on reservoir optimization, i.e. level 2 in Fig. 1. This is a dynamic optimization problem where some economic measure is maximized on a prediction horizon, see Jansen et al. (2008) and references therein for more details.

The reservoir optimization problem is normally solved by a single shooting approach in which an optimization algorithm communicates with a simulator that provides state variable values and sensitivities, or gradients, on the prediction horizon in question. Since high fidelity reservoir models are large constructs with state dimension running into the thousands or even millions, efficient gradient calculation is critical. Gradients are thus either computed by an adjoint method or approximated through an ensemble based method (Jansen et al., 2008; Chen et al., 2009). A reservoir model rarely has internal structure that can be exploited analogeous to the DPO problem. One exception is compartmentalized reservoirs, which are not treated here. There does, however, exist an opportunity to decompose the reservoir optimization problem anyway; along the time axis. This is a feature that is exploited in multiple shooting, a concept that is well known from the Model Predictive Control literature (Diehl et al., 2002). The idea is to divide the prediction horizon into intervals and add continuity constraints on the state variables where the intervals connect. This obviously introduces a large number of new constraints, which seems like an intractable option in most cases. By using a reduced space approach, however, this is indeed feasible as shown in a recent publication (Codas et al., 2015) where it is applied to a two phase reference case with 18500 active cells and 12 wells. It should be noted that the multiple shooting algorithm relies on the fact the continuity constraints need only be satisfied when the multiple shooting algorithm converges to its solution.

There are some interesting advantages by decomposition according to the multiple shooting approach. First, since the state variables are treated as independent variables, one may easily enforce bounds on them. This is in contrary to the single shooting approach in which state variable constraints will appear as nonlinear constraints. Such constraints are usually handled by dualizing the nonlinear constraints by applying an Augmented Lagrangian method. Second, decomposition facilitates parallelization since the simulators for each time interval can be run simultaneously. Further, parallel computation can also be employed at the linear solver level. An obvious downside of the multiple shooting approach is the large increase in independent variables.

7. CONCLUSIONS

This paper recommends the use of formulations that exploit structure for production optimization in oil and gas systems as compared to a black-box approach. Structure exploitation improves efficiency and reliability of the solver and increases flexibility in the formulation of the optimization problem. Among the divide and conquer strategies the surrogate approach where local models use basis functions with compact support is a preferable alternative in some cases.

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9. APPENDIX

The incidence matrix S of the network considered in Sec. 5 is illustrated below. The matrix is in $\mathbb{R}^{|\mathbf{N}|\times|\mathbf{E}|}$ and specifies the couplings in the network. Its structure is

$$S = \begin{bmatrix} -I & 0 & 0 & 0 \\ I & -I & -I & 0 \\ 0 & A & B & C \end{bmatrix} \in \mathbb{R}^{18\times 23} \tag{15}$$

where I is the identity matrix, $I, A, B \in \mathbb{R}^{6\times 6}$, and $C \in \mathbb{R}^{6\times 5}$. S has 46 nonzeros elements and 368 zeros. The sparse structure illustrates that there are potentially many zeros in an input-output map of a production system network. Many gradient calculations can be saved by exploiting this sparsity.

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