Mechanical Design

Prof. Eduardo Camponogara

The four-bar mechanical system is testbed for models and algorithms for designing mechanical systems, such as robotic arms. The four-bar mechanical system is depicted in Figure 1. Given a set of trajectory points $\mathcal{P} = \{(\widehat{x}_{\mathrm{M},i},\, \widehat{y}_{\mathrm{M},i})\}_{i=1}^{I}$, we wish to design the anchor points $\mathrm{O} = (x_{\mathrm{O}}, y_{\mathrm{O}})$ and $\mathrm{E} = (x_{\mathrm{E}}, y_{\mathrm{E}})$, the lengths a, b, c, d, e, and e, and a set $\Phi = \{\varphi_1, \varphi_2, \ldots, \varphi_i\}$ of angles such that the i^{th} coupler position $(x_{\mathrm{M},i},\, y_{\mathrm{M},i})$ is as close as possible to the desired trajectory position.

By using Cartesian coordinates, the problem mechanical design problem can be framed as an MINLP in which the nonlinear terms are all bilinear. For point A, for instance,

$$
\begin{aligned}
x_{\mathrm{A}} &= a \cdot c & s^2 + c^2 &= 1 \\
y_{\mathrm{A}} &= a \cdot s & s, c &\in [-1, 1]
\end{aligned}
$$

in which $s = \sin(\varphi)$ and $c = \cos(\varphi)$.

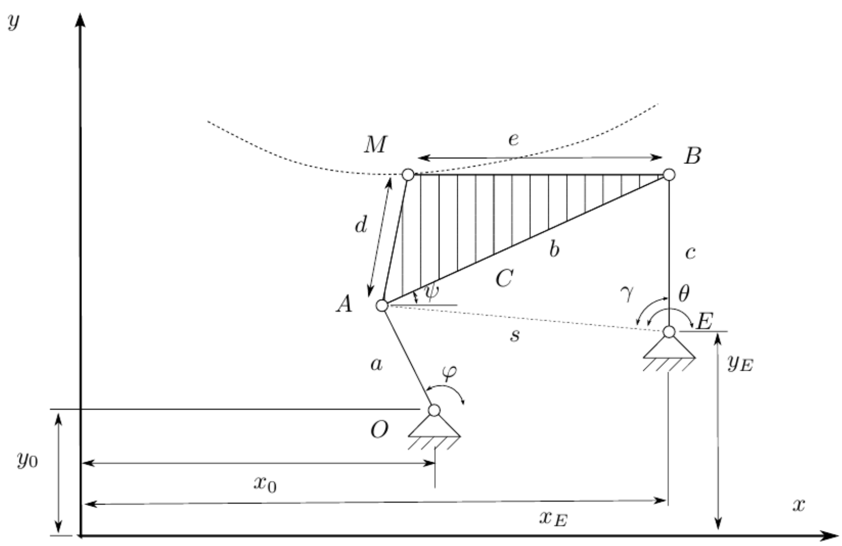

Figure 1: Four-bar mechanism

A simplified version of the mechanical design problem is given below:

$$P: \min \sum_{i=1}^{I} \left[(x_{\mathrm{M},i} - \widehat{x}_{\mathrm{M},i})^2 + (y_{\mathrm{M},i} - \widehat{y}_{\mathrm{M},i})^2\right] \tag{1a}$$

$$\text{s.t.} : \text{for } i = 1, \ldots, I :$$

$$\begin{cases} x_{\mathrm{A},i} - x_{\mathrm{O}} = a \cdot c_i \\ y_{\mathrm{A},i} - y_{\mathrm{O}} = a \cdot s_i \\ c_i^2 + s_i^2 = 1 \\ (x_{\mathrm{A},i} - x_{\mathrm{O}})^2 + (y_{\mathrm{A},i} - y_{\mathrm{O}})^2 = a^2 \\ (x_{\mathrm{A},i} - x_{\mathrm{B},i})^2 + (y_{\mathrm{A},i} - y_{\mathrm{B},i})^2 = b^2 \\ (x_{\mathrm{B},i} - x_{\mathrm{E}})^2 + (y_{\mathrm{B},i} - y_{\mathrm{E}})^2 = c^2 \\ (x_{\mathrm{M},i} - x_{\mathrm{A},i})^2 + (y_{\mathrm{M},i} - y_{\mathrm{A},i})^2 = d^2 \\ (x_{\mathrm{M},i} - x_{\mathrm{B},i})^2 + (y_{\mathrm{M},i} - y_{\mathrm{B},i})^2 = e^2 \end{cases} \tag{1b}$$

$$(x_{\mathrm{E}} - x_{\mathrm{O}})^2 + (y_{\mathrm{E}} - y_{\mathrm{O}})^2 = OE^2 \tag{1c}$$

$$c_i,\ s_i \in [-1, 1],\ i \in \mathcal{I} \tag{1d}$$

$$u^{\min} \leq u \leq u^{\max},\ u \in \mathcal{U} \tag{1e}$$

$$\begin{cases} x_{p,i}^{\min} \leq x_{p,i} \leq x_{p,i}^{\max}, \\ y_{p,i}^{\min} \leq y_{p,i} \leq y_{p,i}^{\max}, \end{cases} \quad p \in \mathcal{P},\ i \in \mathcal{I} \tag{1f}$$

$$\begin{cases} x_{\mathrm{O}}^{\min} \leq x_{\mathrm{O}} \leq x_{\mathrm{O}}^{\max} \\ y_{\mathrm{O}}^{\min} \leq y_{\mathrm{O}} \leq y_{\mathrm{O}}^{\max} \end{cases} \tag{1g}$$

$$\begin{cases} x_{\mathrm{E}}^{\min} \leq x_{\mathrm{E}} \leq x_{\mathrm{E}}^{\max} \\ y_{\mathrm{E}}^{\min} \leq y_{\mathrm{E}} \leq y_{\mathrm{E}}^{\max} \end{cases} \tag{1h}$$

in which:

- $\mathcal{I} = \{1, \ldots, I\}$ is the set of indices of the position angles;
- $\mathcal{P} = \{\mathrm{A, B, M, AM, AB, EA, EB}\}$ is a set of points and vectors;
- $\{(\widehat{x}_{\mathrm{M},i}, \widehat{y}_{\mathrm{M},i})\}_{i \in \mathcal{I}}$ is the set of positions that the coupler should reach;
- $\mathcal{U} = \{a, b, c, d, e, OE\}$.

Tasks:

1. Model the problem above in AMPL and solve it, optimally or approximately, using a MINLP solver such as Baron or Couenne.
2. Propose a MILP approximation formulation by modeling all bilinear terms as two-dimensional piecewise-linear functions. You should identify a suitable range for the domain of the function, and the number of breakpoints.
3. Implement the MILP approximation in AMPL.
4. Perform experiments using a MILP solver such as CPLEX or Gurobi, evaluating the computational cost and quality of approximation as a function of the number of breakpoints. Parameters are given in the file "fourbar25.dat". However, there may be some parameters missing, and it is up to the group to suggest appropriate parameters.

Remarks: This is an open assignment. The groups should figure out suitable function domains and parameters that produce meaningful results. Experimentation and trial-and-error will be part of the investigation.