

Tutorial AMPL

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Summary

Linear Programming

Duality

LP – Example 3

AMPL Model:

- ▶ Consider a factory that produces N products (n) in a serial pipeline.
- ▶ For each product n , a profit p_n is made on each unit sold, there is a weekly demand d_n and a throughput of r_n units per hour.
- ▶ We wish to determine the weekly production x_n , for each product n , so as to maximize the total profit in sales, given that the factory has a total number of T hours per week.
- ▶ Model the problem in linear programming.

LP – Example 3

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- ▶ **Model the problem in linear programming.**

LP – Example 3

Linear programming model:

$$\begin{aligned} \max \quad & \sum_{n=1}^N p_n \cdot x_n \\ \text{s.t. :} \quad & \sum_{n=1}^N \frac{1}{r_n} \cdot x_n \leq T \\ & 0 \leq x_n \leq d_n, \quad n = 1 \dots N \end{aligned}$$

LP – Example 3

- ▶ Complete the AMPL model **example3.mod** according with your mathematical programming model.

```
# Part 1 – Variable Declaration (var, set, param, etc)
param N;
param T;
set Products := {1..N};
set InfoType := {p,r,d};
param data {Products,InfoType};
```

- ▶ Create a file **example3.run**.
- ▶ Use the file **example3.dat**. To this end, just issue the command “**data example3.dat;**” after the command that includes the .mod file in **example3.run**.

LP – Example 3

example3.dat:

```
param N := 4; #Number of products  
param T := 40; #Number of working hours per week  
# Data Matrix  
param data_base:  
p r d :=  
1 1 40 1000  
2 1.5 30 900  
3 1 50 500  
4 1.5 20 800  
;
```


LP – Example 3

example3.mod:

```
# Part 1: Variable Declaration (var, set, param, etc)
param N;
param T;
set Products := {1..N};
set InfoType := {'p','r','d'};
param data_base{Produto,InfoType};
var x{Products} >= 0;
# Part 2: Objective Function
maximize Profit: sum{n in Products} data_base[n,'p']*x[n];
# Part 3: Constraints
subject to available_working_hours:
    sum{n in Products} (1/data_base[n,'r'])*x[n] <= T;
subject to demand{n in Products}: x[n] <= data_base[n,'d'];
```

LP – Example 3

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LP – Example 3

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```

LP – Example 3

example3.run:

```
# Reset Memory
reset ;
# Load Memory
model example3.mod;
# Load Data
data exemple3.dat;
# Change Configurations (optional)
option solver cplex;
# Solve Problem
solve;
# Show Results
display x;
```

LP – Example 3

example3.run:

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LP – Example 3

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```

Dual – Example 4

Obtain the dual model in linear programming for Example 3

$$\begin{aligned} \max \quad & \sum_{n=1}^N p_n \cdot x_n \\ \text{s.t. :} \quad & \sum_{n=1}^N \frac{1}{r_n} \cdot x_n \leq T \\ & 0 \leq x_n \leq d_n, \quad n = 1 \dots N \end{aligned}$$

Dual – Example 4

Dual of Example 3

$$\begin{aligned} \min \quad & T \cdot w + \sum_{n=1}^N d_n \cdot y_n \\ \text{s.t.} \quad & \frac{1}{r_n} \cdot w + y_n \geq p_n, \quad n = 1 \dots N \\ & w, y_n \geq 0 \end{aligned}$$

Create files `example4.mod`, `example4.dat` and `example4.run`

Hint: Copy the content of the file `example3.dat` to `example4.dat`.

Challenge 1

Minimum Cost Network Flow Problem

- ▶ a directed graph $G = (V, A)$ with a set of vertices (nodes) and a set of arcs;
- ▶ unit cost c_{ij} for transportation in arc (i, j) ;
- ▶ lower bound l_{ij} and upper bound u_{ij} for flow in arc (i, j) ; and
- ▶ Flow b_i that must be injected or consumed at node i :
 - ▶ if $b_i > 0$, then i is a supplier node;
 - ▶ if $b_i < 0$, then i is a consumer node; and
 - ▶ if $b_i = 0$, then i is a transshipment node.

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Challenge 1

Model in mathematical programming:

$$\text{Minimize} \quad \sum_{(i,j) \in A} c_{ij} x_{ij}$$

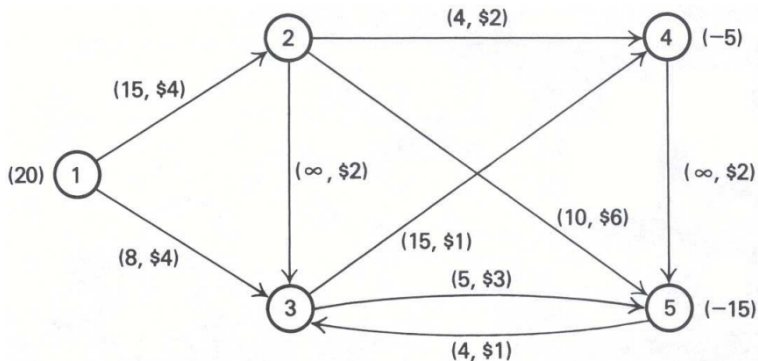
Subject to:

$$\sum_{\{j:(i,j) \in A\}} x_{ij} - \sum_{\{j:(j,i) \in A\}} x_{ji} = b_{ij}, \forall i \in V$$

$$l_{ij} \leq x_{ij} \leq u_{ij}, \forall (i,j) \in A$$

Challenge 1

Describe the following problem in AMPL. Create .dat, .mod and .run files.



Tutorial AMPL

- ▶ Thank you for attending this lecture!!!