Cutting Plane Algorithm, Gomory Cuts, and Disjunctive Cuts

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Cutting-Plane Algorithm

Gomory Cuts

Disjunctive Cuts

Summary

Cutting-Plane Algorithm: Principles

- Assume that that feasible set is $X = P \cap \mathbb{Z}^n$.
- Let $\mathcal{F}$ be a family of valid inequalities for X:

$$\pi^T x \leqslant \pi_0, \ (\pi, \pi_0) \in \mathcal{F},$$

- Typically, $\mathcal{F}$ may contain a large number of elements (exponentially many).
- Thus we cannot introduce all inequalities a priori.
- From a practical standpoint, we don't need a full representation of $conv(X)$, only an approximation around the optimal solution.

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Cutting-Plane Algorithm

Here we present a baseline cutting-plane algorithm for

$$IP: \ \max\{c^T x; x \in X\}$$

, which generates "useful" cuts from the family $\mathcal{F}$.

Cutting-Plane Algorithm

Initialization

Define $t = 0$ and $P^0 = P$

Iteration T

Solve the linear program $\bar{z}^t = \max\{c^T x : x \in P^t\}$

Let x^t be an optimal solution

If $x^t \in \mathbb{Z}^n$, stop since x^t is an optimal solution for IP

If $x^t \notin \mathbb{Z}^n$, find an inequality $(\pi, \pi_0) \in \mathcal{F}$
such that $\pi^T x^t > \pi_0$

If an inequality (π, π_0) was found,
then do $P^{t+1} = P^t \cap \{x : \pi^T x \leqslant \pi_0\}$,
increase t and repeat

Otherwise, stop

Valid Inequalities

- If the algorithm terminates without finding an integer solution, at least

$$P^t = P \cap \{x : \pi_i^T \leqslant \pi_{i0}, i = 1, 2, \ldots, t\}$$

is a "tighter" formulation than the initial formulation P.

- We can proceed from P^t with a branch-and-bound algorithm.

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Cutting-Plane Algorithm with Gomory Cuts

- Here we concentrate in the following integer program:

$$\max \{c^T x : Ax = b, x \geqslant 0 \text{ and integer}\}$$

- The strategy is to solve the linear relaxation and find an optimal basis.
- From the optimal basis, we choose a fractional basic variable.
- Then we generate a Chvátal-Gomory cut associated with this basic variable, aiming to cut it off, that is, eliminate this solution from the relaxation polyhedron.

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Cutting-Plane Algorithm with Gomory Cuts

Given an optimal basis, the problem/dictionary can be expressed as:

$$\begin{aligned}
\max \quad & \bar{a}_{oo} + \sum_{j \in NB} \bar{a}_{oj} x_j \\
\text{s.t.:} \quad & x_{Bu} + \sum_{j \in NB} \bar{a}_{uj} x_j = \bar{a}_{uo} \text{ for } u = 1, \ldots, m \\
& x \geqslant 0 \;\; \text{and integer}
\end{aligned}$$

where:

1. $\bar{a}_{oj} \leqslant 0$ for $j \in NB$,
2. $\bar{a}_{uo} \geqslant 0$ for $u = 1, \ldots, m$, and
3. NB is the set of nonbasic variables, therefore $\{B_u : u = 1, \ldots, m\} \cup NB = \{1, \ldots, n\}$.

Cutting-Plane Algorithm with Gomory Cuts

- If the optimal basic solution x^* is not integer, then there must exist a row u such that $\overline{a}_{uo} \notin \mathbb{Z}$.
- Choosing this row, the Chvátal-Gomory for the row u becomes:

$$x_{Bu} + \sum_{j \in NB} \lfloor \overline{a}_{uj} \rfloor x_j \leqslant \lfloor \overline{a}_{uo} \rfloor \quad (1)$$

- Rewriting (1) so as to eliminate x_{Bu}, we obtain:

$$x_{Bu} = \overline{a}_{uo} - \sum_{j \in NB} \overline{a}_{uj} x_j \quad (2)$$

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Cutting-Plane Algorithm with Gomory Cuts

From (2), we deduce that:

$$\bar{a}_{uo} - \sum_{j \in NB} \bar{a}_{uj} x_j + \sum_{j \in NB} \lfloor \bar{a}_{uj} \rfloor x_j \leqslant \lfloor \bar{a}_{uo} \rfloor$$
$$\Longrightarrow \sum_{j \in NB} \left(\bar{a}_{uj} - \lfloor \bar{a}_{uj} \rfloor \right) x_j \geqslant \bar{a}_{uo} - \lfloor \bar{a}_{uo} \rfloor \quad (3)$$

Cutting-Plane Algorithm with Gomory Cuts

In a more compact form, we can rewrite the cutting plane:

$$\sum_{j \in NB} \left(\bar{a}_{uj} - \lfloor \bar{a}_{uj} \rfloor\right) x_j \geqslant \bar{a}_{uo} - \lfloor \bar{a}_{uo} \rfloor$$

as:

$$\sum_{j \in NB} f_{uj} x_j \geqslant f_{uo} \tag{4}$$

in which:

- $f_{uj} = \bar{a}_{uj} - \lfloor \bar{a}_{uj} \rfloor$ and
- $f_{uo} = \bar{a}_{uo} - \lfloor \bar{a}_{uo} \rfloor$.

Cutting-Plane Algorithm with Gomory Cuts

Remark

Since $0 \leqslant f_{uj} < 1$ and $0 < f_{uo} < 1$, and $x_j^* = 0$ for each variable $j \in NB$ in the solution x^*, the inequality

$$\sum_{j \in NB} f_{uj} x_j \geqslant f_{uo}$$

cuts off the incumbent solution x^*.

Example

Consider the integer program:

$$
\begin{array}{rrcrcll}
z = \max & 4x_1 & - & x_2 & & & \\
\text{s.t.}: & 7x_1 & - & 2x_2 & \leqslant & 14 & \\
 & & & x_2 & \leqslant & 3 & \\
 & 2x_1 & - & 2x_2 & \leqslant & 3 & \\
 & x_1, & & x_2 & \geqslant & 0, & \text{integer}
\end{array}
\tag{5}
$$

Example

After introducing slack variables x_3, x_4 and x_5, we can apply the Simplex method and obtain an optimal solution:

$$
\begin{array}{lllllllll}
\max & \frac{59}{7} & & & - \frac{4}{7}x_3 & - \frac{1}{7}x_4 & & & \\
\text{s.t. :} & & x_1 & & + \frac{1}{7}x_3 & + \frac{2}{7}x_4 & & = & \frac{20}{7} \ (= 2.8571) \\
 & & & x_2 & & + x_4 & & = & 3 \\
 & & & & - \frac{2}{7}x_3 & + \frac{10}{7}x_4 & + x_5 & = & \frac{23}{7} \ (= 3.2857) \\
 & & x_1, & x_2, & x_3, & x_4, & x_5 & \geqslant & 0 \ \text{and integer}
\end{array} \tag{6}
$$

Example

- The optimal solution for the linear relaxation is $x^* = (\frac{20}{7}, 3, \frac{27}{7}, 0, 0) \notin \mathbb{Z}^5_+$.
- Thus, we use the first row of (6), in which the basic variables x_1 is fractional.
- This generates the cut:

$$x_1 + \lfloor \frac{1}{7} \rfloor x_3 + \lfloor \frac{2}{7} \rfloor x_4 \leqslant \lfloor \frac{20}{7} \rfloor \quad \Longrightarrow \quad x_1 \leqslant 2$$

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Example

Introducing a slack variable, we obtain:

$$
\begin{aligned}
& x_1 + s = 2, \\
& x_1 = \tfrac{20}{7} - \tfrac{1}{7}x_3 - \tfrac{2}{7}x_4 \quad \Longrightarrow \quad \tfrac{20}{7} - \tfrac{1}{7}x_3 - \tfrac{2}{7}x_4 + s = 2 \\
& \Longrightarrow \quad s = 2 - \tfrac{20}{7} + \tfrac{1}{7}x_3 + \tfrac{2}{7}x_4 \\
& \Longrightarrow \quad s = -\tfrac{6}{7} + \tfrac{1}{7}x_3 + \tfrac{2}{7}x_4 \\
& \text{with } s, x_3, x_4 \geqslant 0 \text{ and integer.}
\end{aligned}
$$

Example

Introducing the slack variable s and the cut

$$s = -\frac{6}{7} + \frac{1}{7}x_3 + \frac{2}{7}x_4,$$

we obtain a second formulation:

$$\begin{array}{lcccccccccccl}
\max & \frac{15}{2} & & & & & - & \frac{1}{2}x_5 & - & 3s & & & \\
\text{s.t. :} & & x_1 & & & & & & + & s & = & 2 & \\
 & & & x_2 & & & - & \frac{1}{2}x_5 & + & s & = & \frac{1}{2} & \\
 & & & & x_3 & & - & x_5 & - & 5s & = & 1 & \\
 & & & & & x_4 & + & \frac{1}{2}x_5 & + & 6s & = & \frac{5}{2} & \\
 & & x_1, & x_2, & x_3, & x_4, & & x_5, & & s & \geqslant & 0 & \text{and integer.}
\end{array} \tag{7}$$

An optimal solution for the relaxation (7), yields $x = (2, \frac{1}{2}, 1, \frac{5}{2}, 0, 0)$.

Example

The incumbent solution remains fractional because x_2 and x_4 are not integer.

The application of the Chavátal-Gomory cut on the second row yields:

$$
\begin{aligned}
x_2 + \lfloor -\tfrac{1}{2} \rfloor x_5 + \lfloor 1 \rfloor s \leqslant \lfloor \tfrac{1}{2} \rfloor \quad & \Longrightarrow \quad x_2 - x_5 + s \leqslant 0 \\
& \Longrightarrow \quad x_2 - x_5 + s + t = 0, \;\; t \geqslant 0 \\
& \Longrightarrow \quad (\tfrac{1}{2} + \tfrac{1}{2}x_5 - s) - x_5 + s + t = 0 \\
& \Longrightarrow \quad t - \tfrac{1}{2}x_5 = -\tfrac{1}{2}, \;\; t \geqslant 0
\end{aligned}
$$

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& \Longrightarrow \quad x_2 - x_5 + s + t = 0, \ \ t \geqslant 0 \\
& \Longrightarrow \quad (\tfrac{1}{2} + \tfrac{1}{2} x_5 - s) - x_5 + s + t = 0 \\
& \Longrightarrow \quad t - \tfrac{1}{2} x_5 = -\tfrac{1}{2}, \ \ t \geqslant 0
\end{aligned}
$$

Example

After introducing the variable $t \geqslant 0$ and the cut $t - \frac{1}{2}x_5 = -\frac{1}{2}$, we obtain the solution below for the linear relaxation:

$$
\begin{array}{llllllllllll}
\max & 7 & & & & & - & 3s & - & t & & \\
\text{s.t.}: & & x_1 & & & & + & s & & & = & 2 \\
& & & x_2 & & & + & s & - & t & = & 1 \\
& & & & x_3 & & - & 5s & - & 2t & = & 2 \\
& & & & & x_4 & + & 6s & + & t & = & 2 \\
& & & & & x_5 & & & - & t & = & 1 \\
& & x_1, & x_2, & x_3, & x_4, x_5, & & s, & & t & \geqslant & 0 \text{ and integer}
\end{array}
$$

- The obtained solution is optimal because all values are integer.
- The optimal solution is $x^* = (2, 1, 2, 2, 1)$ with objective value $z^* = 7$.

Summary

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Gomory Cuts

Disjunctive Cuts

Disjunctive Cuts

- Let $X = X^1 \cup X^2$ with $X^i \subseteq \mathbb{R}^n_+$.
- That is, X is the disjunction (union) of two sets X^1 and X^2.
- Some important results are given below.

Disjunctive Cuts

Proposition

If $\sum_{j=1}^{n} \pi_j^i x_j \leqslant \pi_0^i$ is a valid inequality for X^i, $i = 1, 2$, then the inequality

$$\sum_{j=1}^{n} \pi_j x_j \leqslant \pi_0$$

is valid for X if:

- $\pi_j \leqslant \min\{\pi_j^1, \pi_j^2\}$ for $j = 1, \ldots, n$; and
- $\pi_0 \geqslant \max\{\pi_0^1, \pi_0^2\}$.

Disjunctive Cuts

Proposition

- if $P^i = \{x \in \mathbb{R}^n_+ : A^i x \leqslant b^i\}$ for $i = 1, 2$ are nonempty polyhedra,
- then (π, π_0) is a valid inequality for $conv(P^1 \cup P^2)$ if, and only if, $u_1, u_2 \geqslant 0$ such that:

$$\begin{aligned}
\pi^T &\leqslant u_1^T A^1 \\
\pi^T &\leqslant u_2^T A^2 \\
\pi_0 &\geqslant u_1^T b^1 \\
\pi_0 &\geqslant u_2^T b^2
\end{aligned}$$

Example

- Consider the following polyhedra:

$$\begin{aligned} P^1 &= \{x \in \mathbb{R}^2 : -x_1 + x_2 \leqslant 1,\ x_1 + x_2 \leqslant 5\} \\ P^2 &= \{x \in \mathbb{R}^2 : x_2 \leqslant 4, -2x_1 + x_2 \leqslant -6, \\ &\qquad x_1 - 3x_2 \leqslant -2\} \end{aligned}$$

- By letting $u_1 = (2, 1)$ and $u_2 = (\frac{5}{2}, \frac{1}{2}, 0)$, and then applying the proposition above, the following results:

$$u_1^T A^1 = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \end{bmatrix} \qquad u_1^T b^1 = 7$$

Example

- We further obtain:

$$u_2^T A^2 = \begin{bmatrix} \frac{5}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & 1 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 3 \end{bmatrix} \qquad u_2^T b^2 = 7$$

- This allows us to obtain the inequality $-x_1 + 3x_2 \leqslant 7$, which is valid for $P^1 \cup P^2$.

Example

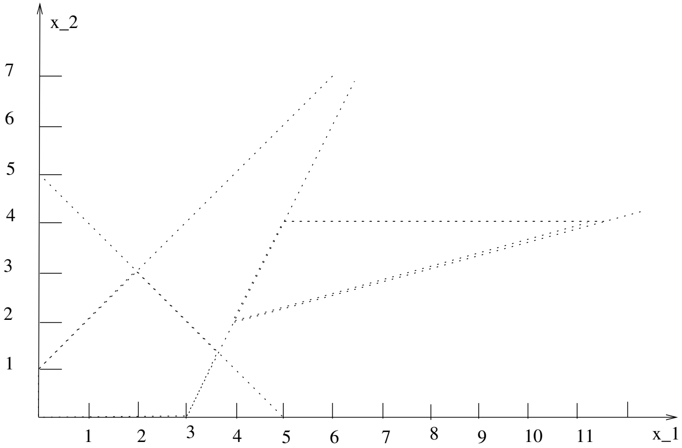

Figura : Disjunctive inequalities

Disjunctive Inequalities for Pure 0-1 Programs

- Specializing even further, we restrict the analysis to pure 0-1 programs, in which:
  - $X = P \cap \mathbb{Z}^n \subseteq \{0,1\}^n$ and
  - $P = \{x \in \mathbb{R}^n : Ax \leqslant b, 0 \leqslant x \leqslant 1\}$.
- Let $P^0 = P \cap \{x \in \mathbb{R}^n : x_j = 0\}$.
- Let $P^1 = P \cap \{x \in \mathbb{R}^n : x_j = 1\}$ for some $j \in \{1, \ldots, n\}$.

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Disjunctive Inequalities for Pure 0-1 Programs

Proposition

The inequality (π, π_0) is valid for $conv(P^0 \cup P^1)$ if there exists $u_i \in \mathbb{R}^m_+$, $v_i \in \mathbb{R}^n_+$, $w_i \in \mathbb{R}^1_+$ for $i = 0, 1$ such that:

$$
\begin{array}{rcl}
\pi^T & \leqslant & u_0^T A + v_0 + w_0 e_j \\
\pi^T & \leqslant & u_1^T A + v_1 - w_1 e_j \\
\pi_0 & \geqslant & u_0^T b + \mathbf{1}^T v_0 \\
\pi_0 & \geqslant & u_1^T b + \mathbf{1}^T v_1 - w_1
\end{array}
$$

Disjunctive Inequalities for Pure 0-1 Programs

Proof

Apply the previous preposition with:

- $P^0 = \{x \in \mathbb{R}^n_+ : Ax \leqslant b, x \leqslant 1, x_j \leqslant 0\}$ and
- $P^1 = \{x \in \mathbb{R}^n_+ : Ax \leqslant b, x \leqslant 1, -x_j \leqslant -1\}$

Example

Consider the following instance of the knapsack problem:

$$\begin{array}{lccccccccc} \max & 12x_1 & + & 14x_2 & + & 7x_3 & + & 12x_4 & & \\ \text{s.t.}: & 4x_1 & + & 5x_2 & + & 3x_3 & + & 6x_4 & \leqslant & 8 \end{array}$$

$$x \in \mathbb{B}^4$$

with optimal linear solution $x^* = (1, 0.8, 0, 0)$.

Example

- Since $x_2^* = 0.8$ is fractional, we choose $j = 2$.
- Defining P^0 and P^1, we look for an inequality (π, π_0) which is violated according to the proposition above.
- To that end, we solve the linear programming problem maximizing $\pi^T x^* - \pi_0$ within the polyhedron that expresses the coefficients for the valid inequalities given by the proposition.

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Example

This linear program is given by:

$$\max \quad 1.0\pi_1 + 0.8\pi_2 - \pi_0$$

$$\text{s.t.}: \quad \begin{cases} \begin{cases} \pi_1 \leqslant 4u^0 + v_1^0 \\ \pi_1 \leqslant 4u^1 + v_1^1 \end{cases} \\ \begin{cases} \pi_2 \leqslant 5u^0 + v_2^0 + w^0 \\ \pi_2 \leqslant 5u^1 + v_2^1 - w^1 \end{cases} \\ \begin{cases} \pi_3 \leqslant 3u^0 + v_3^0 \\ \pi_3 \leqslant 3u^1 + v_3^1 \end{cases} \\ \begin{cases} \pi_4 \leqslant 6u^0 + v_4^0 \\ \pi_4 \leqslant 6u^1 + v_4^1 \end{cases} \\ \begin{cases} \pi_0 \geqslant 8u^0 + v_1^0 + v_2^0 + v_3^0 + v_4^0 \\ \pi_0 \geqslant 8u^1 + v_1^1 + v_2^1 + v_3^1 + v_4^1 - w^1 \end{cases} \\ u^0, u^1, v^0, v^1, w^0, w^1 \geqslant 0 \end{cases}$$

Disjunctive Inequalities for Pure 0-1 Programs

- Aiming to render the space of feasible solutions bound, we should introduce a normalization criterion.
- Two possibilities are:
  a) $\sum_{j=1}^{n} \pi_j \leqslant 1$
  b) $\pi_0 = 1$
- Then we obtain the following cutting plane:

$$x_1 + \frac{1}{4}x_2 \leqslant 1.$$

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Disjunctive Inequalities for Pure 0-1 Programs

- For P^0, the inequality is a combination of the constraints $x_1 \leqslant 1$ and $x_2 \leqslant 0$ with $v_1^0 = 1$ and $w^0 = \frac{1}{4}$, respectively.
- For P^1, the inequality is a combination of the constraints $4x_1 + 5x_2 + 3x_3 + 6x_4 \leqslant 8$ and $-x_2 \leqslant -1$ with $u^1 = \frac{1}{4}$ and $w^1 = 1$, respectively.

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Cutting-Plane Algorithm

- Thank you for attending this lecture!!!