Homework

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1. (1 point) An investor wishes to invest his savings in a set $S = \{1, \ldots, 7\}$ of market stocks. Using 0-1 variables formulate the following constraints in integer (linear) programming.

   (a) the portfolio cannot invest in all stocks;

   (b) at least one stock must be selected;

   (c) stock 1 cannot be selected if stock 3 is selected;

   (d) stock 4 can be selected only if stock 2 can be selected.

   (e) either stocks 1 and 5 are selected simultaneously, or none of the them;

   (f) at least one stock of the set $\{1, 2, 3\}$ or at least two stocks of the set $\{2, 4, 5, 6\}$ mus be selected.

   Remark: your formulation should be efficient.

2. (1 point) Suppose that $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is the vector of decision variables. How would you represent the following constraints in integer linear programming?

$$\|x\|_1 = 1$$
$$\|x\|_\infty = 1$$

3. (3 points) A set of n jobs must be processed in a machine that can handle one job at a time. Task j needs p_j hours to be processed.

   A directed and acyclic graph $G = (V, E)$, with $V = \{1, \ldots, n\}$, establishes a partial order for job processing in the machine. That is, if there exists a path $\delta_{i,j}$ from i to j in G, then job i must be processed before job j.

   Given nonnegative weights $w_j, j = 1, \ldots, n$, in which order should we process the jobs in order to minimize the weighted sum of the start processing time of all jobs, while respecting the precedence order? For the modeling task that follows, s_j is the instant that job j starts to be processed.

   Tasks:

   (a) Formulate the problem in mixed-integer linear programming using discrete and continuous variables.

   (b) Model the problem in AMPL and solve the instance given below, in which $V = \{1, \ldots, 12\}$. Present the results.

j	p_j	w_j	Arcs (j, i)	
1	3	5	(1,3)	
2	2	3		
3	6	7	(3,12),	(3,7)
4	2	6		
5	5	1		
6	4	2	(6,7)	
7	4	8		
8	3	4	(8,6)	
9	10	7		
10	1	1	(10,12)	
11	8	6		
12	7	2		

4. (5 points) Choose a nonlinear, continuous function $f : \mathbb{R} \to \mathbb{R}$ with domain $\mathrm{dom} f = [0,\ 1]$. Your function should have from 4 to 10 local minima/maxima within the chosen domain. Hint: if needed, normalize your function to ensure that its domain is within the given bounds.

   Tasks:

   (a) Develop and implement a piecewise-linear approximation $\widetilde{f}$ for f in $\mathrm{dom} f = [0,\ 1]$ using at least two of the following models:

      i. CC
      ii. DCC
      iii. DLog
      iv. SOS2

   (b) Implement in AMPL two of those models, using a suitable number of breakpoints. You should not use more than 100 points. Plot the piecewise-linear approximation and the original function. Also, solve the following problems:

   $$\begin{aligned} &\min\ \widetilde{f}(x) \\ &\text{s.t.} : x \in [0,\ 1] \end{aligned}$$

   and

   $$\begin{aligned} &\max\ \widetilde{f}(x) \\ &\text{s.t.} : x \in [0,\ 1] \end{aligned}$$

   Question: for the case of a piecewise-linear function $\widetilde{f}(x)$ do you need to solve these problems? Can you obtain the solutions readily?

   (c) Choose 3 values in the range [min $\widetilde{f}(x)$, max $\widetilde{f}(x)$], say y_1, y_2, and y_3. For each y_j, solve the following problem:

   $$\text{find } x \in [0, 1] \text{ such that } \widetilde{f}(x) = y_j$$

   Do the solutions agree for each of the piecewise-linear models?