OFF-EQUILIBRIUM LINEARISATION AND DESIGN OF GAIN SCHEDULED CONTROL WITH APPLICATION TO VEHICLE SPEED CONTROL

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Abstract: In conventional gain scheduled control design, linearisation of a time-invariant nonlinear system and local control design for the resulting set of linear time-invariant systems is performed at a set of equilibrium points. Due to its validity only near equilibrium, such a design may result in poor transient performance. To resolve this problem one can base the control design on a dynamic linearisation about some nominal trajectory. However, a drawback with this approach is that control design for the resulting linear time-varying system is in general a difficult problem. In this paper it is suggested that linearisation and local controller design should be carried out not only at equilibrium states, but also in transient operating regimes. It is shown that this results in a set of time-invariant linearisations which, when they are interpolated, form a close approximation to the time-varying system resulting from dynamic linearisation. Consequently, the transient performance can be improved by increasing the number of linear time-invariant controllers. The feasibility of this approach, and possible improvements in transient performance, are illustrated with results from an experimental vehicle speed control application.

Keywords: Gain scheduling, Transient response, Automotive control.

1. INTRODUCTION

The ability to perform well during transient operation will be an unavoidable requirement in future control systems where one pursues higher control performance and operational flexibility closer to the fundamental limitations of the physical system. The contribution of this paper is a study and experimental evaluation of off-equilibrium linearisations as an approach for explicit design of nonlinear gain scheduled control systems for accurate transient performance. One motivation behind the considerations in this paper is the problem of high-performance speed control design for a truck. The experimental results from this full scale application verify the feasibility of off-equilibrium linearisation and demonstrate the possible improvements in transient performance compared to traditional approaches.

Traditional gain scheduled control system designs typically fall into one of two categories.

Design based on linearisations about a set of equilibrium points: The controllers are typically scheduled on the output, state or some auxiliary variable. This approach is described in e.g. (Shamma and Athans 1990, Rugh 1991, Packard 1994, Apkarian and Gahinet 1995, Lawrence and Rugh 1995, Tanaka and Sano 1994, Wang *et al.* 1996). Application of classical linearisation theory requires that the linearisations correspond to equilibria of the nonlinear system. This is a serious restriction that may lead to poor transient performance of the gain scheduled control system. The system may even become unstable when it is operating far away from the equilibrium manifold.

Design based on dynamic linearisation about a nominal trajectory: Typically, the controllers are scheduled in open loop on the reference trajectory or on auxiliary inputs which are used to define the nominal trajectory, e.g. (Shamma and Athans 1990, Driankov *et al.* 1996). This covers transient operation, but suffers from the drawback that the performance may be poor when the system is operating far away from the reference trajectory, due to the open loop scheduling. Furthermore, control design for the resulting linear time-varying (LTV) systems is considerably more difficult than control design for the linear time-invariant (LTI) systems that result from linearisation about equilibrium points.

Linearisation of the nonlinear system at operating conditions that are not necessarily equilibria and design of local controllers for these operating regimes was suggested in (Hunt and Johansen 1997). The controllers are scheduled on a variable composed of the output, state, control input and auxiliary variables. The transient state linearised LTI local models can be viewed as an approximation to dynamic LTV linearisation about state trajectories, and the local models are highly useful for gain scheduled control design. The benefit of this approach is that the transient (off-equilibrium) dynamics of the control system may be significantly improved.

The importance of the transient behaviour of gain scheduled control systems is also addressed in (Leith and Leithead 1996). They describe how to implement gain scheduled controllers based on classical linearisations (at equilibria only) such that the important "linearisation property" (Kaminer *et al.* 1995) also holds during transient operation.

2. CLASSICAL AND OFF-EQUILIBRIUM LINEARISATION

Consider the time-invariant nonlinear system

$$\dot{x} = f(x,u),\ y = g(x) \tag{1}$$

where $x \in R^n$, $u \in R^r$, $y \in R^m$ and f and g are smooth. The equilibrium manifold for this system is given by

$$\mathcal{E} = \{(x,u) \in R^{n+r} \mid f(x,u) = 0\}$$

Now, for any equilibrium point $(x_0, u_0) \in \mathcal{E}$ the classical linearisation is defined by the LTI system

$$\begin{aligned}\dot{x} &= A_0(x - x_0) + B_0(u - u_0)\\ A_0 &= \frac{\partial f}{\partial x}(x_0, u_0)\\ B_0 &= \frac{\partial f}{\partial u}(x_0, u_0)\end{aligned}$$

One interpretation of the linearised dynamics is that it approximates the dynamics of the nonlinear system for initial values and inputs that deviate slightly from x_0 and u_0 respectively. A major reason for the popularity of the classical linearisation approach is the existence of a complete and very well-known theory for analysis of the local stability properties of equilibria of nonlinear systems in terms of its linearisation, e.g. (Khalil 1992), as well as a theory for design of LTI control systems based on linearisation.

Suppose instead that the dynamics of the nonlinear system are approximated near an arbitrary point $(x_0, u_0) \in R^{n+r}$ (which need not be an equilibrium point):

$$\dot{x} = A_0(x - x_0) + B_0(u - u_0) + f(x_0, u_0) \tag{2}$$

This equation can also be written

$$\dot{x} = A_0 x + B_0 u + d_0 \tag{3}$$

where $d_0 = f(x_0, u_0) - A_0 x_0 - B_0 u_0$. From a mathematical point of view this corresponds to approximating the function f by its tangent plane at the point (x_0, u_0). However, the interpretation of the system (2) is conceptually different when (x_0, u_0) is not an equilibrium point. For example, it makes no sense to consider stability of the point (x_0, u_0), simply because it can only be a transient state of the system rather than an equilibrium point. One interpretation of (2) is that it approximates the (possibly transient) dynamics of the nonlinear system when the trajectory is close to (x_0, u_0). A related interpretation is that the flow of the linearised dynamics approximates the flow of the nonlinear system for states and inputs that deviate slightly from x_0 and u_0. While such an approximation is not of immediate usefulness for stability analysis, it is still very useful for control design, as shown in section 3 and in the examples in sections 4 and 5.

The difference between dynamic linearisation (about a trajectory) and off-equilibrium linearisation (about a set of points) can be seen to depend only on the granularity of the set of points in the off-equilibrium linearisation. The reason for this is that the LTV system resulting from dynamic linearisation depends only on the point the trajectory passes through at a given time. Hence, off-equilibrium linearisation leads to an arbitrarily close approximation of the LTV system in terms of a set of LTI systems, provided there exists an LTI system close to any point on the nominal trajectory of the LTV system, and the LTI systems are interpolated using a sensible interpolation scheme. A formal proof of this can be found in the appendix. Related issues are also discussed in (Driankov *et al.* 1996).

3. CLASSICAL AND OFF-EQUILIBRIUM GAIN SCHEDULED DESIGN

The purpose of a controller is essentially to reshape the dynamic behaviour of the open loop system into more desirable dynamics of the closed loop system, in some sense. For example, unstable dynamics can be reshaped into stable dynamics, nonlinear dynamics into linear dynamics, slow dynamics can be reshaped into fast dynamics, and dynamics that are sensitive to disturbances, noise and unmodelled dynamics can be reshaped into less sensitive dynamics.

In traditional gain scheduling control design, e.g. (Shamma and Athans 1990, Rugh 1991, Packard 1994, Apkarian and Gahinet 1995, Lawrence and Rugh 1995), a family (or finite set) of controllers is designed on the basis of classical linearisations such as (2) about a set of equilibrium points $\mathcal{E}^\star \subset \mathcal{E}$. Compared to a single linear controller, such a gain scheduled control design will typically extend the operating range where the control system performs satisfactorily. However, as with a single linear controller design, the resulting closed loop dynamics when operating far away from the equilibrium manifold is not taken explicitly into account in the design. Hence, the control system may have poor performance or become unstable when the system is operating far away from the equilibrium manifold. At this point it is clear that off-equilibrium linearisations such as (3) will be useful, since they allow shaping of the transient dynamics away from the equilibrium manifold as well, see Figure 1.

In the following, consider the family of linearised dynamics

$$\dot{x} = A(\alpha)x + B(\alpha)u + d(\alpha) \tag{4}$$

$$y = C(\alpha)x + c(\alpha) \tag{5}$$

as an approximation to the nonlinear system (1) based on a finite number of interpolated LTI models

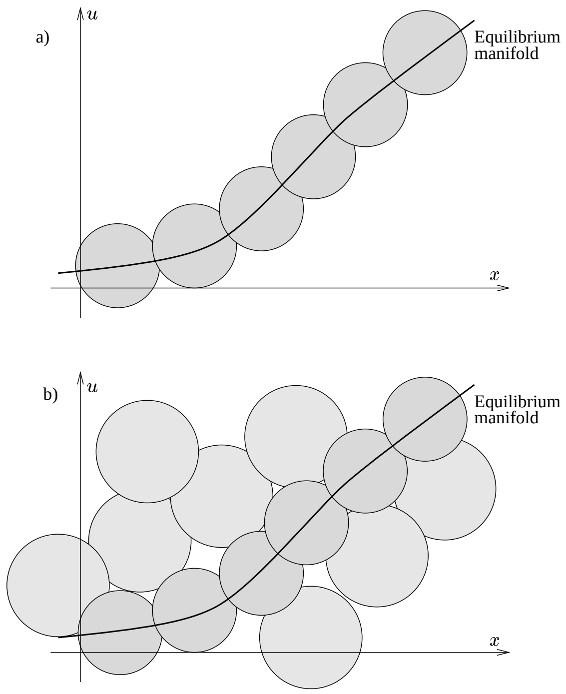


Fig. 1. Classical linearisation and gain scheduled design (part a) versus off-equilibrium linearisation and gain scheduled design (part b).

$$\dot{x} = A_i x + B_i u + d_i \tag{6}$$

$$y = C_i x + c_i \tag{7}$$

with regions of validity characterised by positive definite interpolation functions w_i that satisfy

$$\sum_{i=1}^{N} w_i(\alpha) = 1 \tag{8}$$

Hence, the matrices are defined by

$$A(\alpha) = \sum_{i=1}^{N} A_i w_i(\alpha) \tag{9}$$

$$B(\alpha) = \sum_{i=1}^{N} B_i w_i(\alpha) \tag{10}$$

$$d(\alpha) = \sum_{i=1}^{N} d_i w_i(\alpha) \tag{11}$$

$$C(\alpha) = \sum_{i=1}^{N} C_i w_i(\alpha) \tag{12}$$

$$c(\alpha) = \sum_{i=1}^{N} c_i w_i(\alpha) \tag{13}$$

It is assumed that the point of linearisation is parameterised by the scheduling variable α which depends on the state, input and auxiliary variable θ: $\alpha = \alpha(x, u, \theta)$. This is in contrast to traditional gain scheduling where the scheduling variable is typically a low-dimensional vector that parameterises the equilibrium manifold (e.g.

(Rugh 1991, Lawrence and Rugh 1995)) rather than the full state/input space. Consequently, a possible disadvantage with off-equilibrium linearisation is that the scheduling variable may be of higher dimension than with classical linearisation, and there may be additional difficulties when the full state is not measured. In this case, one might either apply an observer, or seek a sufficiently rich characterisation of the operating point in terms of the input and output for the most significant transient dynamics to be captured. These problems will be further discussed in section 3.4.

In the remainder of this section the various aspects of gain scheduled control design, emphasising aspects related to transient performance, will be discussed. In sections 3.1 and 3.2 closed loop equations for the cases where the gain scheduled controller is based on static state feedback and dynamic output feedback, respectively, will be presented. These equations will form the basis for feedforward design, considered in section 3.3.

3.1 *Static State Feedback*

Let x^r be a state reference trajectory, and u^r a corresponding input trajectory (feedforward). Consider the static state feedback

$$u = u^r + K_c(\alpha)(x - x^r) \tag{14}$$

that is applied when $\alpha = \alpha(x, u, \theta)$. It follows immediately that the closed loop dynamics are given by

$$\dot{x} - \dot{x}^r = (A(\alpha) + B(\alpha)K_c(\alpha))(x - x^r) + \varepsilon \tag{15}$$

$$\varepsilon = -\dot{x}^r + A(\alpha)x^r + B(\alpha)u^r + d(\alpha) \tag{16}$$

The term ε in (15) may or may not be identically zero, depending on how the trajectories x^r and u^r are generated. This problem will be discussed in section 3.3, where feedforward and trajectory generation is discussed for both static state feedback and dynamic output feedback.

The controller $K_c(\alpha)$ should be designed to meet the global stability, robustness and performance requirements, both at equilibria and during transient operation. It is well known that due to the time-varying α, local properties of the closed loop do not carry over to global properties (Rugh 1991, Shamma and Athans 1992). In practice, the feedback $K_c(\alpha)$ is usually designed at a finite number of operating points α, the resulting local controllers are interpolated in some way, and the global properties are verified by extensive nonlinear simulation.

Both equilibrium and off-equilibrium design can be viewed as means to reshape the flow of the open loop vector field

$$A(\alpha)x + B(\alpha)u + d(\alpha)$$

into a closed-loop vector field

$$(A(\alpha) + B(\alpha)K_c(\alpha))(x - x^r) + \varepsilon$$

such that the vector $x - x^r$ "flows" in a way that meets the performance requirements (i.e. towards the origin, with a specific rate) and is robust to various perturbations of the vector field.

3.2 *Dynamic Output Feedback*

In contrast to the state feedback case, assume a smooth reference output trajectory y^r is given, with corresponding state and input trajectories x^r and u^r as above. Now consider the dynamic output feedback

$$\dot{z} = A_c(\alpha)z + B_c(\alpha)(y - y^r) \tag{17}$$

$$u = u^r + K_c(\alpha)z + G_c(\alpha)(y - y^r) \tag{18}$$

with internal state z, which leads to the following closed loop dynamics

$$\begin{pmatrix} \dot{x} - \dot{x}^r \\ \dot{z} \end{pmatrix} = A_0(\alpha) \begin{pmatrix} x - x^r \\ z \end{pmatrix} + \begin{pmatrix} \varepsilon_x \\ \varepsilon_z \end{pmatrix} \tag{19}$$

$$A_0(\alpha) = \begin{pmatrix} A(\alpha) + B(\alpha)G_c(\alpha)C(\alpha) & B(\alpha)K_c(\alpha) \\ B_c(\alpha)C(\alpha) & A_c(\alpha) \end{pmatrix}$$

$$\varepsilon_x = (-\dot{x}^r + A(\alpha)x^r + B(\alpha)u^r + d(\alpha)) + B(\alpha)G_c(\alpha)(C(\alpha)x^r + c(\alpha) - y^r)$$

$$\varepsilon_z = B_c(\alpha)(C(\alpha)x^r + c(\alpha) - y^r)$$

In the examples, dynamic output feedback will be applied, and the controller parameters $A_c(\cdot)$, $B_c(\cdot)$, $G_c(\cdot)$ and $K_c(\cdot)$ are chosen to meet the specifications in terms of closed loop poles and properties of the sensitivity functions at the selected operating point. As with static state feedback, the dynamics of the closed loop can be reshaped by local controllers at transient operating points, and the time-varying nature of α should be taken into consideration. Again, it is observed that the significance of the terms ε_x and ε_z depends on how the trajectories x^r and u^r are generated.

3.3 *Trajectory Generation*

The controls (14) and (17)-(18) can be naturally interpreted as a 2 DOF controller with a feedforward and a feedback term. The term u^r can be viewed as a feedforward from the reference output y^r, while the second term is a gain-scheduled feedback that compensates for model uncertainties and disturbances, c.f. Figure 2. As discussed in sections 3.1 and 3.2, a major issue is how to generate the trajectories x^r and u^r. This also has a significant impact on the transient behaviour of the closed loop.

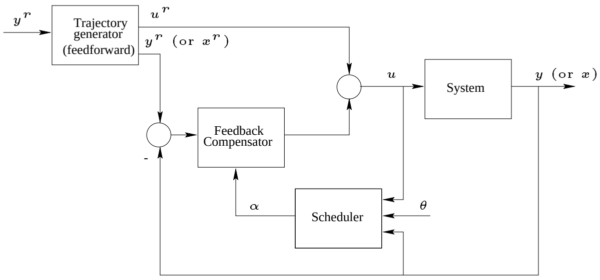


Fig. 2. The basic control structure is viewed as a 2-DOF controller with a trajectory generator (feedforward from the output reference trajectory) and a feedback from the output or state.

Suppose a smooth output reference trajectory y^r is given (both in the state and output feedback cases). There are now several ways to generate the state and input trajectories, leading to different versions of the structure in Figure 2.

- An **Open Loop Dynamic Trajectory Generator** could define x^r and u^r according to
$$\dot{x}^r = A(\alpha^r)x^r + B(\alpha^r)u^r + d(\alpha^r) \quad (20)$$
$$y^r = C(\alpha^r)x^r + c(\alpha^r) \quad (21)$$
where $\alpha^r = (x^r, u^r)$.
- A **Scheduled Dynamic Trajectory Generator** might define x^r and u^r according to
$$\dot{x}^r = A(\alpha)x^r + B(\alpha)u^r + d(\alpha) \quad (22)$$
$$y^r = C(\alpha)x^r + c(\alpha) \quad (23)$$
which actually introduces a feedback loop since the matrices are scheduled on α rather than α^r as in the open loop case. We observe in particular that this is the choice that gives $\varepsilon = \varepsilon_x = \varepsilon_z = 0$ in eqs. (15) and (19).
- An **Open Loop Static Trajectory Generator** may define x^r and u^r according to the steady-state equations
$$0 = A(\alpha^r)x^r + B(\alpha^r)u^r + d(\alpha^r) \quad (24)$$
$$y^r = C(\alpha^r)x^r + c(\alpha^r) \quad (25)$$
where $\alpha^r = (x^r, u^r)$.
- A **Scheduled Static Trajectory Generator** will typically define x^r and u^r according to the steady-state equations
$$0 = A(\alpha)x^r + B(\alpha)u^r + d(\alpha) \quad (26)$$
$$y^r = C(\alpha)x^r + c(\alpha) \quad (27)$$
which is also seen to contain a feedback.
- **No feedforward** can also be applied, i.e. $u^r = 0$, and x^r must be selected to be compatible with y^r. This choice may require integral action to be included in the loop in order to avoid large steady-state error.

The various approaches to trajectory generation have different advantages and disadvantages in terms of applicability, accuracy, computational complexity and transparency. Dynamic trajectory generation will essentially invert the system, which may lead to instability when there are unstable zero-dynamics.

A comprehensive discussion of these aspects is given in (van Nieuwstadt and Murray 1997) where it is illustrated how this problem can be overcome for differentially flat systems, which allow the state and input trajectories to be rewritten as algebraic functions of a suitable output trajectory. In general, the trajectory generation can be formulated as an optimisation problem, and stability can be achieved by introducing a penalty on the generated control signal, as usual. Static trajectory generation obviously avoids this problem, since only the steady-state equations are inverted. However, transient performance will typically have to be sacrificed.

The generated trajectory will usually be influenced by uncertainty with respect to the current state of the system, modelling error and also changes of the nominal trajectory. In particular, there may be error due to incorrect initialisation of $x^r(0)$ in case of dynamic trajectory generation. This may lead to significant transients that must be stabilised by the feedback compensator. Regardless of the trajectory generator strategy selected, it is highly useful that the feedback compensator is designed to operate with high performance in transient operating regimes. Otherwise, the feedback loop might limit the performance of the control system. Off-equilibrium linearisation and design is a useful tool to achieve this, as discussed above.

3.4 *Scheduler*

In traditional equilibrium-based gain scheduled control design, the equilibrium manifold can typically be characterised by r variables, since it is defined by n equations in $n+r$ variables, assuming that the set of equations is not singular. If the number of inputs equals the number of outputs, i.e. $m = r$, the equilibrium point can be characterised by the reference output y^r.

On the other hand, with off-equilibrium based gain scheduled control, information about the full state and input must usually be available, in general. To achieve this, the state must be reconstructed from the output, e.g. using an observer. There will also exist special cases where neglecting a certain state in the scheduling may not have significant impact on the performance. For example, if the linearised system "parameters" $A(\cdot)$, $B(\cdot)$, $d(\cdot)$, $C(\cdot)$, and $c(\cdot)$ do not depend strongly on a certain variable, this variable need not be used for scheduling, since any linearisation can be extrapolated along that particular axis without too much loss of accuracy. Also, if a certain state evolves on a much faster or much slower time-scale than the desired response of the system, it need not be important for scheduling and can be eliminated.

4. SIMULATION EXAMPLE: AN UNSTABLE NONLINEAR SYSTEM

Consider the following unstable nonlinear dynamic system.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^2 + x_2^2 + u \end{aligned} \tag{28}$$

with equilibrium manifold

$$\mathcal{E} = \left\{ (x_1^e, x_2^e, u^e) \mid x_2^e = 0 \text{ and } (x_1^e)^2 + u^e = 0 \right\}$$

A dynamic output feedback from $y = x_1$ will be applied. It is assumed, however, that x_2 is also available for the purpose of scheduling. This is reasonable, since it can be estimated by robust differentiation of y, or by the use of an observer. The input u is not required for scheduling, since the system is linear in this variable (the system matrices $A(\cdot)$ and $B(\cdot)$ do not depend on u). Notice that the dynamic output feedback will introduce additional states in the closed loop. For simplicity, this example will mainly be concerned with the states of the system, even though the controller states are also governed by α-dependent differential equations due to scheduling of linear controllers.

4.1 *Off-equilibrium linearisation*

Consider the point $(x_1, x_2) = (3, 3)$, which is clearly off the equilibrium manifold, since $x_2 \neq 0$. Figure 3b illustrates the flow of the vector field of the dynamical system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 6x_1 + 6x_2 - 18; \end{aligned}$$

that results from this off-equilibrium linearisation with $u = 0$. Comparing with the flow of the nonlinear system (28) in Figure 3a, it is clear that the flow of the linearised system approximates the flow of the nonlinear system in a neighbourhood of the point of linearisation, even though it is not an equilibrium.

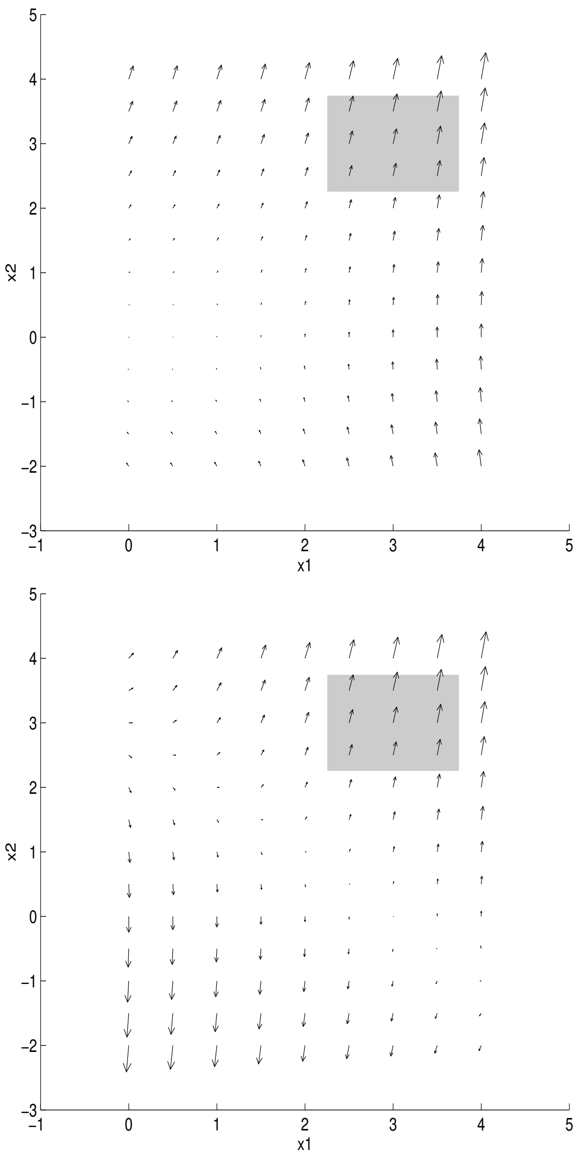


Fig. 3. Part a) illustrates the flow of the nonlinear system, while part b) illustrates the flow of the linearised system at the off-equilibrium point $(x_1, x_2) = (3.0, 3.0)$.

4.2 *The effect of off-equilibrium linearisation in gain scheduling control*

We consider three gain scheduled controllers.

Controller C1 – This gain scheduled controller contains 4 local controllers, designed on the basis of classical linearisations about the equilibrium points

$$\begin{aligned}(x_1^e, x_2^e, u^e) \in \{&(0.5, 0, -0.25), (1.5, 0, -2.25), \\ &(2.5, 0, -6.25), (3.5, 0, -12.25)\}\end{aligned}$$

The partitioning of the state-space into operating regimes is selected such that each local linear model/controller is reasonably accurate in the region it will be applied between the equilibrium points. The local controllers all contain integral action, and are designed such that the local closed loop poles are located at -2.5 and -5.0, and in addition there are observer poles at -10.0 and -20.0. Each local controller is a 2-DOF dynamic output feedback designed with a standard pole placement design algorithm, see (Hunt and Johansen 1997, Åström and Wittenmark 1997) for details. An interpolation algorithm is used to interpolate the local controller parameters in the scheduler. The scheduling variable is $\alpha = y = x_1$, since this is sufficient to characterise the equilibrium manifold. This controller is for most practical purposes equivalent to the gain scheduled controller used in (Gawthrop 1995, Gawthrop 1996), where it is also shown that this controller is superior to a single linear controller designed on the basis of linearisation about a single operating point.

Controller C2 – Contains 16 local controllers, based on off-equilibrium linearisations about the points

$$\begin{aligned}(x_1^o, x_2^o, u^o) \in \{&(0.5,-1,0),(1.5,-1,0),(2.5,-1,0),\\ &(3.5,-1,0),(0.5,0,0),(1.5,0,0),\\ &(2.5,0,0),(3.5,0,0),(0.5,1,0),\\ &(1.5,1,0),(2.5,1,0),(3.5,1,0),\\ &(0.5,4,0),(1.5,4,0),(2.5,4,0),\\ &(3.5,4,0)\}\end{aligned}$$

which also covers a wide range of transient states. The partitioning of the state-space into operating regimes is in this case selected such that at each state there exists a reasonably accurate model/controller (not only equilibrium states as with controller C1). The design parameters are the same as for C1, but here the scheduling variable is $\alpha = (x_1, x_2)$. Notice that it is not necessary to schedule on the input in this case because $A(\cdot)$ and $B(\cdot)$ do not depend on u. Again, each local controller is a 2-DOF dynamic output feedback designed with a standard pole placement design algorithm.

Controller C2-ff – While both C1 and C2 are purely feedback compensators, C2-ff is C2 extended with an additional open loop static trajectory generator that generates the equilibrium value u^r corresponding to the reference signal y^r, i.e. $u^r = -(y^r)^2$.

To compare the performance of these controllers a set of simulation experiments are considered:

- Experiment E1 - Step change in the reference from 0 to 1 at time 0
- Experiment E2 - Step change in the reference from 0 to 3 at time 0.
- Experiment E3 - Step change in the reference from 0 to 4 at time 0.

Simulation results with these 3 experiments using the controllers C1, C2 and C2-ff can be found in Figure 4. From these experiments the following observations are made.

In experiment E1, the performance of C1 and C2 is roughly similar. We see from Part 3 of Figure 4 that the system trajectory moves through transient states that are not very far away from the equilibrium manifold (the x_1-axis).

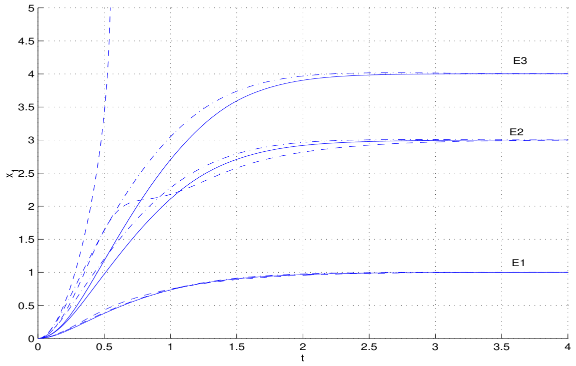
In experiment E2, the performance of C2 is clearly superior to C1. Controller C1 utilises larger control effort and there are some oscillations in both the input and output that are not present with controller C2. The problems of controller C1 in experiment E2 are caused by the fact that its transient states are significantly further away from the equilibrium manifold than in experiment E1.

In experiment E3, the closed loop becomes unstable with controller C1, while good performance and stability are still maintained with controller C2. Viewed differently, the initial state $(0,0)$ is within the region of attraction of the desired final state $(4,0)$ using the controller C2, while it is not with controller C1. The instability of C1 is clearly due to the inadequacy of the equilibrium based design during transient states far off the equilibrium manifold.

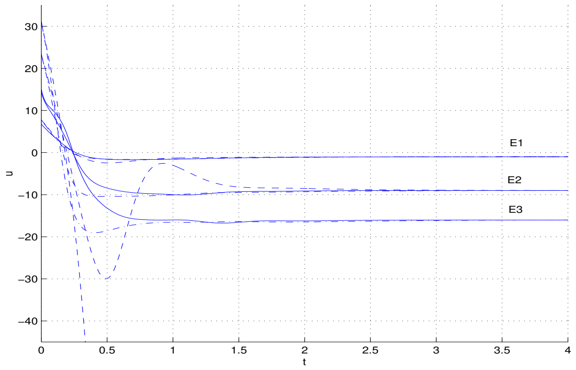
In all experiments the difference between C2 and C2-ff is minor, although the feedforward seems to make the controller meet the specifications (rise-time corresponding to closed loop poles) somewhat better.

It should be mentioned that with off-equilibrium linearisation (C2/C2-ff) the required number of local controllers is significantly increased compared to only equilibrium-based linearisation (C1). The reason for this is that a higher-dimensional scheduling vector is required to characterise the transient operating conditions accurately. Notice however that a larger number of controllers has little effect on real-time computational complexity but may have an effect on real-time computer memory. The reason for this is that a small number of local controllers are active at each time-instant.

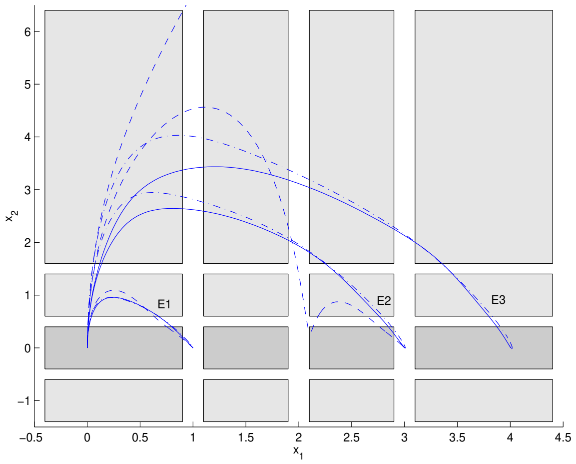
In summary, the simulation example has clearly demonstrated the possible improvements with off-equilibrium gain scheduled control design as opposed to traditional gain scheduled control design limited only to the equilibrium manifold. Improve-



(a) System output $y = x_1$.



(b) System input u.



(c) Phase plane plot (x_1, x_2).

Fig. 4. Simulations results with C1 (dashed lines), C2 (dashed-dotted lines) and C2-ff (solid lines). The different curves are for the experiments E1, E2 and E3, corresponding to setpoint $y^r = 1, 3, 4$. The shaded regions in the phase plane represent the assumed regions of validity of the local controllers. The darkest regions correspond to controller C1, while controllers C2 and C2-ff have local controllers in all regions. Notice that the plane (x_1, x_2) is a projection of the state-space of the closed loop, since the controller contains some states.

ments in both dynamic performance and region of attraction (stability) of the control system is demonstrated.

The example also demonstrates the approximately feedback linearisation property of some gain scheduling control systems (Hunt and Johansen 1997); the response from the reference to the system output is approximately linear. When only classical linearisations (at the equilibrium manifold) are applied, the approximation only holds near the equilibrium manifold, which may lead to poor performance and possibly instability. Including off-equilibrium linearisation and control design will extend the region of linearity to cover the full range of the design, including transient operating regimes. This brings the approximate feedback linearisation achieved by gain scheduling closer to exact feedback linearisation. As discussed in (Hunt and Johansen 1997), the advantages of this approach to approximate feedback linearisation compared to exact feedback linearisation is that unstable zero-dynamics can easily be factorised out rather than inverted, the design computations are numerical rather than symbolic, and certain regularity assumptions are avoided. The main disadvantage is that the approach is only approximate, although in certain cases it is known that the approximation error can be made arbitrarily small.

5. EXPERIMENTAL VERIFICATION: VEHICLE SPEED CONTROL RESULTS

The potential benefit of using off-equilibrium linearisations and controllers in gain scheduled control has been experimentally evaluated. The problem considered concerns the design of a high-performance gain scheduled speed controller for an experimental vehicle, based upon a nonlinear model of the longitudinal dynamics.

5.1 *Problem Definition and Experimental Vehicle*

The experimental vehicle is a middle class 8-tonne truck called OTTO (Optical Truck Transport Optimisation). More details of the vehicle and previous applications are given in (Franke *et al.* 1995, Gehring and Fritz 1997). In this paper, it is used to exemplify the utility of off-equilibrium linearisation and design, and a brief description of the problem is given in this section.

The structure of the speed controller is shown schematically in figure 5. The vehicle's speed y is controlled by manipulation of the throttle angle u. A further possibility is to use the brakes as an additional control signal, but for simplicity the brakes are not used in this work. The desired speed is denoted as y^r. C_y is the nonlinear speed controller; it is designed on the basis of an empirically-derived model of the dynamic relationship between throttle angle u and speed y.

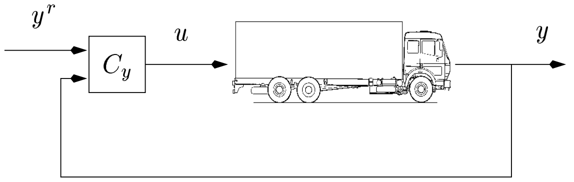


Fig. 5. Structure of the speed-control loop.

The vehicle's nonlinearities come from the various components in the drive-train. The engine produces a torque which is dependent upon the throttle angle and engine speed; this relationship is described by the nonlinear engine characteristics. The experimental vehicle is equipped with an automatic transmission. At low speeds a hydrodynamic torque converter is active. At higher speeds the torque converter is automatically bypassed by activation of a lock-up clutch. When the torque converter is not active, there is a rigid connection between the engine and the drive wheels. The torque transferred to the gear-box input depends nonlinearly upon the engine speed and the converter output speed. Finally, gear changes are dependent upon the throttle angle, vehicle speed and the currently engaged gear. The gear-box produces a hard-switching effect which results in step changes in the drive-train characteristics; it is very important to take account of this in modelling and in controller design.

These considerations lead to the conclusion that the key measurable variables which can be used to characterise the plant's nonlinearity are:

(1) vehicle speed y;
(2) throttle angle u;
(3) gear, denoted as θ.

It will be crucial to utilise this information in the construction of models and in the design of controllers. The main design goals and engineering constraints for this problem can be summarised as follows:

(1) the closed-loop system should give suitably fast disturbance rejection (road incline, rolling resistance, model error and wind resistance are in the main of low-frequency character);
(2) the design should be insensitive to unmodelled dynamics and measurement inaccuracies. These uncertainties are primarily high-frequency, resulting from the throttle actuator (which has a dead-time and a fast underdamped response) and the limited-precision speed sensor;
(3) the closed-loop should have a pre-specified command response;
(4) the closed-loop properties should be consistent over a wide operational envelope (i.e. from low speed in first gear to high speed in top gear) despite the strong system nonlinearities.

In order to meet these demands, it is clear that a high-precision nonlinear model is required.

5.2 *Experimental Data and Identified Models*

For identification of the plant dynamics in each gear, a number of throttle test inputs were applied and the resulting speed was measured. Since the system is strongly nonlinear, a goal in the design of the test inputs was to 'cover' the operational range of the plant to as great an extent as possible. Thus, it is important to investigate both the large-signal and small-signal behaviour of the system. The main input signal types applied were:

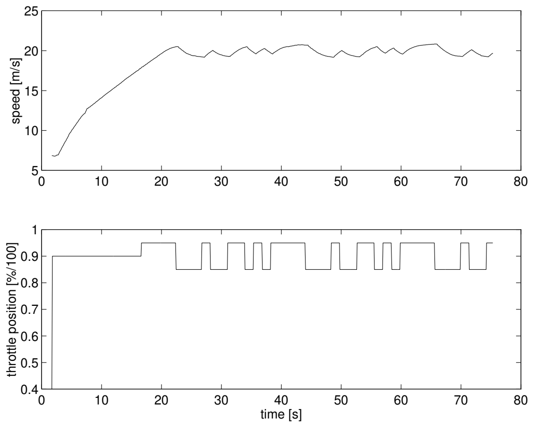


Fig. 6. Test signal and speed response: large-signal (the large step at the beginning of the sequence) and small-signal behaviour (the small amplitude PRBS). The lower part of the figure shows the test input (throttle command); the upper part shows the measured vehicle speed. The test begins by manually taking the plant to the equilibrium close to $(y, u) = (7, 0.4)$.

(1) PRBS-signals (pseudo-random binary sequence) of relatively low magnitude (typically 10%) were applied around a number of equilibria in each gear;
(2) in each gear, large steps of varying magnitude (both positive and negative) with a superimposed low-amplitude PRBS excitation were applied. Before application of these signals the system was driven manually to various equilibria.

One example of the latter test-signal type is shown in figure 6. The trajectory described by this re-

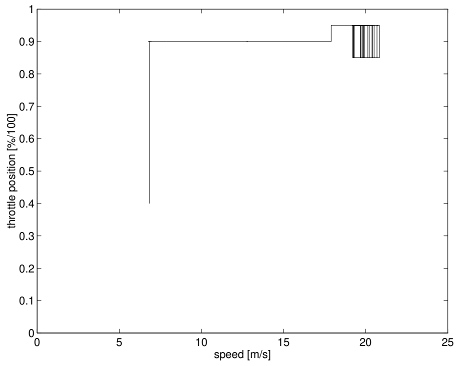


Fig. 7. System trajectory. The plant starts at an equilibrium near the point $(7, 0.4)$, moves through a non-stationary off-equilibrium regime, and converges to an area around the equilibrium at approximately $(20, 0.9)$.

sponse in the (y, u)-plane is shown in figure 7.[1] The full set of data used in this paper, together with the operating regime decomposition and equilibria, are shown in figure 8. These regimes are characterised by throttle, speed and gear, and the equilibrium manifold for each gear is represented by a solid line. The regimes which are associated with off-equilibrium operation of the vehicle can be easily observed.

Notice that the operating regimes are typically not regions with hard boundaries, as illustrated in the figure, but rather regions with soft boundaries (or fuzzy sets). This means that there will be a smooth transition between local controllers when moving between regimes. This is viewed and implemented as an interpolation technique.

Local LTI models of 1st or 2nd order were identified for each operating regime. The nonlinearity of the vehicle can be seen in Figure 9 where the impulse responses of each local LTI model are illustrated. It can be observed that there are significant changes in pole and gain over the operating range, reflecting the nonlinear torque curve of the engine: the response is faster with higher high-frequency gain at intermediate throttle than at very high and low throttle. Also, due to "saturation" in the engine, the response is faster and with higher high-frequency gain near equilibria than far away from equilibria. Different gears also have significantly different dynamics.

5.3 *Experimental Results*

In the experimental results presented here, the local LTI control designs were based upon a 2-DOF dynamic output feedback designed by pole assignment. Interpolation of the controller parameters was carried out on the basis of the operating regime decomposition defined by the three scheduling variables speed, throttle and gear, as in Figure 8. For more details of the control design algorithm, refer to (Hunt and Johansen 1997). The closed-loop poles and observer poles were indirectly chosen by specifying the desired risetime and damping of equivalent 2nd order continuous-time transfer-functions. The feedback controllers contain integral action, but no trajectory generator was applied, i.e. $u^r = 0$.

Figure 10 shows two experimental results in 3rd gear. In each figure the upper plot shows the set-point and measured speed. The lower plot shows the control signal (throttle angle). The response in figure 10(a) shows a linear controller designed for a closed-loop risetime of 7s, critical damping, and a deadbeat observer. The design was based upon one model from the set of identified LTI models (regime R20 in figure 8). The corresponding experimental result for a gain scheduled controller using off-equilibrium local LTI models and controllers is shown in figure 10(b). Each local controller has the same design specification as the single linear controller (risetime 7s, critical damping, deadbeat observer).

Clearly, the linear controller gives a very large overshoot in this test, while the off-equilibrium-based design meets the specification of no overshoot. The reason for this is that the off-equilibrium-based design is able to recover much faster from the low-gain region (regime R17 in Figures 8 and 9) at low throttle since this information is captured by the off-equilibrium local LTI models and controllers, but not in the equilibrium ones. Experience with scheduling on either speed or throttle alone indicates that a full off-equilibrium design with scheduling on both speed and throttle is needed to achieve high performance control without overshoot, although scheduling on throttle alone significantly reduces or eliminates the overshoot for many (but not all) operating conditions.[2]

Figure 11 shows two experimental results in 1st gear. The response in figure 11(a) shows a linear controller designed for a closed-loop risetime of 5s, critical damping, and an observer 'risetime' of 2s. The design was based upon one model from the set of identified LTI models described above (regime R3 in figure 8). The corresponding experimental result for a gain scheduled controller using off-equilibrium models and controllers is shown in figure 11(b). Each local controller has a similar design specification to the single linear controller

[1] Note that for first-order models this plane completely describes the plant's operational range for a given gear. In the first-order case, therefore, this plane must be sufficiently covered by measurement points. In practice, both first and second-order models were used in system identification.

[2] It is known that the main nonlinearities of the vehicle can be associated with the throttle (Hunt *et al.* 1996*a*).

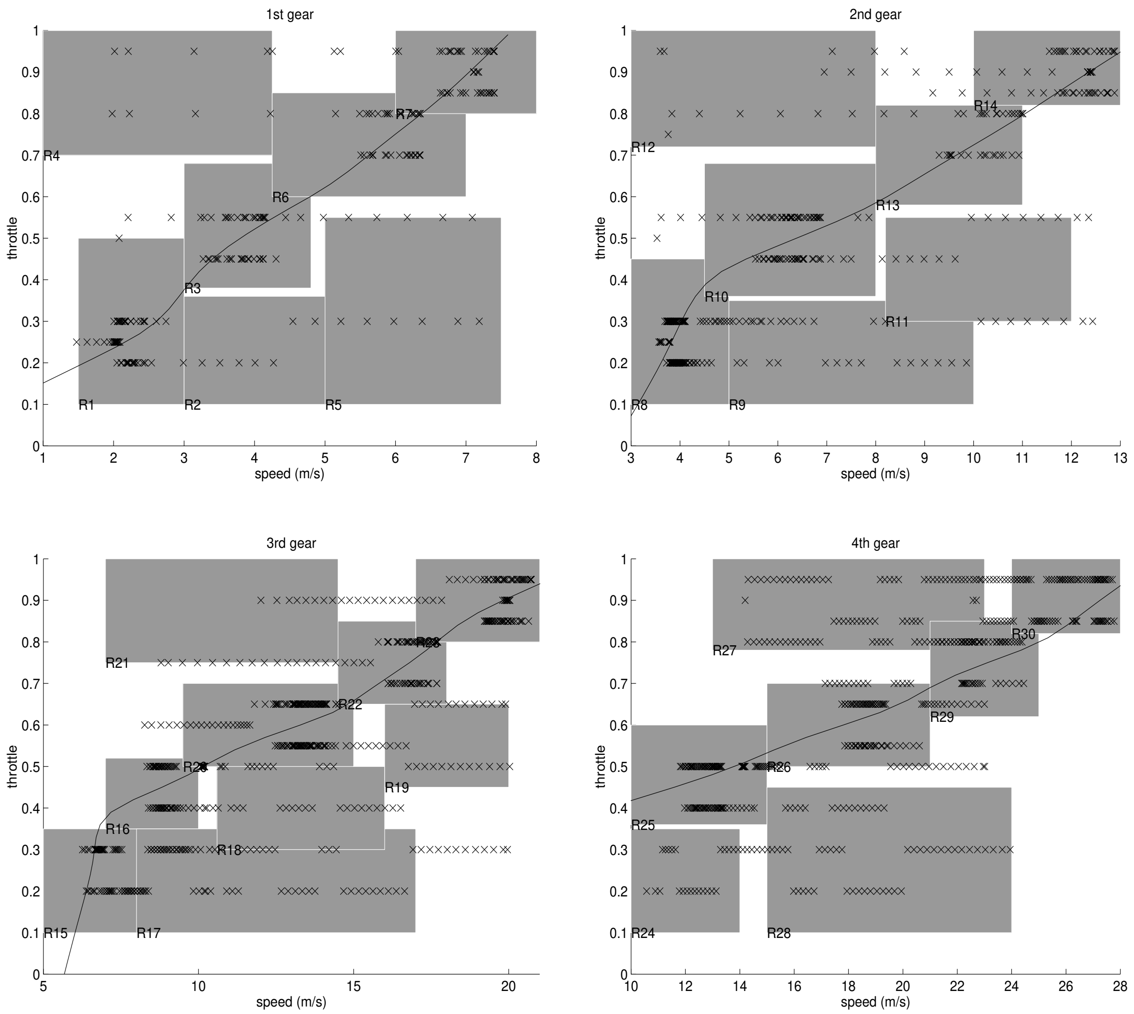


Fig. 8. Operating regimes for OTTO. The regimes are characterised by throttle, speed and gear. The marked points correspond to experimental data, and the solid lines are the estimated equilibrium manifolds for each gear.

(risetime 4s, critical damping, observer 'risetime' of 1.5s).

Both the linear and nonlinear controllers in this case meet the no-overshoot specification. However, the risetime specifications are met only by the nonlinear gain-scheduled controller. The control signal in the linear test shown in figure 11(a) appears to respond linearly, while the nonlinear features of the plant are apparent in the measured speed response; during the step change from 2–7m/s the response is initially very dynamic, but from around 6m/s the system takes a long time to reach the setpoint. The nonlinear controller's response in figure 11(b) is excellent. The very fast risetimes are achieved without overshoot. Notice that in comparison with the linear response the speed goes directly to the setpoint in the desired (approximately linear) manner. On the other hand, the nonlinear effects are now apparent in the control signal, where a definite changeover in controller characteristics can be seen at throttle angle 80%. This approximate feedback linearisation (Hunt and Johansen 1997) is a natural consequence of the fact that the desired (linear) closed loop response is independent of the operating point (for each gear).

A further aspect that should be taken into account (this is not done here because the mass did not vary significantly during the experiments) is that the vehicle dynamics depend strongly on the total mass/load. One approach would be to also include the mass of the vehicle as a scheduling variable. On-line estimation of the vehicle's total mass is a fairly simple task.

5.4 *Other Applications*

Another, but somewhat different, example of off-equilibrium gain scheduled control design can be found in (Hunt *et al.* 1996*b*) where an autonomous steering problem is studied by simulation. The lateral car dynamics are essentially linear for a fixed speed (the lateral car dynamics are usually much faster than the longitudinal dynamics). The idea

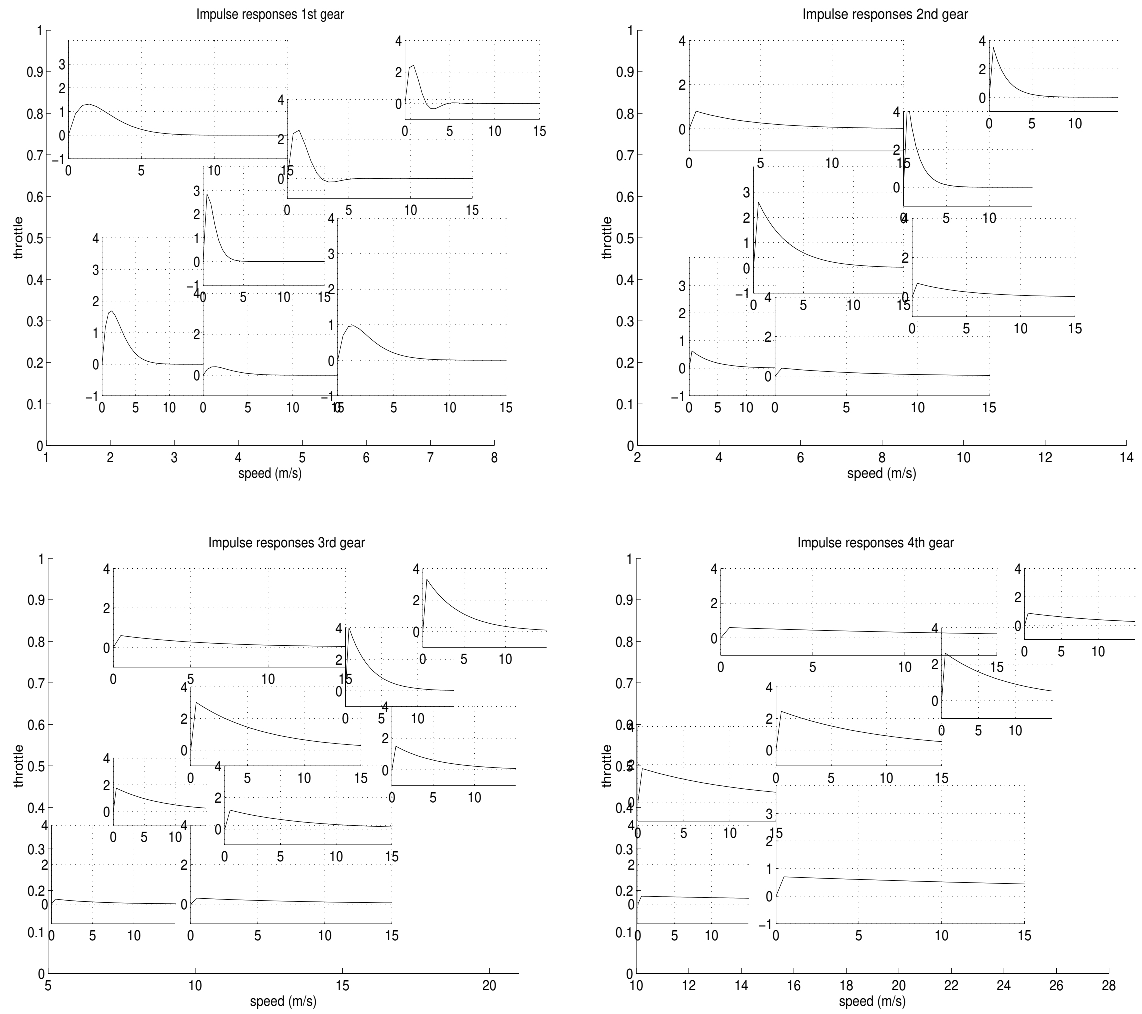
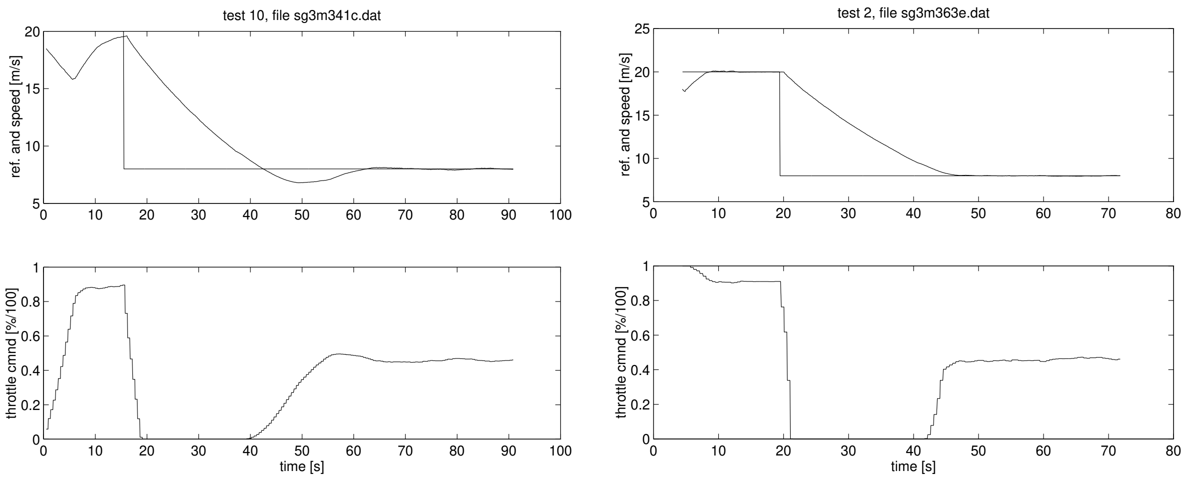


Fig. 9. Impulse responses of the identified local LTI models within each operating regime of OTTO.

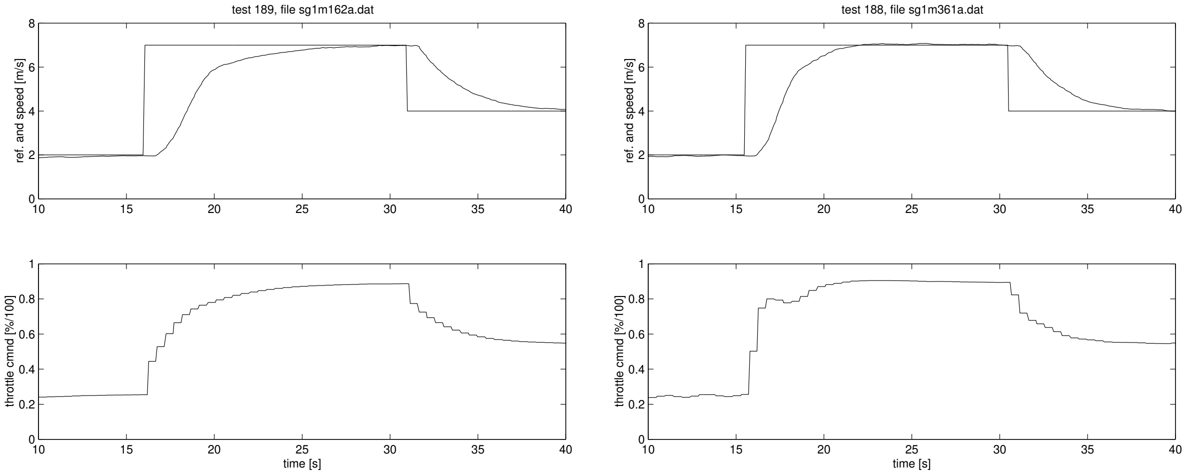


(a) Linear controller.

(b) Gain-scheduled controller.

Fig. 10. Tracking results in 3rd gear. (Experimental results with the test vehicle.)

in that work is that the desired closed loop dynamics should be different in transient states (faster compensation) than at close-to-steady-state (slow compensation). This leads to a good trade-off between safety and comfort.



(a) Linear controller.

(b) Gain scheduled controller.

Fig. 11. Tracking results in 1st gear. (Experimental results with the test vehicle.)

6. CONCLUDING REMARKS

In this paper the potential benefits of the incorporation of off-equilibrium linearisation in addition to classical linearisation in gain scheduling control design have been studied. Both the simulation example and the experimental results demonstrate significant improvements in performance. To conclude, off-equilibrium linearisation is highly recommended when designing gain scheduled controllers for highly nonlinear systems which are subject to major transients.

APPENDIX

Here it proven that a set of LTI linearisations scheduled on a given trajectory (including off-equilibrium linearisations), when interpolated properly, leads to a close approximation to the LTV system resulting from dynamic linearisation about the trajectory. Consider the autonomous nonlinear time-invariant system

$$\dot{x} = f(x), \qquad x(0) = \overline{x} \tag{29}$$

where f if differentiable. Assume a given continuous trajectory x_0 that satisfies $x_0(t) \in X$ is given, and $X \subset R^n$ is a compact subset of the state-space and $\dot{x}_0(t) = f(x_0(t))$, $x_0(0) = \overline{x}$. The dynamic linearisation about the trajectory x_0 is now the LTV system

$$\dot{\xi}_1(t) - \dot{x}_0(t) = A(x_0(t))(\xi_1(t) - x_0(t)) \tag{30}$$

$$\xi_1(0) = \overline{x} \tag{31}$$

where $A(x) = \frac{\partial f}{\partial x}(x)$. As an alternative, suppose linearisations about N points $X^0 = \{x_1, x_2, ..., x_N\} \subset X$ are given in terms of a set of LTI systems

$$\dot{x} = f(x_i) + A(x_i)(x - x_i)$$

An approximation to the dynamic linearisation is now given by the LTV system formed by interpolating (or scheduling) the set of LTIs

$$\dot{\xi}_2(t) = \sum_{i=1}^{N} (f_i(x_i) + A(x_i)(\xi_2(t) - x_i))\mu_i(x_0(t)) \tag{32}$$

$$\xi_2(0) = \overline{x} \tag{33}$$

where the function μ_i is an interpolation function. The choice of interpolation functions is obviously important, but for simplicity only piecewise linear approximations to the system (30) are considered, i.e. the interpolation functions are piecewise constant. Partition the set X into N disjoint connected regions X_i such that $X_1 \cup X_2 \cup \cdots \cup X_N = X$, and suppose $x_i \in X_i$. The function μ_i is now the characteristic function for X_i, i.e. $\mu_i(x) = 1$ if $x \in X_i$, and $\mu_i(x) = 0$ otherwise.

Let use define deviations from the trajectory x_0 by $\Delta\xi_1(t) = \xi_1(t) - x_0(t)$ and $\Delta\xi_2(t) = \xi_2(t) - x_0(t)$. Intuitively, one would expect that the system (32) approximates (30) when "X^0 covers X densely", for example in the sense that the Haussdorf-like distance between X^0 and X is sufficiently small:

$$\delta = \sup_{x \in X} \inf_{\xi \in X^0} ||x - \xi||_2 \tag{34}$$

This is formalised in the following theorem:

Theorem 1. *Assume X is a compact set, and f is smooth. Then*

$$\dot{\Delta\xi}_1(t) = A(x_0(t))\Delta\xi_1(t) \tag{35}$$

$$\dot{\Delta\xi}_2(t) = A(x_0(t))\Delta\xi_2(t) + \varepsilon(\Delta\xi_2(t), x_0(t)) \tag{36}$$

and there exist constants c_1 and c_2 such that ε satisfies

$$||\varepsilon(\Delta\xi_2, x_0)||_2 \leq \delta(c_1 + c_2||\Delta\xi_2||_2) \qquad (37)$$

for all $\Delta\xi_2 \in R^n$ and $x_0 \in X$.

Proof: The expression (35) follows immediately from the definitions. From (32)

$$\dot{\xi}_2(t) - \sum_{i=1}^{N} f(x_i)\mu_i(x_0(t)) = \sum_{i=1}^{N} A(x_i)(\xi_2(t) - x_i)\mu_i(x_0(t))$$

which gives

$$\dot{\xi}_2(t) - f(x_0(t)) = A(x_0(t))(\xi_2(t) - x_0(t)) + \varepsilon(\Delta\xi_2(t), x_0(t))$$

(equivalent to (36)) and the following expression for ε:

$$\begin{aligned}\varepsilon(\Delta\xi_2, x_0) &= \sum_{i=1}^{N} \left(f(x_i)\mu_i(x_0) - f(x_0)\right) \\ &+ \sum_{i=1}^{N} \left(A(x_i)(\xi_2 - x_i) - A(x_0)(\xi_2 - x_0)\right)\mu_i(x_0) \\ &= \sum_{i=1}^{N} \left(f(x_i)\mu_i(x_0) - f(x_0)\right) \\ &+ \sum_{i=1}^{N} \left((A(x_i) - A(x_0))\Delta\xi_2 + A(x_i)(x_0 - x_i)\right)\mu_i(x_0)\end{aligned}$$

The parameters of the interpolated system are piecewise constant approximations to the parameters of the LTV system at each point $x_0(t)$ on the trajectory. Hence, from the regularity assumptions on f and A, the compactness of X and (34) it is clear that there exists constants K_1, K_2 and K_3 such that

$$\begin{aligned}\sup_{x\in X}\left|\left|A(x) - \sum_{i=1}^{N} A(x_i)\mu_i(x)\right|\right| &\leq K_1\delta \\ \sup_{x\in X}\left|\left|f(x) - \sum_{i=1}^{N} f(x_i)\mu_i(x)\right|\right|_2 &\leq K_2\delta \\ \sup_{x\in X}\left|\left|x - \sum_{i=1}^{N} x_i\mu_i(x)\right|\right|_2 &\leq \delta \\ \sup_{x\in X}||A(x)|| &\leq K_3\end{aligned}$$

where $||\cdot||$ is the induced matrix norm. The result follows from the Schwartz and triangle inequalities.

ACKNOWLEDGEMENT

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