IPTC 11728

Challenges in Parameter Estimation of Models for Offshore Oil and Gas Production Optimization

S. M. Elgsaeter, SPE, NTNU, O. Slupphaug, SPE, ABB, T.A. Johansen, NTNU

Abstract

Some authors have proposed decomposing real-time optimization of offshore oil and gas production into production optimization, maximization of value from the daily production of reservoir fluids, and reservoir management, optimization of injection and reservoir drainage on the time scales of months and years. Each subproblem may consider different models which are less complex than an all-purpose model.

Any model used in either reservoir management or production optimization must be fitted to production data, a set of historical measurements, consisting of test separator measurements and possibly others such as measured total rates. If the information content in this set of production data is low, it may cause uncertainty. In recent years, explicit treatment of uncertainty in reservoir models has received much attention, but little attention has been paid to uncertainty in models for production optimization.

In this paper we make model-independent assertions about uncertainty in production optimization through a case study of actual production data from a North Sea oil and gas field. As the information content in production data describing normal day-to-day operations was observed to be low, we propose that an explicit treatment of uncertainty may be as relevant for production optimization as it is for reservoir management.

We highlight three challenges for further research based on our observations that have received little attention in the literature until now. Firstly, the expected losses incurred due to uncertainty should be quantified to assess their significance. Secondly, costs and values of uncertainty mitigation should be estimated to allow proposed actions to be evaluated through structured business cases. Thirdly, strategies for making decisions in day-to-day operations under uncertainty should be investigated.

Introduction

Production in the context of offshore oil and gas fields can be considered the total output of wells, a mass flow with components including hydrocarbons, for simplicity often lumped into oil and gas, often in addition to water, CO_2, H_2S, sand and possibly other components. Production travels from wells through flow lines to a processing facility for separation, as illustrated in Figure 1.

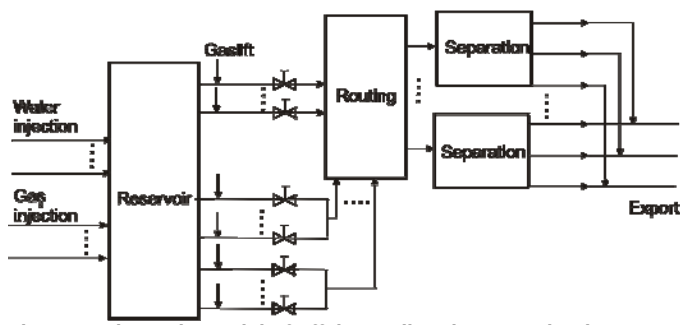

Fig 1- A schematic model of offshore oil and gas production

Production is constrained by several factors. On the field level, the capacity of the facilities to separate components of production and the amount of available lift gas is limited by the amount of produced gas and by compressor capacity. The production of groups of wells may travel through shared flow lines or inlet separators which have a limited capacity. The production of individual wells may be constrained due to slugging, other flow assurance issues or due to reservoir production constraints.

The production of a well is manipulated by changing the settings of production equipment. Decision variables are a subset of all possible settings through which production is influenced. Different decision variables may manipulate the same equipment. We may for instance choose to influence the settings of a gas lift choke by deciding a target lift gas rate, a target annulus pressure or a target gas lift choke opening. On short timescales individual wells can be influenced by production choke settings, by gas lift choke settings and possibly routing settings.

In the context of hydrocarbon processing, *real-time optimization* (RTO) has been defined in [1] as a process of measure-calculate-control cycle at a frequency, which maintains the system's optimal operating conditions within the time-constant constraints of the system. Some authors have

suggested dividing real-time optimization into subproblems on different time scales to limit complexity, and to consider separately *reservoir management*, optimization of injection and reservoir drainage on the time scales of months and years, and *production optimization*, maximization of value from the daily production of reservoir fluids [1]. Reservoir management can specify constraints on production optimization to link these problems. We will refer to models for production optimization as *production models* and models for reservoir management as *reservoir models.*

Parameters of production models must be fitted to production data through *parameter estimation* to compensate for un-modelled disturbances and to set reasonable values for physical parameters which cannot be measured directly or determined in the laboratory. Erosion of production chokes is an example an umodeled disturbance.

Planned excitation is some planned variation in one ore more decision variables designed to reveal information on production through measurements, for instance a multi-rate well test. The information flow in production optimization is illustrated in Figure 2.

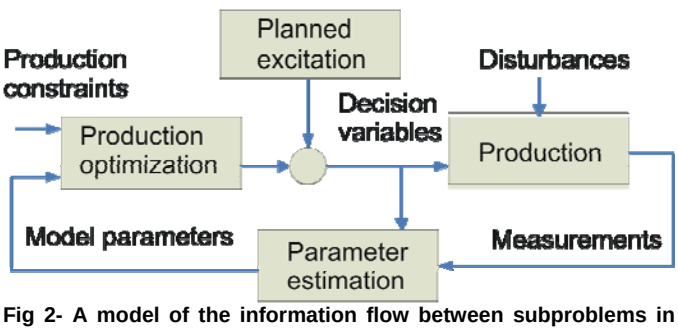

Fig 2- A model of the information flow between subproblems in production optimization.

Prior work

Quantifying uncertainty in reservoir models has attracted much interest in recent years. Authors have described fitting multiple models to data [3], describing measurement uncertainties [4] and sensitivity analysis using prior knowledge of parameter uncertainty [5]. Very few references have been found which attempt to quantify uncertainty in production models. An estimate of uncertainty derived from well tests for wells with rate-independent gas-oil ratio and water-oil ratios is derived in [6], and a strategy for production optimization of such wells that treats uncertainty explicitly is suggested in [7]. The lack of attention to the subject of uncertainty in production optimization seems paradoxical as production models are fitted to the same production data as reservoir models. As quantified uncertainty depends on the model used, it is hard to discern from studies of uncertainty in reservoir models whether observed uncertainties are a result of the low information content in production data or of the models. No references have been found which address the characteristics of typical production data itself and what implications these characteristics will have in terms of modeling.

The concept of estimating the value of information in uncertain systems has roots in stochastic programming, which uses the concept of value of perfect information, and the idea of assigning a value on information have found applications in reservoir modeling [8]. No references have been found which address the value of information in the context of production optimization.

Problem formulation

This paper rests on the assumption that any production model will require fitting to production data, and therefore the characteristics of production data will have implications for production optimization. We will seek to make model-independent assertions on the significance of uncertainty in production optimization through a case study of production data.

We first describe some preliminary considerations on the relationship between production data and production optimization A case study of a North Sea oil and gas field is used to illustrate the typical state of real-life production data. Finally key challenges motivated by the observations made are outlined.

The relationship between production optimization, models and data

As the number of logged measurements from oil and gas production on one field may run into the thousands, we must focus on selected measurements in our case study. In this section we will describe the requirements for production data in the context of production optimization, which will motivate the focus of the case-study in the following section.

Production optimization

One possible formulation of the production optimization problem is

$$\hat{u}^* = \arg\max_u M(\hat{x}, u, d) \quad (1)$$

$$0 = f(\dot{\hat{x}}, \hat{x}, u, d, \theta) \quad (2)$$

$$0 \le c(\hat{x}, u, d), \quad (3)$$

where u is a vector of decision variables to be determined, d is a vector of measured disturbances which are independent of u. x is a vector of rates of each modeled component produces for each well considered. M is an economic objective function, (2) is a production model and (3) are production constraints. Throughout this paper variables with a hat (ˆ) will denote estimates and variables with a bar (¯) will denote measurements.

Normally x is not measured. Instead, the total rates of oil is measured with a fiscal measurement, while total rates of gas and water can often be determined by a mass balance of several topside rate measurements. In addition, wells are periodically routed to a test separator and rates measured. On some fields, multiphase rate measurements at each well may be available. Let y be a vector of measured variables, which may include separator rate measurements, fiscal or other total rate measurements or multiphase rate measurements where available. To describe the relationship between rates x and measurements y, and to capture the typically time-varying nature of routing, we define a routing matrix R of ones and

zeros such that when measurement uncertainties can be neglected,

$$\bar{y}[t] \triangleq \bar{R}[t]x[t] \qquad (4)$$

Production optimization employs a production model (2), which may be implicit or explicit and which includes a vector of fitted parameters $\hat{\theta}$. This paper is a study of production data and what implications the state of production data will have for the design of the model. We have purposely attempted to keep the discussion as independent of the choice of production model as possible. In practice, production models are either commercial simulators of well and pipeline, or a lookup-table or other proxy-model derived from such simulators [9]. Which components to model is a design question, which depends on the choice of c(x, u, d) and M(x, u, d), and typically at least oil, gas and water production is modeled.

The parameter estimation problem is to determine $\hat{\theta}$ from a set of historical production data

$$Z^N = \begin{pmatrix} \bar{y}[1] & \bar{d}[1] & \bar{u}[1] & \bar{y}[2] & \bar{d}[2] & \bar{u}[2] & \dots \\ & & & & \bar{y}[N] & \bar{d}[N] & \bar{u}[N] \end{pmatrix}. \qquad (5)$$

Let the residuals of Z^N for a given model structure and estimate $\hat{\theta}$ be given by

$$\varepsilon[t] \triangleq \bar{y}[t] - \hat{y}[t], t \in \{1, \dots, N\}. \qquad (6)$$

Fitted parameters can be found by minimizing the sum of squared residuals

$$\hat{\theta} = \arg\min_{\theta} \sum_{t=1}^{N} w[t]\varepsilon[t], \qquad (7)$$

where w[t] is a user-specified weighting.

In general the time interval spanned by the tuning set Z^N is limited by un-modelled disturbances, as it is assumed that all variations observed in Z^N can be described by the chosen model structure. To update the model as changing states or disturbances cause production to change, the window spanned by Z^N can be moved as new data becomes available, a moving-horizon approach [10].

Production models, information and excitation

In this paper we aim to reach conclusions about the implications production data have on production modeling which are as independent of the choice of model structure as possible. Selection of variables, both of measured variables y and decision variables u, is a design choice [11]. The emphasis of this paper is not to discuss the selection of variables, but for our discussion we will still have to choose what variables to consider y and u.

As our emphasis is on the optimization of wells and flow lines, rather than the reservoir or topside facilities, the main settings through which we can influence production are related to routing, gas lift and production chokes. Routing is accounted for through the matrix R in (4). For simplicity we consider one decision variable for gas lift and production choke settings of each well and choose relative production valve opening z and gas lift rates q_{gl} as our decision variable u = [z q_{gl}]. The only modeled disturbance we will consider is routing d = R.

In this paper when discussing information content in production data Z^N, we will be referring to the information in production data on sensitivities

$$\frac{dy}{du} = R\frac{dx}{du}. \qquad (8)$$

This idea is illustrated in Example 1.

Example 1 (Sensitivities) Consider that the objective of production optimization (1) – (3) is to find the gas lift rate u = q_{gl}^1 of a single gas-lifted well which maximizes oil production, M(x, u, d)= q_o^1 , and assume that the rate of produced oil is measured y = q_o^1. Assume that the relationship between gas lift rate and produced oil is as shown in Figure 3, and that production is currently at (q_{gl}^1,q_o^1). To solve the production optimization problem, the production model should express how changing gas lift rates will influence change in produced oil. Tthe model may be found through physical modeling, through expiermentation or a combination thereof. Ideally the production model should predict the response in oil production Δq_o^1 to all conceivable changes Δq_{gl}^1 in gas lift correctly, but as a minimum, Z^N should allow at least, a form of (8)*, to be determined***.**

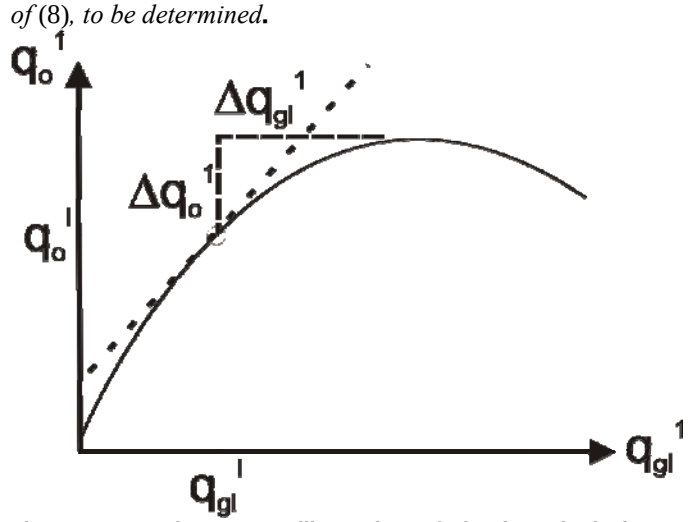

Fig 3 - Example 1: An illustration of the hypothetical true performance curve (solid), along with the sensitivity $\frac{dq_o^1}{dq_{gl}^1}$ illustrated by the dotted line.

Information content in production data is related to changes in u, which we will refer to as excitation. If one or more elements in u do not change or change only very little in Z^N , then the data set is "weak" in some way, sensitivities cannot be determined from production data, (7) will be ill-conditioned and determining certain elements of uniquely may become diffcult.

It is possible in principle to introduce planned excitation to increase the information content in Z^N, for instance by performing multi-rate well tests at intervals for all wells. As planned excitation may have both a cost and a risk, it would be

preferable to fit production models to production data stemming from normal day-to-day operations if possible.

The objective of decomposing production optimization into reservoir management and production optimization is that each subproblem may consider simpler models considering different time scales [1]. Reservoir management may consider a coarse production model and a detailed reservoir model, while production optimization may consider a coarse reservoir model and a detailed production model.

The length of the tuning set Z^N is limited by the interval for which the chosen model can describe observed variations in Z^N, and for production models with a coarse description of reservoir dynamics this may mean that Z^N must span a relatively short time interval. The information content in Z^N will decrease with decreasing length in practice, which introduces the possibility that (7) may be subject to significant uncertainties.

On auxiliary measurements

In this paper we will use the term *auxiliary measurement* to refer to measured variables which may vary with production but which are dependent on u. Measurements which are dependent on the choice of u may for instance be the measured pressure or temperature upstream of a production choke when production choke opening is an element in u. It is conceivable that the ability of the model to describe the data set (5) can be improved by exploiting such auxiliary measurements.

The disadvantage of including axuilliary measurements y_a in the production model is that a second model

$$y_a = g_a(\dot{\hat{x}}, \hat{x}, u, d) \qquad (9)$$

is needed to close the production model when being applied for in production optimization (1)-(3).

A key difference between models for production monitoring and models for production optimization, is that models for production monitoring do not need to close (9). The disadvantage of introducing a second model (9) for closure is that it increases the chance of introducing structural model errors, and measured pressures and temperatures may themselves be subject to significant measurement uncertainties. For these reasons, we will not consider measured pressures or temperatures in our case study, although we cannot rule out that model performance could improve by including such measurements.

Production data: A case-study

In this section contains a study of a set of real-world production data. The emphasis of the study is to highlight characteristics of production data that may have implications for production optimization.

Field description

We will consider one field in this case study:

> **Field A**: A mature North Sea field in decline with 20 platform production wells producing oil, gas and water, all with the aid of gas-lift. A data set spanning 240 days is considered.

We have also studied production data from another field, but as we made similar observations for it, we will omit a detailed description here for brevity. No multiphase flow meters are available while one test separator with rate measurements of oil, water and gas are available. The total rate of produced oil is measured by fiscal measurements, while total rates of gas and water can be estimated by aggregating several measured rates in the processing facilities. No allocated rates are used in this paper; we attempt to draw conclusions based entirely on measurements. The owner of the production data requested it be kept anonymous; therefore all figures display normalized variables.

Field factor

Total production is measured continuously by rate measurements in the processing facilities for the field considered. In addition, an estimate of total production can be found by summing the rates of each measured component at the most recent well test for each well. When the field is produced close to the settings of the most recent well tests and disturbances since the last well tests are small, these two sets of rates should have comparable values. We define the ratio between these two sets of rates as field factors ($F_o[t]$, $F_g[t]$, $F_w[t]$)

$$F_o[t] \triangleq \frac{\bar{q}_o^{tot}[t]}{\sum_{i=1}^{n_w} z^i \bar{q}_o^{test,i}[t]} \qquad (10)$$

$$F_g[t] \triangleq \frac{\bar{q}_g^{tot}[t]}{\sum_{i=1}^{n_w} z^i \bar{q}_g^{test,i}[t]} \qquad (11)$$

$$F_w[t] \triangleq \frac{\bar{q}_w^{tot}[t]}{\sum_{i=1}^{n_w} z^i \bar{q}_w^{test,i}[t]} \qquad (12)$$

where a feld factor equal to one means that measurements of total production match the sum of the last well tests perfectly. $q_o^{test,i}[t]$, $q_g^{test,i}[t]$ and $q_w^{test,i}[t]$ are the rates of oil, gas and water, respectively, that have been measured for well i at the most recent well test at time t. z^i is the relative production choke opening for well i, and n_w is the number of wells. A field factor different from 1 may be an indication of the presence of measurement uncertainties.

Data

The average number of single-rate and multi-rate well test per year was 4.87 and 0.29 respectively.

Figure 4 shows how produced rates of oil and liquid vary with changing gas lift rates measured during single-rate well tests for one chosen well. Figure 5 shows gas-oil ratios and water cut measured at the test separator as a function of time. Figure 6 shows an example of an observed response in

measured total rates as the gas lift rate of a well is changed. Figure 7 illustrate the information content of gas lift rates for a time interval, compared with the number of well tests in the same interval. Figure 8 shows a comparison of summed test separator rates and measured total rates and the estimates of the field factors.

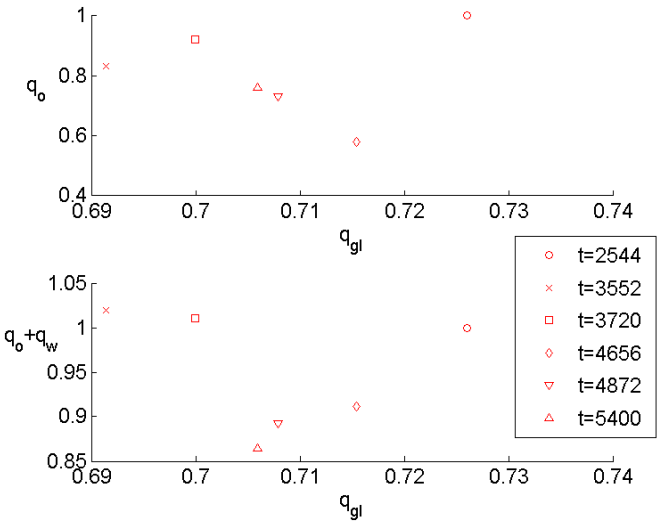

Fig 4- Case study (Field A): well tests for a representative well. Top graph: normalized measured oil rate q_o against normalized measured sum of oil oil and gas rate q_o+q_w.

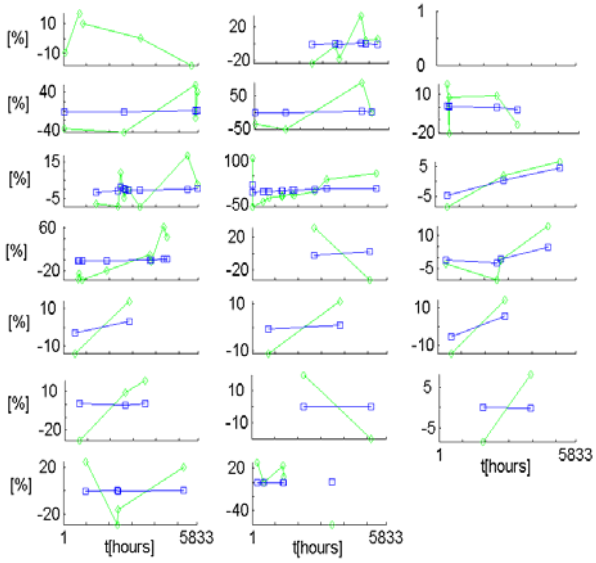

Fig 5 - Case study (Field A): change in gas-oil ratios (diamond) and watercut (square) relative to the average as measured at the test separator.

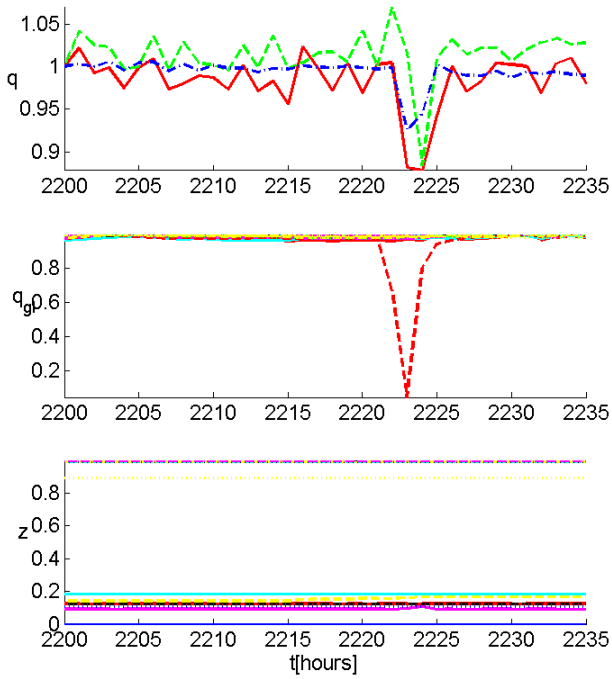

Fig 6- Case study (Field A): the observed response in total oil (solid), total gas (dashed) and total water rates (dashdot) to changes in gas lift rates q_{gl}, while all production valve openings z are kept constant.

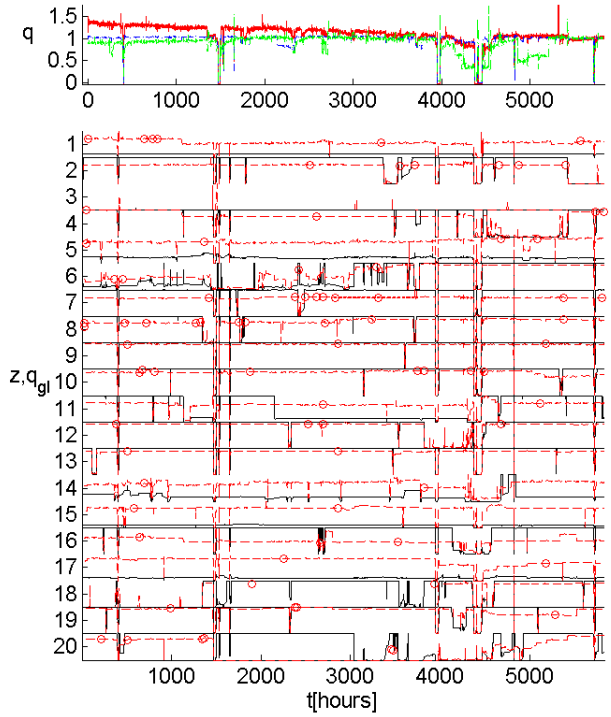

Fig 7 - Case study (Field A): top graph: measured total oil production q_o^{tot}(solid), measured total water production q_w^{tot} (dashdot) and measured total gas production q_g^{tot} (dashed). Bottom graph: gas lift rates (dashed) q_{gl}, relative choke opening z, and well tests (circles) for all wells plotted in ascending order according to well index (well index along y-axis).

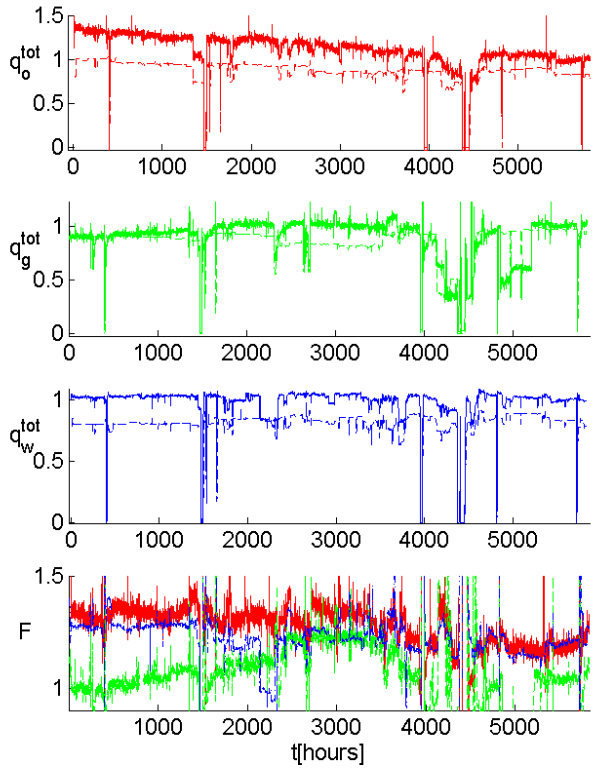

Fig 8 - Case study (Field A): top three graphs: a comparison of summed rates measured at the test separator (dashed) to measured total rates of oil, gas and water (solid). Bottom graph: field factors of oil, gas and water.

Observations

Oftentimes, only single-rate well tests are performed: Single-rate well tests do not directly yield information on sensitivities, yet such test far outnumber multi-rate well tests. This observation can be attributed to the cost of performing a multi-rate well test. The low frequency of multi-rate well tests will make it difficult to verify sensitivities in production models and that parameters may be uncertain.

It may be diffcult to determine sensitivities by aggregating single-rate well tests: In principle it is feasible to combine pairs of single-rate well tests to determine sensitivities. Figure 4 illustrates that single-rate well tests are often performed over a narrow interval of decision variable values. When determining sensitivities by aggregating single-rate well tests performed in this manner, estimates may become very sensitive to disturbances that occurred in the time interval between those well-tests.

Some well characteristics remain similar for days, weeks and even months , while other characteristics vary in ways that may be difficult to model and may require limiting the length of the tuning set: Figure 5 illustrates variations in gas-oil ratios and watercuts, as measured by well tests for each invidivual well.

The absence of variation in ratios may aid production modeling. For the field in question it may be reasonable to make the simplifying assumption that watercut is time-invaraint for the timescales considered.

When variation in ratios are observed, this may be attributable to time-varying well characteristics, measurement uncertainty or a combination of the two. For the field in question and on the timescales considered, variations of gas-oil ratios on the order of ±20% relative to the average are observed. If we attribute this variation to well dynamics, we must either include a model of these dynamics in the production model or treat these dynamics as an un-modeled disturbance and limit the permissible length of the tuning set to intervals short enough for this disturbance to be negligable. In general, the shorter the time interval spanned by the tuning set, the lower the information content and the more significant uncertainty will be in (7).

As the operator reports that there is no free gas on this field, and as the gas-oil ratio depend on gas rate measurements that may be prone to a higher measurement uncertainty than liquid rate measurements, we may speculate that observed variation in gas-oil ratios is a result of measurement uncertainty. If this is the case, we may be unable to uniquely determine the gas-oil ratio from historical well tests, resulting in uncertainty in the production model.

Measurements of total rates contain information on production of individual wells: Figure 6 illustrates the observed response in measured total rates to a change in the gas lift rate of a single well. This response reveals information on sensitivities which may be exploited in production modeling. The response considered is large and isolated and could easily be determined through manual inspection, but oftentimes several wells are excited simultaneously or responses are far less obvious, and in such instances it may be advantageous to extract information using a parameter estimation procedure on the form (7).

The information content of total production rate measurements is a significant addition to the information in well test measurements: Figure 7 allows the frequency at which decision variables are changed to be compared with the wellt test frequency. In principle each variation in z or q_{gl} produces a response in measured total rates, although not all such responses may be discernible. Figure 7 illustrates that the frequency at which z or q_{gl} are changed during normal operations is high comparable to the frequency at which well tests are performed. Furthermore, most well tests are single-rate, giving no direct information on sensitivities, but changes z or q_{gl} do yield information on sensitivities.

The overall information content of data, both of total rates and of test separator measurements, is still quite low: We can see from Figure 7 that gas lift rates q_{gl} and production choke openings z on some wells remain constant for months. When considering tuning sets spanning short time intervals, this may mean that some uncertainty should still be expected in (7) even if both total rate measurements and test separator measurements are included in y.

Production varies within a narrow band around what is believed to be optimum production: Figure 7 illustrates that, if shutdowns are excluded, gas lift rates vary within a relatively small range. This may mean that we are only able to verify a production model for a narrow range of decision variable values. Unceratinty in a fitted model should be expected to be larger outside the range of decision variable values observed

in Z^N, and production optimization may need to take this into account when suggesting changes in decision variables.

Significant field-factors indicate the presence of measurement uncertainties: Figure 8 illustrate that the field factors of oil and water are in the region of 1.3 and in the region of 1.1 for gas on average. Field factors can be seen as an indication of measurement uncertainty, but will also be affected by decline, well dynamics and change in decision variables since the time of the last well test. As single-rate well tests are performed relatively frequently, and relatively few changes are made in decision variables on this field, as illustrated by Figure 7, the observed field factors can be interpreted as indication of the presence of measurement uncertainty on the field considered.

Seemingly predictable reservoir dynamics on short timescales: The measured total oil rate illustrated in Figure 7 show a slow and steady decline in total oil rates. For the field considered, this may indicate that a relatively simple description of depletion may be required in a production model valid for short time intervals.

Discussion

Our intent in writing this paper has been to document our experiences in assessing production data with regards to modeling for production optimization, and to illustrate these using data for a particular field. We do not expect all observations made in this paper to be field-independent, but in our experience the difficulties posed by the state of production data with regards to information content and measurement uncertainty are not unique to this field.

A measurement uncertainty which we have not considered in this paper, but which may be significant is uncertainty in measurement of settings z and q_{gl}. The sensitivities of relative choke openings on production may vary with time as a production chokes are eroded. In this paper we have considered measured gas lift rates q_{gl}, but these rates may be improved through model-based reconciliation with auxiliary measurements, which we have not considered.

A visible seemingly random or “noisy” component is visible in measurements y. This component may be due to measurement uncertainties or un-modelled disturbances in the topside facilities. The signal-to-noise ratio may become low as the number of wells increase, as the relative change in total measured rates caused by altering production of an individual well will diminish, while disturbances stemming from processing facilities and flow dynamics will not diminish with increasing number of wells.

Challenges in production optimization

Based on the observations made in the previous section, it seems reasonable to assume that production models will be subject to a degree of uncertainty, due both to measurement uncertainties and the low information content in production data. What challenges do these observations raise in the context of production optimization, and what requirements do they pose for new production optimization technology?

Determine the significance of uncertainty

The significance of production model uncertainty depends on how model uncertainty influences the optimal decision variable value determined by production optimization. If model uncertainty causes production optimization to suggest settings which are suboptimal, a loss in production profit will result. The significance of model uncertainty should be expressed in terms of this loss. An explicit treatment of model uncertainty will only be warranted on fields where this loss is estimated to be significant. Further research could focus on devising methods to assess the significance of uncertainty on a case-by-case basis.

Business cases for uncertainty mitigation

Mitigating uncertainty can take on many forms, but will usually incur a cost. A truly structured approach to uncertainty mitigation would be to formulate a business case, a structured proposal for business change that is justified in terms of costs and benefits, for each candidate action. As it may not be cost-efficent to completey eliminate uncertainty, the costs and value of a proposed action should be weighed.

Uncertainty due to low information content in production data can be mitigated through systematic excitation planning. The cost of excitation is linked to the possible temporary reduction in production profit that may result from exciting production, the risks of triggering a shutdown and possibly costs associated with using the test separator. The value of excitation is the increase in production profits that results from re-optimizing production with a production model that has been updated with information gained from a particular excitation.

Measurement uncertainty can be mitigated through the careful calibration of measurements. by installing improved measurement equipment or by installing redundant measurement equipment to allow data reconciliation [13], and each actions will also have costs and values which should be weighed.

Estimating costs and values of uncertainty mitigation to allow proposed actions to be evaluated through structured business cases is a topic for further research.

Decision making under uncertainty

Through analysis of structured business cases for uncertainty mitigation, it is expected that it will not be cost-efficient to mitigate all uncertainty. In that case day-to-day operational decisions will have to be made under uncertainty.

If a framework which describes uncertainty in production models has been developed and uncertainties are quantified, this framework may be exploited to reduce the consequences of uncertainty in day-to-day decision making. One example of explicit uncertainty handling is [7], which deals with prioritizing wells under uncertainty. Decision making under uncertainty should be a topic of further research.

Conclusion

In the case study we observed that production data may have a low information content and be subject to significant measurement uncertainties. Based on these observations, it seems reasonable to assume that production optimization may be subject to significant uncertainty.

We highlighted three challenges for further research, which until now have all received little attention in the literature. Firstly, the expected losses incurred due to

uncertainty should be quantified to asses their signifcance. Secondly, costs and values of uncertainty mitigation should be estimated to allow proposed actions to be evaluated through structured business cases. Thirdly, strategies for making decisions in day-to-day operations under uncertainty should be investigated.

Acknowledgements

The authors would like to thank the Norwegian Research Council, Norsk Hydro and ABB for funding this work.

Nomenclature

ε	*residual, the difference between modeled and measured y*
θ	*vector of parameters to be fitted to* Z^N
d	*modeld disturbance vector, independent of u*
d_u	*vector of disturbances dependent of u*
F_o	*field factor oil*
F_g	*field factor gas*
F_w	*field factor water*
i	*index of well*
n_w	*number of wells*
q_o^{tot}	*total produced rate of oil*
q_g^{tot}	*total produced rate of gas*
q_w^{tot}	*total produced rate of water*
$q_o^{test,i}$	*most recent test separator measurement of produced oil rate of well i*
$q_g^{test,i}$	*most recent test separator measurement of produced gas rate of well i*
$q_w^{test,i}$	*most recent test separator measurement of produced waterl rate of well i*
R	*matrix of ones and zeros which define routing between y and x*
t	*index of sample*
u	*decision variable vector*
y	*vector of measured rates*
x	*vector of all modeled production rates*
Z^N	*set of production data used for fitting* θ
z^i	*production choke valve opening for well i*

References

1 L.A. Saputelli, S. Mochizuki, L. Hutchkins, R. Cramer, M.B. Anderson, J.B. Mueller, A. Escoricia, A.L. Harms, C.D.Sisk, S. Pennebaker, J.T. Han, A. Brown, C.S. Kabir, R.D. Reese, G.J. Nuñez, K.M Landgren, C.J. McKie, and C. Airlie. *"Promoting real-time optimization of hydrocarbon producing systems*" paper SPE 83978 presented at 2003 SPE Offshore Europe, Aberdeen, Sept. 2-5.

2 H.P Bieker, O. Slupphaug, and T.A Johansen. *"Real-time production optimization of offshore oil and gas production systems: Technology survey*." paper SPE 99446 presented at the 2006 SPE Intelligent Energy Conference and Exhibition, Amsterdam, April 11-13.

3 A. Diab, B.K Griess, and R. Schulze-Riegert. "*Application of global optimization techniques for model validation and prediction scenarios of a north african oil field.*" paper SPE 100193 presented at 2006 SPE Europec/EAGE Annual Conference and Exhibition, Vienna, June 12-15.

4 A.J. Little, H.A. Jutila, and A. Fincham. *"History matching with production uncertainty eases transition into prediction*." paper SPE 100206 presented at 2006 SPE Europec/EAGE Annual Conference and Exhibition, Vienna, June 12-15.

5 A.P.A. Costa, D.J. Schiozer, and C.A. Poletto. "*Use of uncertainty analysis to improve production history matching and the decision-making process*" paper SPE 99324 presented at 2006 SPE Europec/EAGE Annual Conference and Exhibition, Vienna, June 12-15.

6 H.P. Bieker, O. Slupphaug, and T.A. Johansen. "*Optimal well-testing strategy for production optimization: A monte carlo simulation approach*" paper SPE 104535 presented at 2006 SPE Eastern Regional Meeting, Canton, Ohio Canton, Oct. 11-13.

7 H.P. Bieker, O. Slupphaug, and T.A. Johansen. "Well management under uncertain gas or water oil ratios" paper SPE 106959 presented at 2007 SPE Digital Energy Conference and Exhibition, Houston, April 11-12.

8 C.C.M Branco, Antônio C.C. Pinto, Paulo M.F. Tinoco, Paulo M.B. Vieira, Alexandre M. Sayd, Renato L.A. Santos, and Fabio Prais. "*The role of the value of information and long horizontal wells in the appraisal and development studies of a brazilian o.shore heavy-oil reservoir*" paper SPE 97846. presented at 2005 SPE/PS-CIM/CHOA International Thermal Operations and Heavy Oil Symposium, Calgary, Alberta, Nov 1-3.

9 G. Zangl, T. Graf, and A. Al-Kinani. "*Proxy modeling in production optimization*" paper SPE 100131 presented at 2006 SPE Europec/EAGE Annual Conference and Exhibition, Vienna, June 12-15.

10 M. Nikolaou, A.S. Cullick, and L. Saputelli. "*Production optimization -a moving-horizon approach*" paper SPE 99358. Presented at 2006 SPE Intelligent Energy Conference and Exhibition, Amsterdam, April 11-13.

11 I.J. Halvorsen, S. Skogestad, J. Morud, and V. Alstad. *"Optimal selection of controlled variables*" Ind. Eng. Chem. Res., 42(14):3273–3284, 2003.

12 L. Saputelli, M Nikolaou, and M. J. Economides. "*Self-learning reservoir management."* paper SPE 84064 presented at 2003 SPE Annual Technical Conference and Exhibition, Denver, Oct. 5-8.

13 C. M. Crowe. "*Data reconciliation — progress and challenges*" Journal of Process Control, 6:89–98, 1996.