Regularized and Adaptive Nonlinear Moving Horizon Estimation of Bottomhole Pressure during Oil Well Drilling

Marcel Paasche * **Tor A. Johansen** * **Lars Imsland** *

* *Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway.*

Abstract: With the restrictions on online downhole measuring during petroleum drilling, mathematical reconstruction of the bottomhole pressure (BHP) is desired to improve control of the pressure profile throughout the well. With sufficient pressure management, increasingly tight pressure margins, that are imposed by the worsening accessibility of the remaining oil reservoirs, might be met. A regularized and adaptive Moving Horizon Estimation (MHE) approach is proposed to estimate the BHP. The observer comes with abilities to adapt to the degree of excitation via thresholded singular value decomposition, and means for filtering using an a priori estimate in the cost function. The employed observer model is nonlinear, as required by the process, and chosen to be third order to mitigate computational load. The algebraic BHP equation includes two unknown parameters and an unmeasured state. Accuracy of the model is enhanced by estimation of selected parameters. Testing on a data set from a North Sea well, conducted to investigate the suitability of the approach, has delivered promising results.

Keywords: Managed Pressure Drilling; Moving Horizon Estimation; Regularization.

1. INTRODUCTION

Oil well drilling is primarily about driving drill pipes, with a drill bit attached, several km into the ground to create a wellbore. The drill pipes come in stands of about 27 m and, as drilling proceeds, pipe segments regularly get added to what forms the drill string. At surface level, the drill string is handled within the derrick, and the top drive is mounted onto the last pipe section to provide the needed torque. A mud circulation system is established with the main pumps forcing the drilling fluid ("mud") down the drill pipe, through the drill bit and up again in the annulus, i.e. the wellbore. The mud balances the pressure and transports cuttings from the drill bit to the surface.

To ensure stable and efficient operation, the pressure profile of the well must be kept between the pore and collapsing pressure on one side and fracturing pressure on the other. Nygaard (2006) formulated the requirement as

$$max(p_{coll}(t,x), p_{pore}(t,x)) < p_{well}(t,x) < p_{frac}(t,x) \quad (1)$$

It is intended to prevent dangers like wellbore fracturing, reservoir influx and surface blow outs that might involve the loss of assets, time, and possibly human lives. The open-to-atmosphere approach of conventional oil drilling attempts to meet inequality (1) by adjusting only the pump speed, and the density of the drilling mud. The resulting weight of the mud volume determines the hydrostatic pressure which constitutes the major part of the pressure profile. On a smaller scale, pressure loss due to friction is involved that depends on mud flow, i.e. the pump rates. Since these depend on the current drilling operation, other inputs are used for pressure control. Still, it effects the pressure more rapidly than density changes. There are also scenarios where volume changes add disturbingly to the pressure dynamics. For a more comprehensive introduction to oil well drilling, please refer to Devereux (1999).

Recently, Managed Pressure Drilling systems have introduced additions to improve actuation and its dynamics (Malloy (2007)). The annulus is, then, tightly sealed at the top, and release of the drilling mud is controlled via a choke valve that can be rapidly closed to increase the pressure in the annulus. Some systems include a backpressure pump to better maintain the pressure, e.g. under loss of circulation and leakage. A check valve above the drill bit prevents drilling mud from re-entering the drill string. Hence, the pressure in the drill string can be released for connection of new pipe segments. To decouple most of the well pressure-wise and further stabilize the borehole mechanically, steel casings are run down along the walls of the wellbore. Therefore, concerns about the annulus pressure profile can be reduced to one of the BHP. Since, eventually, control of the BHP is desired, good online knowledge of this variable is needed for use as feedback.

During drilling operation, the BHP is measured at the drill bit and transmitted via mud pulse telemetry to the surface. Since the data travel as acoustic pulses through the mud, noise, general delays of $\sim 1\,\mathrm{s/km}$ and a slow update rate of $\sim 0.05\,\mathrm{Hz}$ are to be expected as well as complete signal loss during pump-down phases. The topside measurements, most notably including the pump and annulus pressure, usually come in higher sampling rates of $\sim 1\,\mathrm{Hz}$. However, the configuration of measurements available online can vary significantly from case to case with only the BHP transmitted via telemetry as a worst case. Due to the unreliable and not persistent nature of the bottomhole measurement, estimation is a promising tool for the highly desirable, continuous reconstruction of the BHP.

There are a few previous works on the estimation of the BHP with some unknown parameters. Stamnes et al. (2008) and Zhou and Nygaard (2010) both employ a nonlinear model-based adaptive observer to estimate the BHP along with the parameters for annulus friction and density. The former bases results on data from a North Sea well, the latter uses a sophisticated simulation framework and additionally applies a simple PI controller to track the desired pressure. Breyholtz et al. (2010) utilize an alternative telemetry system to circumvent the need for estimation of the BHP. Model-predictive control is applied on the multivariable problem to control BHP and hook position via pump flows and drill string velocity. Zhou et al. (2010) present a switch controller with kick/loss detection that controls bottom hole pressure under normal operation and otherwise attenuates kicks/losses that are detected via estimation of the drill string and annulus flow rates and the reservoir pore pressure.

With a MHE approach, the cost function is crucial since it defines how the various errors map to the scalar cost which the solver tries to minimize. Proposed in Sui and Johansen (2010), the regularized observer applied in this work has a potent cost function that can be flexibly tuned. Starting with the sum of error squares, it can be enhanced via the observer's design parameters to add a filtering effect as proposed in Alessandri et al. (2008). Furthermore, the error squares are weighted by a matrix which can be set to adapt to the degree of excitation in the data. A sensitivity threshold, finally, can freeze estimates that are not exciting enough and thus inhibit unwanted influence of disturbances and model errors.

To the author's knowledge, this paper presents the first case of MHE for the application. The described attributes of the approach are believed to cope particularly well with the application where distinct modes of operation cause different sets of unexcited states and parameters. Additional estimation of parameters plays a central role to improve model accuracy. Isolating different sets of parameter via the thresholded adaptive weighting matrix under the lack of persistent excitation can increase the total number of estimated variables, over time, in compliance with limited observability. Hence, there is more flexibility with respect to model parameterization.

2. THE MODEL

The model requires to match a trade-off between accuracy and computational load. Drill string and annulus are considered as two separate control volumes that are connected through the drill bit's check valve. The model is based on a mass balance for the two control volumes each, and a momentum balance at the drill bit. This one-phase third-order model is taken from Kaasa (2007) and is given by the state equations

$$\frac{V_d}{\beta_d}\dot{p}_p = q_p - q_{bit} \tag{2a}$$

$$\frac{V_a}{\beta_a}\dot{p}_c = q_{bit} - q_c + q_{bck} + \dot{V}_a \tag{2b}$$

$$M\dot{q}_{bit} = p_p - p_c - \theta_1 q_{bit} - \theta_2|q_{bit}|q_{bit} + (\rho_d - \rho_a)gh_{bit}. \tag{2c}$$

and the algebraic model equation

$$p_{bit} = p_c + \theta_1 q_{bit} + \rho_a g h_{bit}. \tag{3}$$

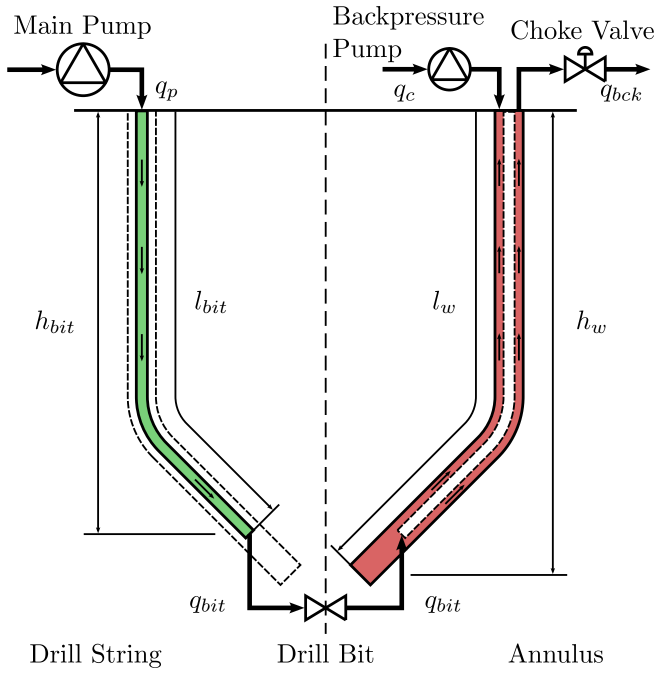

Fig. 1. Well modelled as two control volumes connected through the drill bit.

Herein, p_p, p_{bit}, p_c denote the pressures at the main pump, drill bit and choke valve (bar), and q_p, q_{bit}, q_c the volumetric flows ($^{\mathrm{m}^3}/_{\mathrm{s}}$), respectively. The backpressure pump is considered with q_{bck}. For regard of reservoir influx, see Zhou et al. (2008); it is not considered in this work. The change of annulus volume $\dot{V}_a$ ($^{\mathrm{m}^3}/_{\mathrm{s}}$) is neglectably small and the drill string volume only changes in discrete lengths of the pipe segments added. However, volume (m^3) and bulk modulus (bar) are given as V_a and β_a for the annulus and V_d and β_d for the drill string. Together they implement the transition from differences in volumetric flow to pressure changes. Likewise, ρ_a and ρ_d define the densities ($^{10^5\mathrm{kg}}/_{\mathrm{m}^3}$) while θ_1 and θ_2 denote the friction factors. Equation (2c) shows laminar flow is being assumed for the annulus, and turbulent flow for the drill string. M ($^{10^5\mathrm{kg}}/_{\mathrm{m}^4}$) is given by $M = M_a + M_d$ with

$$M_i = \rho_i \int_0^{l_i} \frac{1}{A_i(x)}dx, \quad i = a, d. \tag{4}$$

Finally, h_{bit} (m) is the true vertical depth of the drill bit and $g = 9.81\,^{\mathrm{m}}/_{\mathrm{s}^2}$ the gravitational acceleration.

Since the choke flow q_c is usually not being measured, it is calculated from the choke opening $z_c \in [0,1]$ and the differential choke pressure $p_c - p_0$ following the model

$$q_c^{sim} = \gamma_c f_c(z_c)\sqrt{p_c - p_0} \tag{5}$$

with pressure p_0 downstream of the valve (Green (1997)). The choke characteristic $f_c = \sqrt{2/\rho_a}CA(z_c)$ is a combination of the nearly constant annulus density ρ_a, the orifice flow coefficient C and the cross-sectional area A of the orifice hole as a function of z_c. It has been fitted to match the data set by using a third order polynomial and some manual adjustments. The factor γ_c is used for parameter estimation and will be explained further in section 5. The inputs available for manipulation are z_c and q_{bck} (if available). The flow rate serves a lot of purposes and thus can not be set arbitrarily just to manipulate the

BHP. Influence of the drill string velocity is not considered. The focus here lies on the algebraic model equation since it includes the sought after BHP p_{bit}. At this stage is already evident that the friction term $\theta_1 q_{bit}$ is going to vanish with $q_{bit} \to 0$ leaving p_c to ideally compensate for the loss. Also visible is the distinct nature of parameters. Some parameters, like the friction and density parameters, characterize the steady state behaviour, others, like β_a, β_d and M, only have impact during transient conditions. This will be important for parameter estimation.

3. THE OBSERVER

On the described application, Nonlinear MHE is applied as proposed in Sui and Johansen (2010). The regularized observer sports an adaptive output weighting matrix and an open loop term for filtering. For estimation, the above model is considered as

$$x_{t+1} = f(x_t, u_t) \tag{6a}$$
$$y_t = h(x_t) \tag{6b}$$

with the state vector $x_t = (p_p, p_c, q_{bit})^T$, vector of inputs $u_t = (q_p, q_{bck}, z_c)^T$ and output vector $y_t = (p_p, p_c, p_{bit})^T$ at discrete time t. The function f describes the propagation of x_t over observer time step Δt and under input u_t to the next state vector x_{t+1} whereas h maps x_t to the corresponding y_t for each time step. To encompass estimation of parameters, in addition to the states defined above, x is augmented with the desired parameter vector p with $p_{t+1} = p_t$. Hence $n = \dim(x_t) = n_x + n_p$ where $n_x = 3$, and n_p is the number of estimated parameters.

The core task of the MHE problem is to estimate the state and corresponding output vector at any observer time step $t > N$. As described by the term *moving horizon*, the observer uses the N-information vector

$$I_t = col(y_{t-N}, \dots, y_t, u_{t-N-1}, \dots, u_{t-1}) \tag{7}$$

comprised of a fixed number $N+1$ of the last measurement and input vectors. These can be summarized in a single vector for measurement and input

$$Y_t = \begin{bmatrix} y_{t-N} \\ y_{t-N+1} \\ \vdots \\ y_t \end{bmatrix}, \qquad U_t = \begin{bmatrix} u_{t-N} \\ u_{t-N+1} \\ \vdots \\ u_t \end{bmatrix}. \tag{8}$$

A one-step prediction $\bar{x}_{t-N,t}$ on the preceding time step's solver solution $\hat{x}^o_{t-N-1,t-1}$ is reached via

$$\bar{x}_{t-N,t} = f(\hat{x}^o_{t-N-1,t-1}, u_{t-N-1}). \tag{9}$$

With this a priori estimate and the N-information vector, the observer derives a chain of estimates $\hat{x}_{t-N,t}, \dots, \hat{x}_{t,t}$ of the state vectors $x_{t-N}, \dots, x_t$ by iterative propagation spanning from $t-N$, at a N sample times back, to the current observer time step t. With equation (6b), this estimated trajectory section results in a sequence of estimated output vectors $\hat{Y}_t$. Since each estimated state vector complies with equation (6a), $\hat{Y}_t$ can be written as

$$\hat{Y}_t = H(\hat{x}_{t-N,t}, U_t)$$
$$= H_t(\hat{x}_{t-N,t}) = \begin{bmatrix} h(\hat{x}_{t-N,t}) \\ h \circ f^{u_{t-N}}(\hat{x}_{t-N,t}) \\ \vdots \\ h \circ f^{u_{t-1}} \circ \dots \circ f^{u_{t-N}}(\hat{x}_{t-N,t}) \end{bmatrix} \tag{10}$$

wherein all estimated output vectors are functions of the window's initial estimate $\hat{x}_{t-N}$ and the corresponding input vectors. Varying $\hat{x}_{t-N,t}$, the observer tries to match the estimated trajectory section with the real one of the plant by comparing the estimated output vectors to the measurements. This is implemented as minimization of the regularized least-squares criterion

$$J(\hat{x}_{t-N,t}, \bar{x}_{t-N,t}, I_t) = \|W_t(H_t(\hat{x}_{t-N,t}) - Y_t)\|^2 + \|V(\hat{x}_{t-N,t} - \bar{x}_{t-N,t})\|^2 \tag{11}$$

which maps weighted differences between the vectors to a scalar penalty and is a special case of the description found in Sui and Johansen (2010). At an optimal estimate $\hat{x}^o_{t-N,t}$, the minimum penalty $J^o_t = \min_{\hat{x}_{t-N,t}} J(\hat{x}_{t-N,t}, \bar{x}_{t-N,t}, I_t)$ is reached leaving the estimation error to

$$e_{t-N} = x_{t-N} - \hat{x}^o_{t-N,t}. \tag{12}$$

While the first term of criterion (11) is the conventional output error term with an output weighting matrix W_t, the second term is a penalty for a difference between the currently evaluated initial estimate $\hat{x}_{t-N,t}$ and the a priori open loop estimate $\bar{x}_{t-N,t}$. The weighting matrix V allows for the intensity of the filtering effect to be tuned for each single state separately. As mentioned above, the solution of the solver $\hat{x}^o_{t-N,t}$ is the initial entry in the observer window; the optimal estimate $\hat{x}^o_{t,t}$ is only indirectly derived from $\hat{x}^o_{t-N,t}$ by iterative propagation as in (10).

Regularization is achieved, in this MHE approach, through the adaptive matrix W_t. It tries to avoid the influence of noise when there is little or no excitation in the data, thus better retaining the, otherwise lost, performance of the observer. Without such regularization, unexcited estimates may be more likely to drift off.

Assuming small estimation errors e_{t-N}, Sui and Johansen (2010) design the adaptive law of W_t as

$$\left\| \left(W_t \frac{\partial H}{\partial x}(\hat{x}^o_{t-N,t}, U_t) \right)^+ \right\| = \alpha \tag{13}$$

with the Jacobian $\frac{\partial H}{\partial x}(\hat{x}^o_{t-N,t}, U_t)$ describing the sensitivity of output changes towards changes of the different states. Using the singular value decomposition

$$\frac{\partial H}{\partial x}(\hat{x}^o_{t-N,t}, U_t) = \tilde{U}_t S_t \tilde{V}_t^T, \tag{14}$$

the sensitivities of the plant's modes are available in the diagonal matrix S_t. Any of those that are zero or close to zero indicate that the mode in question is not observable or the input not exciting since the output measurements are not sensitive to it. A threshold design parameter $\delta > 0$ is defined. By

$$\frac{1}{\sigma_{\delta,i,t}} = \begin{cases} \dfrac{1}{\sigma_{i,t}}, & \text{if } \sigma_{i,t} \geq \delta \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

the inverse of small singular values is adjusted in the pseudo-inverse $S^+_{\delta,t} = \mathrm{diag}(1/\sigma_{\delta,1,t}, \dots, 1/\sigma_{\delta,n_x,t})$ so that unobservable or unexciting modes are effectively disabled, and the weight is reduced to zero in the newly adapted matrix

$$W_t = (1/\alpha)\tilde{V}_t S^+_{\delta,t} \tilde{U}_t^T. \tag{16}$$

The approach promises to be well suited for the application since its regularization should well cope with estimation of parameters that depend on the characteristic steady and transient conditions.

4. EXPERIMENTS AND RESULTS

For testing of the observer's capabilities and suitability for the application, a real data set from an oil field in the North Sea has been employed. It spans across 3 h of measurements and two pipe connection events. The site has no backpressure pump ($q_{bck} = 0$) and lacks means to measure q_c; hence, the choke model (5) is utilized. Three cases of estimation are examined

a) Only estimation of states p_p, p_c, q_{bit} and the output p_{bit}
b) As case a) plus added parameter estimation of γ_c
c) As case b) plus added parameter estimation of θ_1, ρ_a

If not estimated, the friction and density parameters of the model are tuned offline to steady state information as available in the data set. Geometric values (V_a, V_d, h_{bit}) are supplied in the logged data. Parameters β_a, β_d and M, relevant for the dynamics, are partly calculated, partly tuned offline to fit the data. As such, they are candidates for parameter estimation during transient phases of the process (see Paasche (2010)). The values used for non-estimated parameters and the observer's standard tuning for the application are on display in table 1.
Estimates are generally set to start 5 % off of the (suspected) real value. If no special scaling is applied, all estimates are normalized before entering the solver cost function (11). This is important to balance the solver's impact on the individual estimates. Since the weighting in W_t can not be influenced directly, except for the scalar α, scaling is a tool to prioritize outputs and states as deemed appropriate. This tuning parameter α balances the output error term towards the overall filtering and has been chosen to attain a compromise between responsiveness and smoothness. The filtering via matrix V is set equal for all states (case a), b)). With its effects, adding constraints to the optimization process was not felt necessary or effective.
The data is sampled at 1 Hz for the topside measurements (p_p, p_c, ...) and 0.05 Hz for p_{bit}, transmitted by mud pulse telemetry. Since the observer estimates upon arrival of new measurements, its window size has been chosen as $N + 1 = 40$ to encompass 40 s and therefore two p_{bit} samples at all times.
The results of case a) can be seen as state estimates in Figure 2 and flow rates in Figure 3. Case b) is displayed similarly in Figures 4 and 5 with the parameter estimate for the choke model added to the latter. Case c) again adds estimates for ρ_a and θ_1 to Figure 7, while Figure 6 shows its state estimates. The opening of the choke valve is identical in all three cases and can be seen in Figure 8. Solid black line in the figures denotes (suspected) real values, dashed red line indicates estimates. The data set also contains logged measurements of p_{bit} during pump-down that would generally not be available online. For reference, these measurements are added to the plots of p_{bit} as dotted green lines. Figure 9 shows the singulary values of $\frac{\partial H}{\partial x}$ and the threshold δ for case a). This is important for observability since it is given if $\frac{\partial H}{\partial x}$ is well conditioned and of full rank, i.e. $\sigma_{i,t} > 0$, for all i. In cases b) and c) the estimation error of the BHP stays below

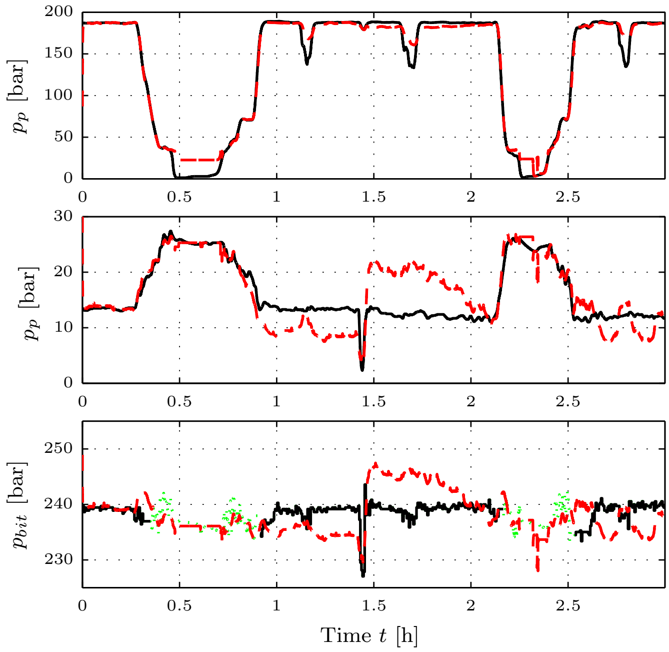

Fig. 2. *Case a):* Pressures

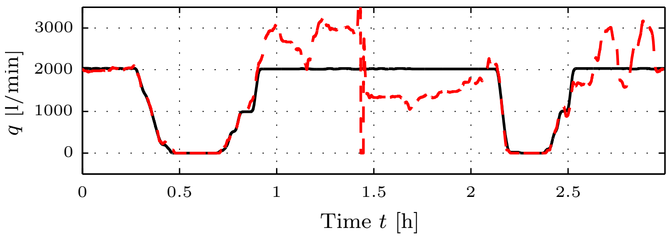

Fig. 3. *Case a):* Flow rates — q_p *(solid)*, q_c^{sim} *(dashed)*

2 bar. With ceasing excitation, the freezing effect that the observer's δ-thresholded adaption process of W_t has on the p_{bit} estimate can be observed during phases of pump-down.

Table 1. Parameter settings

Variable	Value	Description
∂t	0.5 s	Integration timestep
$N+1$	40	Observer window size
δ	0.2	SVD threshold
α	1	W_t gain
V	diag(2, 2, 2)	Filtering weights
ρ_a	1180 $\mathrm{kg/m^3}$	Annulus density
ρ_d	1130 $\mathrm{kg/m^3}$	Drill string density
θ_1	443 $\mathrm{bar\ s/m^3}$	Annulus friction parameter
θ_2	131350 $\mathrm{bar\ s^2/m^6}$	Drill string friction parameter
β_a	8000 bar	Annulus bulk modulus
β_d	9836 bar	Drill string bulk modulus
M	$6537 \cdot 10^5$ $\mathrm{kg/m^4}$	Drill string bulk modulus
p_0	4 bar	Choke downstream pressure

5. DISCUSSION

Without good model parameters, the model is not going to be accurate and the estimates' quality will degrade. Thus either good tuning as in cases a) and b), or reliable parameter estimation as in case c) is essential.
Depending on the accuracy of the observer model, pure state estimation might be limited in its results. With

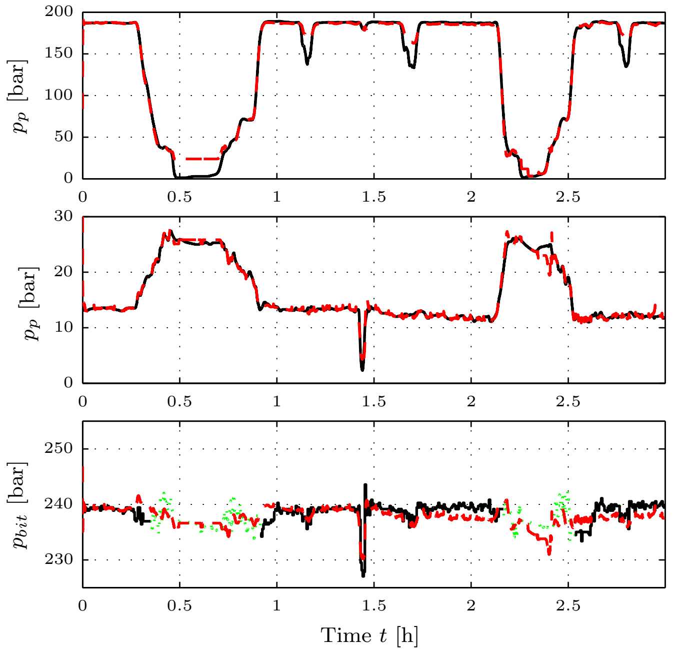

Fig. 4. *Case b):* Pressures

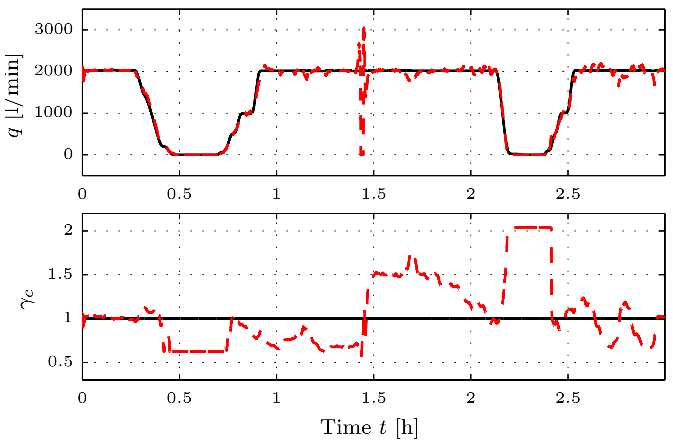

Fig. 5. *Case b):* Flow rates *(t.)*; parameter estimate *(b.)*

parameter estimation, a powerful tool is given to improve model accuracy. As far as observability allows, badly tuned parameters might be corrected and time-varying ones tracked. However, parameter estimates might also compensate for other insufficiencies in the model like structural errors that neglect certain dynamics in the real plant. In the case of the used data set, there are clogging effects altering the characteristics of the choke valve that the simple model (5) can not comprehend. Over time, the choke valve gets gradually clogged by the mud and cuttings flowing through it. Thus the choke valve needs to be opened more to maintain the flow rate. Just before $t \approx 1.5\,$h, a designated choke clearing event is launched after which the choke opening is reduced again to achieve the same flow rate.

For case a) (Figures 2 and 3), the insufficiently accurate choke model creates an significantly off choke flow q_c^{sim}, between $t \approx 1\,$h and $t \approx 2\,$h, which the observer can not handle. Even so, the well tuned ρ_a in combination with the, nonetheless, good estimation of p_c during phases of pump-down deliver a reliable estimate of p_{bit} during these

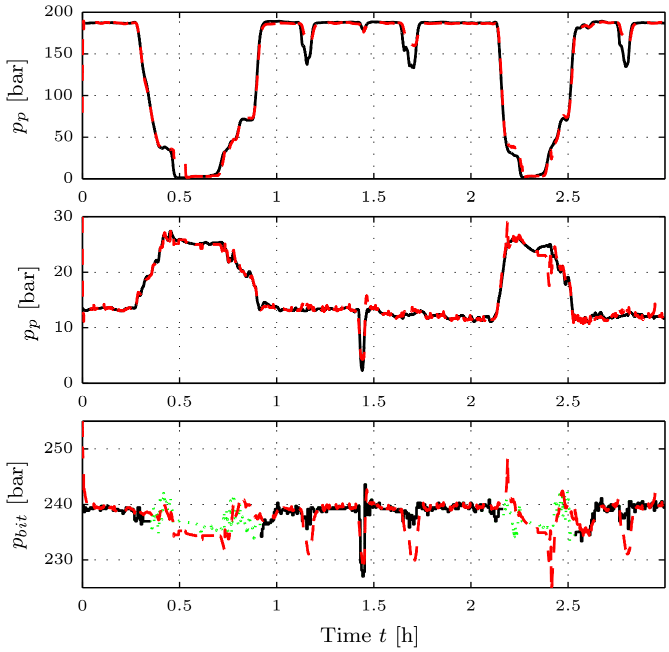

Fig. 6. *Case c):* Pressures

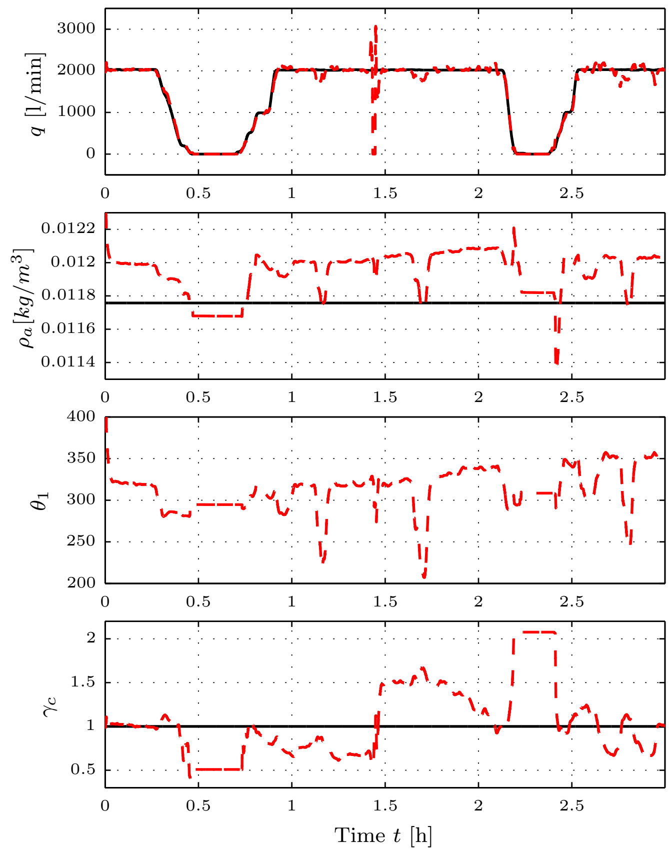

Fig. 7. *Case c):* Flow rates *(t.)*; parameter estimates *(b.)*

crucial phases.

When estimated as in case b) (Figures 4 and 5), the virtual choke parameter γ_c is intended to compensate for the unmodelled clogging effect and improve on the estimate $\hat{p}_c$ via a more appropriate q_c^{sim}. Hence, the quality of the estimate $\hat{p}_{bit}$ is improved largely during non-critical steady state phases. However, Figure 5 shows that

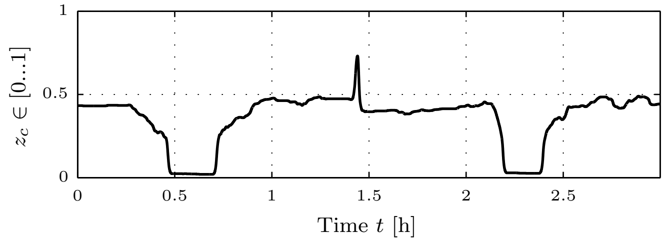

Fig. 8. *Cases a), b), c)*: Choke opening

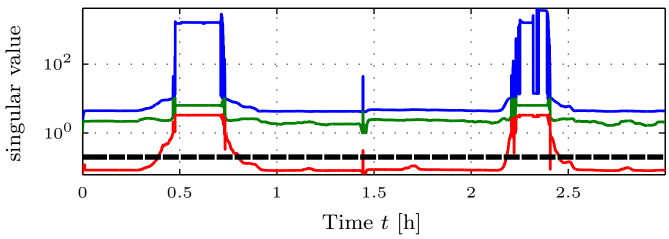

Fig. 9. *Case a):* Singular values *(b,g,r)*, threshold *(dashed)*

during transients it seems to also pick up other dynamic errors caused, for example, by the transient bulk modulus parameters that are not estimated.

Due to its direct contribution to p_{bit} (see equation (3)), estimation of the annulus density is highly desirable and, in a way, necessary since the tuning of ρ_a as in cases a) and b) depends on logged measurements of p_{bit} not available online. Then again, the applied tuning of θ_1 depends on ρ_a to be tuned as described above thus suggesting the combination of ρ_a and θ_1 for parameter estimation as in case c). In this case (Figures 6 and 7), the observer was tuned to attenuate the density estimate by decreasing the scaling of this particular estimate in the solver cost function and increasing the filtering weight corresponding to the estimate. This is needed to consider the sensitivity of the model to changes in the annulus density ρ_a. However, alternate ways (e.g. measurement) might be found to get a better view on ρ_a. As opposed, ρ_d should be known from the mixture of drilling mud used. Looking at the results, case c) might not seem much of an improvement over case b) but the achievement, here, really is the reduced use of information and offline tuning.

As mentioned in Section 4, estimation of parameters β_a, β_d and M during transient conditions promises to further improve observer performance. If q_c is available as measurement, especially estimation of β_a should be easily feasible. However, accurate (unbiased and undelayed) measurements would be needed since the difference in flow is, in any case, expected to be very small due to the relatively low compressibility of the drilling fluid and the rigid structure of the well assembly.

6. CONCLUSION

In this paper, regularized MHE has been applied to a third order model to estimate the BHP. Simulations employing real data deliver good results provided the observer model is carefully tuned or selected parameters are estimated. Estimating the choke parameter γ_c efficiently compensates for clogging effects in the model of the choke valve. Dependence on prior knowledge can be reduced by enlarging the set of parameters estimated. The thresholded adaption of the observer's weighting matrix W_t reliably freezes estimates on lack of excitation while the open-loop term provides for the necessary filtering. Challenge lies in the careful choice of the model, its tuning and the parameters to be estimated. Differences between the model and the real plant are inevitable but it is important to understand how they manifest themselves in the estimation process.

ACKNOWLEDGEMENTS

This work was supported by the Strategic University Program on Control, Information and Communication Systems for Environmental- and Safety-Critical Systems funded by the Research Council of Norway. Statoil ASA contributed supplying the utilized data sets kindly provided by Glenn-Ole Kaasa. Sui Dan and Øyvind Nistad Stamnes have aided the authors with their experience on the observer and the process model respectively.

REFERENCES

Alessandri, A., Baglietto, M., and Battistelli, G. (2008). Moving-horizon state estimation for nonlinear discrete-time systems: New stability results and approximation schemes. *Automatica*, 44, 1753–1765.

Breyholtz, O., Nygaard, G., and Nikolaou, M. (2010). Automatic control of managed pressure drilling. In *American Control Conference, Baltimore*, 442–447.

Devereux, S. (1999). *Drilling technology in nontechnical language*. PennWell.

Green, D.W. (1997). *Perry's Chemical Engineers' Handbook, 7th Edition*. McGraw-Hill.

Kaasa, G.O. (2007). A simple dynamic model of drilling for control. Technical report, StatoilHydro Research Centre, Porsgrunn.

Malloy, K. (2007). Managed pressure drilling — what is it anyway? *World Oil*, March, 27–34.

Nygaard, G. (2006). *Multivariable Process Control in High Temperature and High Pressure Environment Using Non-Intrusive Multi Sensor Data Fusion*. Ph.D. thesis, Norwegian University of Science and Technology.

Paasche, M. (2010). *Nonlinear Moving Horizon Estimation of Bottomhole Pressure during Oil Drilling*. Master's thesis, University of Stuttgart.

Stamnes, O., Zhou, J., Kaasa, G.O., and Aamo, O. (2008). Adaptive observer design for the bottomhole pressure of a managed pressure drilling system. In *IEEE Conference on Decision and Control, Cancun*, 2961–2966.

Sui, D. and Johansen, T. (2010). Regularized nonlinear moving horizon observer for detectable systems. In *IFAC NOLCOS, Bologna*.

Zhou, J. and Nygaard, G. (2010). Control and estimation of downhole pressure in managed pressure drilling operations. In *International Symposium on Communications, Control and Signal Processing, Limassol*, 1–6.

Zhou, J., Nygaard, G., Godhavn, J., Breyholtz, O., and Vefring, E. (2010). Adaptive observer for kick detection and switched control for bottomhole pressure regulation and kick attenuation during managed pressure drilling. In *American Control Conference, Baltimore*, 3765–3770.

Zhou, J., Stamnes, O., Aamo, O., and Kaasa, G.O. (2008). Adaptive output feedback control of a managed pressure drilling system. In *IEEE Conference on Decision and Control, Cancun*, 3008–3013.