Design of Reduced Dimension Explicit Model Predictive Controller for a Gas-Liquid Separation Plant

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***Abstract*-- Exact or approximate solutions to constrained linear model predictive control (MPC) problems can be pre-computed off-line in an explicit form as a piecewise linear state feedback defined on a polyhedral partition of the state space. However, the complexity of the polyhedral partition often increases rapidly with the dimension of the state vector, and the number of constraints. Recently, several approaches for reducing the dimension of the explicit solution to constraint MPC problems have been developed. This paper considers the design of a reduced dimension explicit model predictive controller for a gas-liquid separation plant.**

***Index terms*-- Constrained linear model predictive control, Multi-parametric quadratic programming, Piecewise linear controllers.**

I. INTRODUCTION

Model predictive control (MPC) is an efficient methodology to solve complex constrained multivariable control problems. The requirement to perform on-line optimization however limits the applicability of MPC mostly to slowly varying processes. Recently, several methods for explicit solution of MPC problems have been developed. The main motivation behind explicit MPC is that an explicit state feedback law avoids the need for real-time optimization, and is therefore potentially useful for applications with fast sampling where MPC has not traditionally been used. In [1] it was recognized that the constrained linear MPC problem is a multi-parametric quadratic program (mp-QP), when the state is viewed as a parameter to the problem. It was shown that the solution (the control input) has an explicit representation as a piece-wise linear (PWL) state feedback on a polyhedral partition of the state space, see also [2], [3], [4], and they develop an mp-QP algorithm to compute a representation of this function. However, the complexity of the polyhedral partition often increases rapidly with the dimension of the state vector, and the number of constraints. This led to the investigation of efficient implementation of piecewise linear function evaluation [5], [6], as well as input trajectory parameterization [6] and restrictions on the active constraint switching [7] in order to reduce the complexity. In [8], a method for reducing the dimension of the mp-QP solutions to the explicit constrained LQR problems has been investigated.

Recently, several approximate algorithms for solving mp-QP problems have been developed, [9], [10], [11], [12], with significant reduction in complexity. The algorithms in [10], [11], [12] determine an approximate explicit PWL state feedback solution by imposing an orthogonal search tree structure on the partition, thus leading to more efficient real-time computations.

In [13], an approach for reducing the dimension of the approximate explicit solution to linear constraint MPC problems has been proposed and it can be considered as an extension of the approximate mp-QP algorithm in [10], [11]. The idea is to build partition in a sub-space of the state space, where each region is characterized by a set of PWL feasible full-state feedback laws, since the unique optimal active set can not be determined from a reduced-dimension state, in general. In this paper, the approach in [13] is applied to design a reduced dimension explicit model predictive controller for a gas-liquid separation plant.

II. EXPLICIT MPC AND EXACT MP-QP

Formulating a linear MPC problem as an mp-QP is briefly described below, see [1] for further details. Consider the discrete-time linear system:

$$\begin{aligned} \boldsymbol{x}(t+1) &= \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \end{aligned} \quad (1)$$

where $\boldsymbol{x}(t)\in \boldsymbol{R}^n$, $\boldsymbol{u}(t)\in \boldsymbol{R}^m$, and $\boldsymbol{y}(t)\in \boldsymbol{R}^p$ are the state, input and output variable. Also, $\boldsymbol{A}\in \boldsymbol{R}^{n\times n}$, $\boldsymbol{B}\in \boldsymbol{R}^{n\times m}$,

$\boldsymbol{C} \in \boldsymbol{R}^{p \times n}$ and ($\boldsymbol{A}$,$\boldsymbol{B}$) is a controllable pair. It is assumed that a full measurement of the state $\boldsymbol{x}(\boldsymbol{t})$ is available at the current time $\boldsymbol{t}$. Then, for the current $\boldsymbol{x}(\boldsymbol{t})$, MPC solves the optimization problem:

$$\boldsymbol{V}^*(\boldsymbol{x}(\boldsymbol{t})) = \min_{\boldsymbol{U} \equiv \{\boldsymbol{u}_t, \ldots, \boldsymbol{u}_{t+N-1}\}} \boldsymbol{J}(\boldsymbol{U}, \boldsymbol{x}(\boldsymbol{t})) \qquad (2)$$

subject to:

$$\boldsymbol{y}_{\min} \le \boldsymbol{y}_{t+k|t} \le \boldsymbol{y}_{\max}, \ \boldsymbol{k} = 1, \ldots, \boldsymbol{N} \qquad (3)$$

$$\boldsymbol{u}_{\min} \le \boldsymbol{u}_{t+k} \le \boldsymbol{u}_{\max}, \ \boldsymbol{k} = 0, 1, \ldots, \boldsymbol{N}-1 \qquad (4)$$

$$\boldsymbol{x}_{t|t} = \boldsymbol{x}(\boldsymbol{t}) \qquad (5)$$

$$\boldsymbol{x}_{t+k+1|t} = \boldsymbol{A}\boldsymbol{x}_{t+k|t} + \boldsymbol{B}\boldsymbol{u}_{t+k}, \ \boldsymbol{k} \ge 0 \qquad (6)$$

$$\boldsymbol{y}_{t+k|t} = \boldsymbol{C}\boldsymbol{x}_{t+k|t}, \ \boldsymbol{k} \ge 0 \qquad (7)$$

with the cost function given by:

$$\boldsymbol{J}(\boldsymbol{U}, \boldsymbol{x}(\boldsymbol{t})) = \sum_{k=0}^{N-1} \left[\boldsymbol{x}^T{}_{t+k|t} \boldsymbol{Q} \boldsymbol{x}_{t+k|t} + \boldsymbol{u}^T{}_{t+k} \boldsymbol{R} \boldsymbol{u}_{t+k} \right] + \boldsymbol{x}^T{}_{t+N|t} \boldsymbol{P} \boldsymbol{x}_{t+N|t} \qquad (8)$$

and symmetric $\boldsymbol{R} > 0$, $\boldsymbol{Q} \ge 0$, $\boldsymbol{P} > 0$. The final cost matrix $\boldsymbol{P}$ may be taken as the solution of the algebraic Riccati equation. With the assumption that no constraints are active for $\boldsymbol{k} \ge \boldsymbol{N}$ this corresponds to an infinite horizon LQ criterion [14]. This and related problems can by some algebraic manipulation be reformulated as:

$$\boldsymbol{V}_z^*(\boldsymbol{x}) = \min_{z} \frac{1}{2} \boldsymbol{z}^T \boldsymbol{H} \boldsymbol{z} \qquad (9)$$

subject to:

$$\boldsymbol{G}\boldsymbol{z} \le \boldsymbol{W} + \boldsymbol{S}\boldsymbol{x} \qquad (10)$$

where $\boldsymbol{z} \equiv \boldsymbol{U} + \boldsymbol{H}^{-1}\boldsymbol{F}^T\boldsymbol{x}$. Note that $\boldsymbol{H} > 0$ since $\boldsymbol{R} > 0$. The vector $\boldsymbol{x}$ is the current state, which can be treated as a vector of parameters. For ease of notation we write $\boldsymbol{x}$ instead of $\boldsymbol{x}(\boldsymbol{t})$. The number of inequalities is denoted $\boldsymbol{q}$ and the number of free variables is $\boldsymbol{n_z} = \boldsymbol{m} \cdot \boldsymbol{N}$. Then $\boldsymbol{z} \in \boldsymbol{R}^{n_z}$, $\boldsymbol{H} \in \boldsymbol{R}^{n_z \times n_z}$, $\boldsymbol{G} \in \boldsymbol{R}^{q \times n_z}$, $\boldsymbol{W} \in \boldsymbol{R}^{q \times 1}$, $\boldsymbol{S} \in \boldsymbol{R}^{q \times n}$. The solution of the optimization problem (9)-(10) can be found in an explicit form $\boldsymbol{z}^* = \boldsymbol{z}^*(\boldsymbol{x})$, [1]:

Theorem 1 *Consider the mp-QP (9)-(10) and suppose* $\boldsymbol{H} > 0$. *The solution* $\boldsymbol{z}^*(\boldsymbol{x})$ *(and* $\boldsymbol{U}^*(\boldsymbol{x})$*) is a continuous PWL function of* $\boldsymbol{x}$ *defined over a polyhedral partition of the parameter space, and* $\boldsymbol{V}_z^*(\boldsymbol{x})$ *is a convex (and therefore continuous) piecewise quadratic function.*

III. APPROXIMATE REDUCED DIMENSION APPROACH

In [10], [11], [12], algorithms that determine an approximate explicit PWL state feedback solution of possible lower complexity are developed. The idea is to require that the state space partition is represented as a binary search tree (quad-tree partition [10], [11] or *k*-*d*-tree partition [12], cf. Fig. 1), i.e. to consist of orthogonal hypercubes organized in a hierarchical data-structure. This allows extremely fast real-time search.

The approximate mp-QP algorithm in [10], [11] is guaranteed to terminate with an approximate solution that satisfies a specified maximum allowed error in the cost function, while the algorithm in [12] respects the specified approximation tolerance in the solution (the control input).

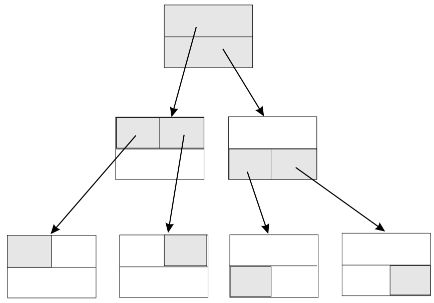

Fig. 1. *k*-*d*-tree partition of a rectangular region in a 2-dimensional state-space.

Like the exact solution, the number of regions with the approximate solution increases significantly with the order $\boldsymbol{n}$ of the system. Therefore, it is reasonable to search for modification of the approximate approach that would reduce significantly the complexity of partition. In [13], a reduced dimension approximate mp-QP algorithm has been developed that is an extension of the approximate approach in [10], [11]. The idea is to build partition in a sub-space of the state space, where each region will be characterized by a set of PWL feasible full-state feedback laws, since the unique optimal active set can not be determined from a reduced-dimension state, in general. In the real-time implementation of the explicit MPC, it will be necessary to check which of the feasible feedback laws associated to the current region is the optimal one (minimizes the performance index), see also [7]. However, keeping the number of these feedback laws small and searching in a reduced dimension partition, will guarantee an efficient real-time implementation.

Let the full state vector be $\boldsymbol{x} \in \boldsymbol{R}^n$, the reduced state vector on which the partition will be made be $\tilde{\boldsymbol{x}} \in \boldsymbol{R}^{\tilde{n}}$, where $\tilde{\boldsymbol{n}} < \boldsymbol{n}$, and the vector with the remaining part of the state variables be $\breve{\boldsymbol{x}} \in \boldsymbol{R}^{n-\tilde{n}}$. Denote the hypercube defining the whole region of interest in the full state space as $\boldsymbol{X}$ and the corresponding hypercube in the reduced state space as $\tilde{\boldsymbol{X}}$, where $\tilde{\boldsymbol{X}} \subset \boldsymbol{X}$. Let also for convenience call the state variables $\tilde{\boldsymbol{x}}$ "partitioned" and the state variables $\breve{\boldsymbol{x}}$ "non-partitioned". The idea of the reduced dimension approach is illustrated on Fig. 2, where the full state vector is $\boldsymbol{x} = [\boldsymbol{x}_1 \ \boldsymbol{x}_2 \ \boldsymbol{x}_3]^T$, the partitioned state variables are $\tilde{\boldsymbol{x}} = [\boldsymbol{x}_1 \ \boldsymbol{x}_2]^T$ and the non-partitioned state variable is $\breve{\boldsymbol{x}} = \boldsymbol{x}_3$.

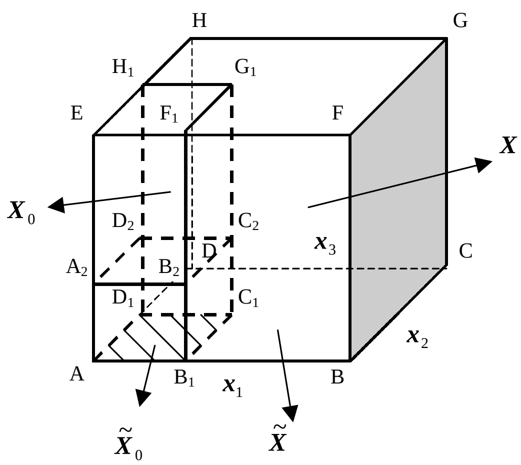

Fig. 2. Illustration of the reduced dimension approach.

The hypercube $\boldsymbol{X}$ with vertices $\{\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D},\boldsymbol{E},\boldsymbol{F},\boldsymbol{G},\boldsymbol{H}\}$ denotes the whole region of interest in the full state space and $\tilde{\boldsymbol{X}}$ with vertices $\{\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D}\}$ denotes the whole region in the reduced state space. Consider a region $\tilde{\boldsymbol{X}}_0$ in the partition of $\tilde{\boldsymbol{X}}$, with vertices $\{\boldsymbol{A},\boldsymbol{B}_1,\boldsymbol{C}_1,\boldsymbol{D}_1\}$. The corresponding region $\boldsymbol{X}_0$ in the full state space has vertices $\{\boldsymbol{A},\boldsymbol{B}_1,\boldsymbol{C}_1,\boldsymbol{D}_1,\boldsymbol{E},\boldsymbol{F}_1,\boldsymbol{G}_1,\boldsymbol{H}_1\}$. Let us consider the set of vertices $\{\boldsymbol{A},\boldsymbol{B}_1,\boldsymbol{C}_1,\boldsymbol{D}_1\}$ corresponding to the minimal allowed value of the non-partitioned state variable $\boldsymbol{x}_3$. Then, the candidate approximate solution $\hat{\boldsymbol{z}}_1(\boldsymbol{x})=\hat{\boldsymbol{K}}_1\boldsymbol{x}+\hat{\boldsymbol{g}}_1$ is chosen as the best fit to the optimal solutions at the vertices $\{\boldsymbol{A},\boldsymbol{B}_1,\boldsymbol{C}_1,\boldsymbol{D}_1\}$, while satisfying all constraints at the vertices $\{\boldsymbol{A},\boldsymbol{B}_1,\boldsymbol{C}_1,\boldsymbol{D}_1,\boldsymbol{E},\boldsymbol{F}_1,\boldsymbol{G}_1,\boldsymbol{H}_1\}$. By choosing another value for $\boldsymbol{x}_3$ within the admissible range $[\boldsymbol{x}_3^{\boldsymbol{l}};\boldsymbol{x}_3^{\boldsymbol{u}}]$, we will have another set of vertices $\{\boldsymbol{A}_2,\boldsymbol{B}_2,\boldsymbol{C}_2,\boldsymbol{D}_2\}$ and therefore another candidate feedback law $\hat{\boldsymbol{z}}_2(\boldsymbol{x})=\hat{\boldsymbol{K}}_2\boldsymbol{x}+\hat{\boldsymbol{g}}_2$ that will be the best fit to the optimal solutions at these vertices. Both candidate control laws $\hat{\boldsymbol{z}}_1(\boldsymbol{x})$ and $\hat{\boldsymbol{z}}_2(\boldsymbol{x})$ will be feasible in the region $\boldsymbol{X}_0$ in the full state space. We therefore need to consider all candidate control laws corresponding to all possible values of the non-partitioned state variable $\breve{\boldsymbol{x}}=\boldsymbol{x}_3$. The details of the computation of feasible candidate control laws are given in [13].

Also in [13], the approximate reduced dimension mp-QP algorithm is described together with the algorithm for real-time implementation of the resulting explicit MPC controller.

IV. DESIGN OF REDUCED DIMENSION EXPLICIT MPC FOR A GAS-LIQUID SEPARATION PLANT

We consider a sub-process within a semi-industrial installation which is used for reduction of NO_x in effluent gasses and technological waste water treatment by means of neutralisation with CO_2 contained in flue gasses [15]. The role of the separation unit (Fig. 3 from [15]) is to capture flue gasses under low pressure from effluent channels by means of water flow and to carry them over under high enough pressure to the downstream (neutralisation) stage. The flue gasses coming from the effluent channels are "pooled" by the water flow into the water circulation pipe through the injector $\boldsymbol{I_1}$. The water flow is generated by the pump $\boldsymbol{P_1}$ (water ring). The speed of the pump is kept constant. The pump feeds the mixture of water and gas into the separator $\boldsymbol{R_1}$ where gas is separated from water. Hence the accumulated gas in $\boldsymbol{R_1}$ forms a sort of "gas cushion" with increased internal pressure. Owing to this pressure, flue gas is blown out from $\boldsymbol{R_1}$ into the next neutralisation unit. On the other side the "cushion" forces water to circulate back to the reservoir $\boldsymbol{R_2}$. The quantity of water in the circuit is constant. If for some reason additional water is needed, the water supply path through the valve $\boldsymbol{V_5}$ is utilised.

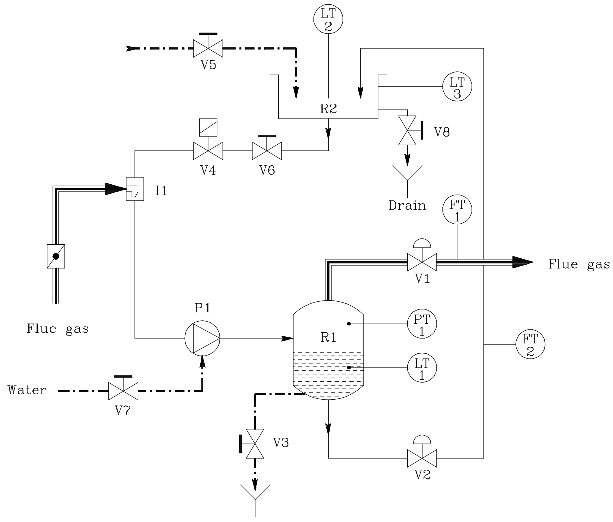

Fig. 3. Process scheme of the separation unit.

The complete non-linear model of the gas-liquid separator is given in [15]. A linearized model can be obtained from the existing non-linear model:

$$\begin{bmatrix}\Delta\dot{\boldsymbol{p}}_1\\ \Delta\dot{\boldsymbol{h}}_1\end{bmatrix}=\boldsymbol{A}_c\begin{bmatrix}\Delta\boldsymbol{p}_1\\ \Delta\boldsymbol{h}_1\end{bmatrix}+\boldsymbol{B}_c\begin{bmatrix}\Delta\boldsymbol{v}_1\\ \Delta\boldsymbol{v}_2\end{bmatrix} \qquad (11)$$

where $\Delta\boldsymbol{p_1}$ and $\Delta\boldsymbol{h_1}$ denote the change of separator gas pressure $\boldsymbol{p_1}$ and liquid level $\boldsymbol{h_1}$ from the steady-state values ($\Delta\boldsymbol{p}_1=\boldsymbol{p}_1-\boldsymbol{p}_{1s}$, $\Delta\boldsymbol{h}_1=\boldsymbol{h}_1-\boldsymbol{h}_{1s}$), and $\Delta\boldsymbol{v_1}$ and $\Delta\boldsymbol{v_2}$ are respectively the changes in the positions $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ of the two valves ($\Delta\boldsymbol{v}_1=\boldsymbol{v}_1-\boldsymbol{v}_{1s}$, $\Delta\boldsymbol{v}_2=\boldsymbol{v}_2-\boldsymbol{v}_{2s}$). The linear model corresponds to the following steady state:

$$\boldsymbol{p}_{1s}=0.5\,\boldsymbol{bar},\boldsymbol{h}_{1s}=1.4\,\boldsymbol{m},\boldsymbol{v}_{1s}=0.4152,\boldsymbol{v}_{2s}=0.7462 \qquad (12)$$

and the way to compute the elements of the matrices $\boldsymbol{A}_c$ and $\boldsymbol{B}_c$ is given in details in [15]. From the continuous-time model, a linear discrete-time model corresponding to sampling interval $\boldsymbol{T}_s=1\boldsymbol{s}$ is obtained, with the following state and control matrices:

$$\boldsymbol{A}=\begin{bmatrix}0.9719 & -0.0001\\ -0.0006 & 0.9999\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}-0.0832 & -0.0041\\ 0 & -0.0023\end{bmatrix} \qquad (13)$$

The state variables are $\boldsymbol{x}_1=\Delta\boldsymbol{p}_1\,[\boldsymbol{bar}]$ and $\boldsymbol{x}_2=\Delta\boldsymbol{h}_1\,[\boldsymbol{m}]$, and the control variables are $\boldsymbol{u}_1=\Delta\boldsymbol{v}_1$ and $\boldsymbol{u}_2=\Delta\boldsymbol{v}_2$. The following input constraints are imposed on the valve positions $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$:

$$0\le\boldsymbol{v}_1\le 1,\ 0\le\boldsymbol{v}_2\le 0.8625 \qquad (14)$$

which by taking into account the steady state values (12) are represented as the following constraints on the control inputs $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$:

$$\begin{aligned}&-0.4152\le\boldsymbol{u}_1(\boldsymbol{t}+\boldsymbol{k})\le 0.5848\\ &-0.7462\le\boldsymbol{u}_2(\boldsymbol{t}+\boldsymbol{k})\le 0.1163\\ &\boldsymbol{k}=0,1,...,\boldsymbol{N}-1\end{aligned} \qquad (15)$$

In order to avoid the steady state offset of the model predictive controller, two more states are added to the model (13), which take into account the integral error:

$$\boldsymbol{x}_3(\boldsymbol{t}+1)=\boldsymbol{x}_3(\boldsymbol{t})+\boldsymbol{T}_s\boldsymbol{x}_1(\boldsymbol{t}),\ \boldsymbol{x}_4(\boldsymbol{t}+1)=\boldsymbol{x}_4(\boldsymbol{t})+\boldsymbol{T}_s\boldsymbol{x}_2(\boldsymbol{t}) \qquad (16)$$

Thus, the linear discrete-time model of the gas-liquid separation unit becomes:

$$\boldsymbol{A} = \begin{bmatrix} 0.9719 & -0.0001 & 0 & 0 \\ -0.0006 & 0.9999 & 0 & 0 \\ \boldsymbol{T_s} & 0 & 1 & 0 \\ 0 & \boldsymbol{T_s} & 0 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -0.0832 & -0.0041 \\ 0 & -0.0023 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (17)$$

The approximate reduced dimension mp-QP approach [13] is applied to design an explicit MPC controller for the gas-liquid separation plant. The MPC minimizes the cost function (8) subject to the system equation (17) and the input constraints (15). In (8), $\boldsymbol{P}$ is chosen as the solution of the discrete algebraic Riccati equation and the cost matrices are:

$$\boldsymbol{Q} = \mathrm{diag}\{0.05, 100, 0.005, 0.0001\}, \ \boldsymbol{R} = \mathrm{diag}\{1, 1\} \quad (18)$$

The horizon is $\boldsymbol{N} = 500$ and the time instants at which the input variables can change are:

$$\boldsymbol{N}_{\boldsymbol{u}_1} = [1\ 5\ 10\ 15\ 20\ 25\ 30\ 35\ 40\ 45\ 50\ 100\ 102\ 104\ 106\ 108\ 110\ 300\ 302\ 304\ 306\ 308\ 310] \quad (19)$$

$$\boldsymbol{N}_{\boldsymbol{u}_2} = [1\ 5\ 10\ 15\ 20\ 25\ 30\ 35\ 40\ 45\ 50\ 100\ 300] \quad (20)$$

which makes totally 36 optimization variables.

The full state space to be considered is 4-dimensional and is defined by $\boldsymbol{X}$=[-0.5,0.5]×[-0.2,0.2]×[-3,3]×[-10,60]. The reduced state space $\tilde{\boldsymbol{X}}$ (in which the partition is made) includes the state variables $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, and is defined by $\tilde{\boldsymbol{X}}$ = [-0.5,0.5]×[-0.2,0.2]. The choice of these state variables is made according to the method described in [8].

The partition in the reduced state space is shown in Fig. 4 and it has 88 regions with up to 12 candidate PWL control laws for each region. In Fig. 5 to 8 the closed-loop performance of the approximate explicit MPC controller is shown, where the solid line corresponds to the approximate MPC and the dotted line corresponds to the exact MPC. The set point is $\boldsymbol{p}_1^* = \boldsymbol{p}_{1s} = 0.5\,\boldsymbol{bar}$ and $\boldsymbol{h}_1^* = \boldsymbol{h}_{1s} = 1.4\,\boldsymbol{m}$. In Fig. 4, the two curves represent the approximate and the exact trajectories in the reduced state space $(\boldsymbol{x}_1, \boldsymbol{x}_2)$. It can be seen that the approximate MPC controller designed in the reduced state space gives feasible trajectories and the sub-optimality is acceptable.

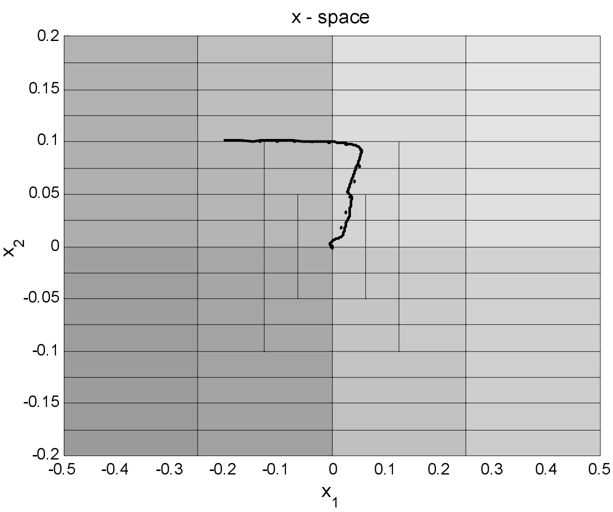

Fig. 4. Partition in $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ space.

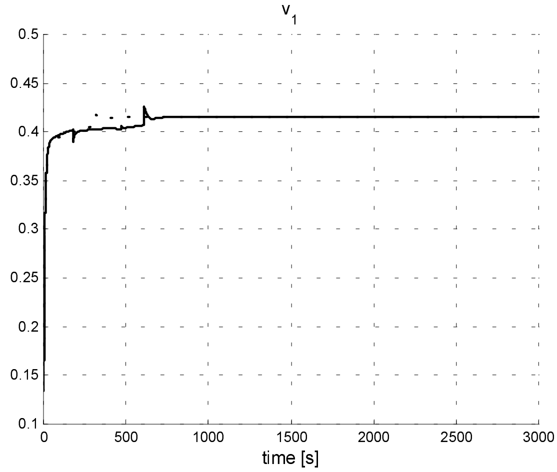

Fig. 5. Trajectory of $\boldsymbol{v}_1$ (position of valve 1).

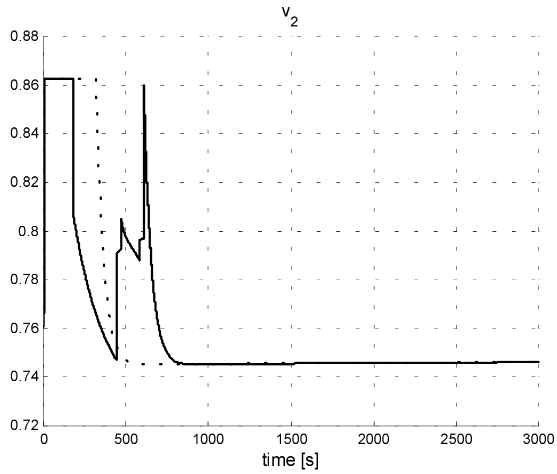

Fig. 6. Trajectory of $\boldsymbol{v}_2$ (position of valve 2).

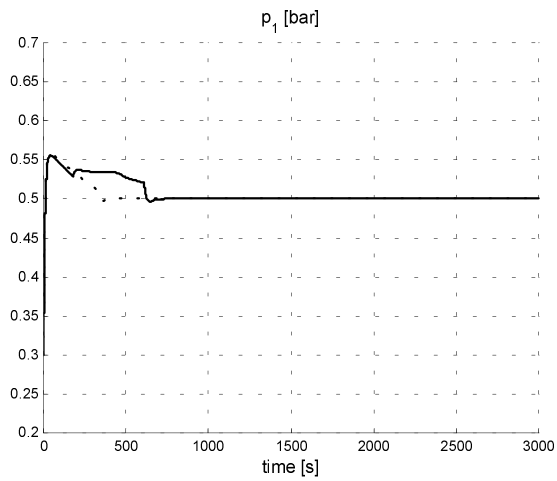

Fig. 7. Trajectory of $\boldsymbol{p}_1$ (pressure in the separator).

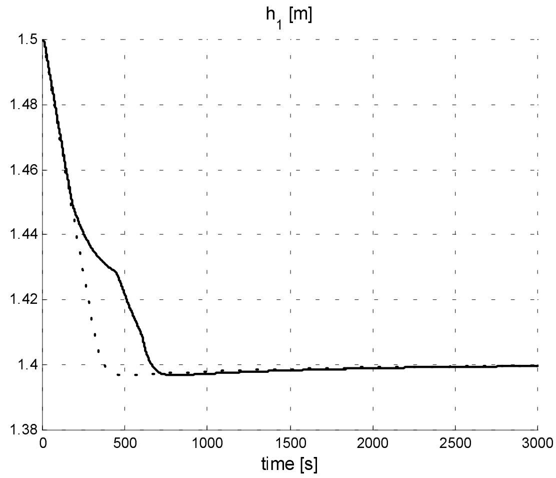

Fig. 8. Trajectory of $\boldsymbol{h}_1$ (liquid level in the separator).

V. CONCLUSIONS

In this paper, a reduced dimension approximate mp-QP approach is applied to design an explicit MPC controller for a gas-liquid separation plant. The controller represents a piecewise linear state feedback defined on an orthogonal partition of the reduced (two-dimensional) state space, where each region is characterized by a set of candidate feasible feedback laws. Results show that the approximate explicit MPC controller gives a satisfactory performance.

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