Mass Flow Estimation with Model Bias Correction for a Turbocharged Diesel Engine

Tomáš Polóni*, Boris Rohal-Ilkiv

*Institute of Automation, Measurement and Applied Informatics
Faculty of Mechanical Engineering, Slovak University of Technology
812 31 Bratislava, Slovakia
Email: tomas.poloni@stuba.sk, boris.rohal-ilkiv@stuba.sk*

Tor Arne Johansen

*Center for Autonomous Marine Operations and Systems (AMOS)
Department of Engineering Cybernetics
Norwegian University of Science and Technology
N-7491 Trondheim , Norway
Email: tor.arne.johansen@itk.ntnu.no*

Abstract

A systematic design method for mass flow estimation with correction for model bias is proposed. Based on an augmented observable Mean Value Engine Model (MVEM) of a turbocharged Diesel engine, the online estimation of states with additional biases is performed to compute the mass flows for different places. A correction method is applied, that utilizes estimated biases which are in a least-square sense redistributed between the correction terms to the uncertain mass flow maps and then added to the estimated mass flows. An Extended Kalman Filter (EKF) is tested off-line on production car engine data where the combination of an intake manifold pressure sensor, exhaust manifold pressure sensor and turbocharger speed sensor is compared and discussed in different sensor fusions. It is shown that the correction method improves the uncorrected estimated air mass flow which is validated against the airflow data measured in the intake duct.

Keywords: Diesel engine, Mass flow estimation, Bias estimation, Kalman filtering, Mean value engine model, Model correction method

*Corresponding author

1. Introduction

Accurate information about the air charge of turbocharged Diesel engines is important for the fueling control where the injected amount of fuel can cause a visible smoke due to incomplete combustion, if the air charge quantity is overestimated. On the other side, the performance can suffer if the fueling is too conservative due to underestimating the air charge quantity. The mass charge induced by the cylinders is defined by the air quantity from the compressor flow and the exhaust gas recirculation (EGR) mass flow. The quality of the induced mass charge is defined by the air-fuel ratio. The information about the true amount of air charge is dimmed by premixing the EGR mass flow to the cylinders, containing the burned gas and unburned fresh air. The percentage of the fresh air in the EGR mass flow can be estimated via knowledge of burned gas fraction dynamics (Diop et al., 1999; Wang, 2008). The most desirable estimation technique combines the mass flow estimation (especially the EGR flow) and the oxygen concentration estimation to estimate the complete information about the air charge for a turbocharged Diesel engine (Kang et al., 2011) or for direct injection spark ignition engine (Stotsky and Kolmanovsky, 2002). These two problems of in-cylinder mass flow estimation and in-cylinder oxygen concentration estimation are often treated as independent estimation problems. The estimation of air charge for turbocharged Diesel engines with no EGR system with measured intake pressure (MAP) and intake temperature is reported in Storset et al. (2004). For the same kind of engine, the estimation of air charge through the adaptation of volumetric efficiency based on the upstream compressor airflow measurement (MAF) and MAP measurement is documented in Stefanopoulou et al. (2004). The compressor flow to the intake manifold can be estimated with MAP sensor and exhaust manifold pressure (EXMP) sensor (van Nieuwstadt et al., 2000; Kolmanovsky et al., 2000; Polóni et al., 2012), where van Nieuwstadt et al. (2000) base the EGR control loop on the estimated compressor flow information. The improved mass flow estimation (in-cylinder mass flow, compressor mass flow and EGR mass flow) accuracy is achieved with additional measurement of the turbocharger speed (NTC) (Höckerdal et al., 2009; Polóni et al., 2012). The MAF sensor adaptation based on the MAF measurement and turbocharger speed signal is studied in Höckerdal et al. (2011), where the measured air-mass flow with a production

MAF sensor is imprecise and requires corrections. A promising approach to estimate the quantity of the induced charge is in application of the in-cylinder pressure measurement, however so far the experiments are reported under steady state conditions (Desantes et al., 2010).

The precision of the engine’s mass flow estimation is characterized by the magnitude of the error. A model-based mass flow estimation algorithms relies on engine model precision which is sometimes uncertain due to map-based steady state Mean-Value-Engine-Model (MVEM) identification principles (Jung, 2003; Eriksson et al., 2010) and engine aging. The adaptation scheme (Stefanopoulou et al., 2004), for engines without the EGR, considers the adaptation of the volumetric efficiency where adaptation scheme for a naturally aspired spark ignition engine with EGR in Kolmanovsky et al. (2006) suggests to adapt in-cylinder mass flow based on the MAF and MAP measurements. An approach founded on the linearized state-space model augmentation with variable structure of model bias terms is proposed in Höckerdal et al. (2009) where the augmentation with a given structure of model bias terms is proposed in Polóni et al. (2012). The augmentations (Höckerdal et al., 2009; Polóni et al., 2012) aim to improve the estimation of the intake manifold pressure, exhaust manifold pressure and turbocharger speed by additional estimation of the bias terms entering the nonlinear state-space model.

For the air mass flow estimation problem, different possibilities exist:

- the use of a MAF sensor
- the use of a MAF and MAP sensor with an observer/model,
- the use of an Extended Kalman Filter (EKF) and MVEM with fusion of different sensor sets (MAP, EXMP, NTC) (as presented in this paper)
- the use of an exhaust lambda sensor (EGO) and injection signal for estimating the air mass flow
- other combinations of the previous ones

This work presents novelty in utilization of the estimated biases for the mass flow computation and presents the correction method that is experimentally tested on the turbocharged Diesel engine data. As we have focused on the bias modeling and redistribution of the bias error to mass flow maps, the optimal choice of estimation algorithm is not emphasized in our research.

Figure 1: Schematic turbocharged Diesel engine model representation

The Extended Kalman Filter (EKF) is chosen for estimation since it is a widely established and well known engineering tool. The computation of the mass flow correction vector is formulated as a least-squares problem under the assumption that map-based mass flow computations are the most uncertain parts of the model. Further contribution of this paper is the methodology of using the EKF, MVEM and the sensor fusion for on-board estimation of (not only) upstream compressor airflow, allowing to omit the MAF sensor.

2. Turbocharged engine system description and Mean-Value Engine model

The simplified schematic turbocharged Diesel engine setup considered in this study is shown in Figure 1. According to this figure, the engine is modeled with the compressor (c), turbine (t), final volume intake manifold (i), final volume exhaust manifold (x), intercooler (ic), EGR cooler (co), EGR pipe and the engine cylinders (e). The model has five states. The first four states represent the mass dynamics and they are: p_i intake manifold pressure [kPa], m_i intake mass [kg], p_x exhaust manifold pressure [kPa] and m_x exhaust mass [kg]. The fifth state is the turbocharger speed n_{tc} [rpm]. The inputs are EGR valve position X_{egr} [% open], VGT actuator position X_{vgt} [% open], injected fuel mass W_f [kg.s⁻¹] and the engine speed n_e [rpm].

The engine model is represented by the following mass and energy balance differential equations in the intake and exhaust manifolds, and by the torque

balance at the turbocharger shaft (Kolmanovsky et al., 1998)

$$\dot{p}_i = \frac{R\kappa}{V_i} \left(W_{ci}T_{ci} - W_{ie}T_i + W_{xi}T_{xi} - \frac{\dot{Q}_1}{c_p} \right) \quad (1)$$

$$\dot{m}_i = W_{ci} - W_{ie} + W_{xi} \quad (2)$$

$$\dot{p}_x = \frac{R\kappa}{V_x} \left((W_{ie} + W_f)T_e - (W_{xi} + W_{xt})T_x - \frac{\dot{Q}_2}{c_p} \right) \quad (3)$$

$$\dot{m}_x = W_{ie} + W_f - W_{xi} - W_{xt} \quad (4)$$

$$\dot{n}_{tc} = \left(\frac{60}{2\pi} \right)^2 \frac{1}{J} \left(\frac{P_t - P_c}{n_{tc}} \right) \quad (5)$$

The symbols in these equations are either input variables, state variables, constants or functions of the five states and inputs. The index associated with each variable defines the location of the variable. In the case of two indexes the first one is the upstream and the second one is the downstream location. The mass flows are denoted as W [$kg.s^{-1}$]. The EGR mass flow is modeled as W_{xi} where alternatively the backflow can be considered as W_{ix} however, under standard operating conditions $W_{ix} = 0$. The temperatures [K] in the intake and exhaust manifolds are T_i and T_x . The differences of the static and dynamic pressures and temperatures are neglected because of the low mass flows. The constant parameters for the model are the intake manifold volume V_i [m^3], exhaust manifold volume V_x [m^3], specific heats at constant pressure and volume [$J.kg^{-1}.K^{-1}$] c_p , c_v , isentropic exponent of air $\kappa = \frac{c_p}{c_v}$, specific gas constant of air $R = c_p - c_v$ and the turbocharger inertia J [$kg.m^2$]. The heat losses in the Eq. (1), $\dot{Q}_1$ and Eq. (3), $\dot{Q}_2$ are assumed to be zero. For the intake manifold, this is a reasonable assumption since the temperature is not much different than the ambient temperature. However in the case of exhaust manifold the heat losses are significant. The exhaust gas heat loss is implicitly captured in the steady state (look-up table) out-flow engine temperature model T_e that is discussed later.

The following equations summarize the dependencies of intermediate variables in Eq. (1)-(5). Some of these dependencies are expressed as the look-up tables obtained by fitting the steady-state experimental engine data to a second order polynomial surface with parameter vector Θ . The others are obtained by physical relations. The air mass flow from compressor to the intake manifold W_{ci} , pressure after compressor p_c and the compressor efficiency

η_c are mapped as (Jung, 2003)

$$W_{ci} = f_{W_{ci}}(p_i, n_{tc}, \Theta_{W_{ci}}) \quad (6)$$

$$p_c = f_{p_c}(W_{ci}, n_{tc}, \Theta_{p_c}) \quad (7)$$

$$\eta_c = f_{\eta_c}(W_{ci}, p_i, \Theta_{\eta_c}) \quad (8)$$

with fitted polynomial surfaces characterizing the turbocharged Diesel combustion engine. The look-up tables are 2-D functions with a linear interpolation capability with grid data generated from the fitted polynomial surfaces.

The temperature after the compressor is given by

$$T_c = T_a + \frac{1}{\eta_c} T_a \left[\left(\frac{p_c}{p_a} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right] \quad (9)$$

where T_a is the ambient temperature and p_a is the ambient pressure. The following mappings for the temperature after the intercooler T_{ci} and the engine volumetric efficiency η_v are

$$T_{ci} = f_{T_{ci}}(W_{ci}, T_c, \Theta_{T_{ci}}) \quad (10)$$

$$\eta_v = f_{\eta_v}(n_e, p_i, \Theta_{\eta_v}) \quad (11)$$

with fitted polynomial surfaces. The mass flow from the intake manifold to the engine cylinders is

$$W_{ie} = \frac{m_i n_e V_d}{V_i 120} \eta_v \quad (12)$$

The temperatures in the intake and the exhaust manifolds are

$$T_i = \frac{p_i V_i}{m_i R} \quad (13)$$

$$T_x = \frac{p_x V_x}{m_x R} \quad (14)$$

The EGR flow W_{xi} is modeled by the orifice equation that is characterized by the effective flow area

$$A_{egr} = a_2 X_{egr}^2 + a_1 X_{egr} \quad (15)$$

where a_2 and a_1 are polynomial coefficients and by nonlinear term Ψ_{egr} that reflects the pressure conditions upstream and downstream at the orifice (Heywood, 1988)

$$\Psi_{egr} = \begin{cases} \sqrt{\frac{2\kappa}{\kappa-1} \left[\left(\frac{p_i}{p_x}\right)^{\frac{2}{\kappa}} - \left(\frac{p_i}{p_x}\right)^{\frac{\kappa+1}{\kappa}} \right]} & \text{if } \frac{p_i}{p_x} \geq \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \\ \sqrt{\left(\kappa \frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}} & \text{otherwise} \end{cases} \quad (16)$$

$$W_{xi} = \frac{A_{egr} p_x}{\sqrt{RT_x}} \Psi_{egr} \quad (17)$$

The temperature of the mass that flows out of the cylinders T_e , EGR mass temperature T_{xi} and the mass flow from the exhaust manifold to the turbine W_{xt} are

$$T_e = f_{T_e}(W_f, W_{ie}, \Theta_{T_e}) \quad (18)$$

$$T_{xi} = f_{T_{xi}}(T_x, W_{xi}, \Theta_{T_{xi}}) \quad (19)$$

$$W_{xt} = f_{W_{xt}}(X_{vgt}, p_x, \Theta_{W_{xt}}) \quad (20)$$

with fitted polynomial surfaces. The power of the compressor is given by

$$P_c = W_{ci} c_p \frac{1}{\eta_c} T_a \left[\left(\frac{p_c}{p_a}\right)^{\frac{\kappa-1}{\kappa}} - 1 \right] \quad (21)$$

The power of the turbine is calculated as

$$P_t = W_{xt} c_p \frac{1}{\eta_t} T_x \left[1 - \left(\frac{p_t}{p_x}\right)^{\frac{\kappa-1}{\kappa}} \right] \quad (22)$$

where the turbine efficiency

$$\eta_t = f_{\eta_t} \left(\frac{p_t}{p_x}, n_{tc}, \Theta_{\eta_t} \right) \quad (23)$$

is mapped with the turbine pressure

$$p_t = b_3 W_{xt}^3 + b_2 W_{xt}^2 + b_1 W_{xt} + b_0 \quad (24)$$

where $b_0, \dots, b_3$ are the polynomial coefficients. More details on model validation can be found in (Polóni et al., 2012).

3. Model bias augmentations

The precision of the MVEM depends on its structure and parameters. The imprecision is given by the finite mean-value model structure that with limited accuracy gives a true picture of physical relations between different sub-systems of a turbocharged Diesel engine. The identification of the maps of the MVEM is experimentally performed at the engine steady state conditions. The identification in steady state conditions introduces certain systematic errors due to neglecting the dynamic effects of the modeled variables.

The open loop observer indicates with which state equations the modeling errors are associated with (Polóni et al., 2012). One way to remove these errors is to consistently inspect the parameter identification of the maps, to inspect the methods and conditions of the experiments (Wahlström and Eriksson, 2011) under which the maps were obtained and identify more accurate maps. This is a considerable task and does not account for structural model errors or errors due to aging of the engine. Another way to compensate for these modeling errors is to augment the model with bias parameters to be identified on-line by the observer or to introduce adaptive models (Höckerdal et al., 2011). Allocating bias terms to the state equations with the largest open loop modeling errors provides a first guess. In the experimental evaluation presented in section 5, careful validation considering alternative combination of bias term allocations showed that this provides good solutions. However, as each model and engine type may have very different modeling error characteristics, we recommend that careful validation with alternative combinations of bias terms is always performed. The number of such bias parameters is limited by the observability i. e. by the number of measurements available and by the structure of the equations (Höckerdal et al., 2009).

A compact form of an augmented state-space model is

$$\dot{x}_s = \tilde{f}_c(x_s, w, u) + z_x \quad (25)$$

$$\dot{w} = z_w \quad (26)$$

where the continuous function $\tilde{f}_c$ has the mapping properties $\tilde{f}_c : \mathbb{R}^{n_s} \times \mathbb{R}^{n_w} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_s}$, n_s is the size of the state vector x_s , n_w is the size of the bias vector w and n_u is the size of the input vector u . The state process noise vector is z_x and the augmented state (bias) process noise vector is z_w . The

augmented state vector $x \in \mathbb{R}^{n_s+n_w}$ is defined by

$$x = \begin{bmatrix} x_s \\ w \end{bmatrix} \quad (27)$$

The state and bias equations can be combined as

$$\dot{x} = f_c(x, u) + z \quad (28)$$

where the continuous function f_c has the mapping properties $f_c : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_x}$, n_x being the dimension of the augmented state vector x and $z = [z_x, z_w]^T$. The observation equation may be written as

$$y = h_c(x, u) + v \quad (29)$$

where $y \in \mathbb{R}^{n_y}$ is the vector of measurements, $h_c : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_y}$ is the continuous measurement function and n_y is the number of measured states. The measurement errors are modeled with the noise term $v \in \mathbb{R}^{n_y}$. The most frequent situation encountered in practice is when the system is governed by continuous-time dynamics and the measurements are obtained at discrete time instances. For the estimator formulation we consider the numerically discretized dynamic nonlinear system described by the equations

$$x_{t+1} = f(x_t, u_t) + z_t \quad (30)$$

$$y_t = h(x_t, u_t) + v_t \quad (31)$$

for $t = 0, 1, \dots$, where $x_t \in \mathbb{R}^{n_x}$ is the state vector, $u_t \in \mathbb{R}^{n_u}$ is the input vector and $z_t \in \mathbb{R}^{n_z}$ is the process noise vector. The numerical discretization algorithm is discussed in the Appendix A. The state vector is observed through the measurement equation (31) where $y_t \in \mathbb{R}^{n_y}$ is the observation vector and $v_t \in \mathbb{R}^{n_y}$ is a measurement noise vector.

In the following presentation, the additive bias terms have been chosen as a lumped representation of modeling errors since it allows straightforward use of the EKF for estimation, and straightforward distribution of the modeling error to the modeled mass flows. We do not claim that additive bias terms will in general lead to more accurate models than e.g. multiplicative parameters, but this choice makes the approach simple and the results shows that it is an effective choice allowing model accuracy to be improved in the case study.

3.1. Two-sensor setup with sensors for p_i and p_x

Two measured variables will be processed by the two-sensor setup observer which means that, at most, two biases can be added. Instead of a mathematical analysis of observability we note that the system is open loop asymptotically stable and the estimates would converge if there were no model errors. Two measurements give two additional equations that allow two unknowns (two bias variables) to be introduced and solved. A possible augmentation of the model where $x_s = [p_i, m_i, p_x, m_x, n_{tc}]^T = [x_1, x_2, x_3, x_4, x_5]^T$ is with bias placed on the intake manifold pressure and bias placed on the exhaust manifold pressure (Alt. 1).

Alt. 1: Biases w_i and w_x . The augmented state-space model with the biases w_i and w_x is

$$\dot{x}_1 = \frac{R\kappa}{V_i} (W_{ci}T_{ci} - W_{ie}T_i + W_{xi}T_{xi}) + w_i + z_1 \quad (32)$$

$$\dot{x}_2 = W_{ci} - W_{ie} + W_{xi} + z_2 \quad (33)$$

$$\dot{x}_3 = \frac{R\kappa}{V_x} [(W_{ie} + W_f)T_e - (W_{xi} + W_{xt})T_x] + w_x + z_3 \quad (34)$$

$$\dot{x}_4 = W_{ie} + W_f - W_{xi} - W_{xt} + z_4 \quad (35)$$

$$\dot{x}_5 = \left(\frac{60}{2\pi}\right)^2 \frac{1}{J} \left(\frac{P_t - P_c}{n_{tc}}\right) + z_5 \quad (36)$$

$$\dot{w}_i = z_6 \quad (37)$$

$$\dot{w}_x = z_7 \quad (38)$$

The open loop observer simulation shows that the model is most imprecise on the intake manifold pressure state and turbocharger speed state (Polóni et al., 2012). The augmentation with bias terms placed on derivatives of these states is therefore logical with the two-sensor setup (Alt. 2).

Alt. 2: Biases w_i and w_{tc} . The augmented state-space model with the biases w_i and w_{tc} is

$$\dot{x}_1 = \frac{R\kappa}{V_i} (W_{ci}T_{ci} - W_{ie}T_i + W_{xi}T_{xi}) + w_i + z_1 \quad (39)$$

$$\dot{x}_2 = W_{ci} - W_{ie} + W_{xi} + z_2 \quad (40)$$

$$\dot{x}_3 = \frac{R\kappa}{V_x} [(W_{ie} + W_f) T_e - (W_{xi} + W_{xt}) T_x] + z_3 \quad (41)$$

$$\dot{x}_4 = W_{ie} + W_f - W_{xi} - W_{xt} + z_4 \quad (42)$$

$$\dot{x}_5 = \left(\frac{60}{2\pi}\right)^2 \frac{1}{J} \left(\frac{P_t - P_c}{n_{tc}}\right) + w_{tc} + z_5 \quad (43)$$

$$\dot{w}_i = z_6 \quad (44)$$

$$\dot{w}_{tc} = z_7 \quad (45)$$

3.2. Three-sensor setup with sensors for p_i , p_x and n_{tc}

The third measured variable, the turbocharger speed, is additionally introduced to study its impact on overall estimation precision with the bias terms placed on the intake manifold pressure, exhaust manifold pressure and turbocharger speed (Alt. 3).

Alt. 3: Biases w_i , w_x and w_{tc} . The augmented state-space model with the biases w_i , w_x and w_{tc} is

$$\dot{x}_1 = \frac{R\kappa}{V_i} (W_{ci}T_{ci} - W_{ie}T_i + W_{xi}T_{xi}) + w_i + z_1 \quad (46)$$

$$\dot{x}_2 = W_{ci} - W_{ie} + W_{xi} + z_2 \quad (47)$$

$$\dot{x}_3 = \frac{R\kappa}{V_x} [(W_{ie} + W_f) T_e - (W_{xi} + W_{xt}) T_x] + w_x + z_3 \quad (48)$$

$$\dot{x}_4 = W_{ie} + W_f - W_{xi} - W_{xt} + z_4 \quad (49)$$

$$\dot{x}_5 = \left(\frac{60}{2\pi}\right)^2 \frac{1}{J} \left(\frac{P_t - P_c}{n_{tc}}\right) + w_{tc} + z_5 \quad (50)$$

$$\dot{w}_i = z_6 \quad (51)$$

$$\dot{w}_x = z_7 \quad (52)$$

$$\dot{w}_{tc} = z_8 \quad (53)$$

4. Mass flow correction method

The basic strategy to correct the mass flows is shown in Figure 2. The jointly estimated state vector x_s and bias vector w is used to compute the

Figure 2: Mass flow estimation scheme with error-bias correction

mass flow vector and the correction vector respectively. The estimated mass flow vector $\hat{W}$ is corrected by the correction vector $\hat{e}$ to be explained in this section. The corrected mass flow vector is denoted $\hat{W}^{[c]}$.

The estimated bias vector w represents the information about errors of the state equations. This error is called a model bias, although it is not a constant value but modeled by a Wiener process. This bias information can be redistributed in a least-square sense among the modeled mass flow terms W_{ci} , W_{ie} , W_{xi} , W_{xt} . The basis for the least-squares problem is that the mass flow maps W_{ci} , W_{ie} , W_{xi} and W_{xt} contains the most significant contributions to model error, as other sources are neglected in the redistribution.

The least-square error equations are formed by the subtraction of each state equation from the corrected state equation. For example, the subtraction of the following two equations derived from Eq. (32) leads to the first least-square error equation given by the Eq. (56)

$$\frac{V_i}{R\kappa} \dot{x}_1 = W_{ci}T_{ci} - W_{ie}T_i + W_{xi}T_{xi} + \frac{V_i}{R\kappa} w_i \quad (54)$$

$$\frac{V_i}{R\kappa} \dot{x}_1 = (W_{ci} + e_{ci})T_{ci} - (W_{ie} + e_{ie})T_i + (W_{xi} + e_{xi})T_{xi} \quad (55)$$

The least-square error equations are formed below each state equation as

follows

$$\begin{aligned}\frac{V_i}{R\kappa}\dot{x}_1 &= W_{ci}T_{ci} - W_{ie}T_i + W_{xi}T_{xi} + \frac{V_i}{R\kappa}w_i \\ 0 &= e_{ci}T_{ci} - e_{ie}T_i + e_{xi}T_{xi} - \frac{V_i}{R\kappa}w_i\end{aligned}\quad (56)$$

$$\begin{aligned}\dot{x}_2 &= W_{ci} - W_{ie} + W_{xi} + \frac{V_i}{T_{Si}R\kappa}w_i \\ 0 &= e_{ci} - e_{ie} + e_{xi} - \frac{V_i}{T_{Si}R\kappa}w_i\end{aligned}\quad (57)$$

$$\begin{aligned}\frac{V_x}{R\kappa}\dot{x}_3 &= (W_{ie} + W_f)T_e - (W_{xi} + W_{xt})T_x + \frac{V_x}{R\kappa}w_x \\ 0 &= e_{ie}T_e - e_{xi}T_x - e_{xt}T_x - \frac{V_x}{R\kappa}w_x\end{aligned}\quad (58)$$

$$\begin{aligned}\dot{x}_4 &= W_{ie} + W_f - W_{xi} - W_{xt} + \frac{V_x}{T_{Sx}R\kappa}w_x \\ 0 &= e_{ie} - e_{xi} - e_{xt} - \frac{V_x}{T_{Sx}R\kappa}w_x\end{aligned}\quad (59)$$

$$\begin{aligned}\frac{J}{\left(\frac{60}{2\pi}\right)^2}\dot{x}_5 &= W_{xt}\frac{c_pT_x}{n_{tc}\eta_t}\left[1 - \left(\frac{p_t}{p_x}\right)^{\frac{\kappa-1}{\kappa}}\right] - \\ &\quad - W_{ci}\frac{c_pT_a}{n_{tc}\eta_c}\left[\left(\frac{p_c}{p_a}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] + \frac{J}{\left(\frac{60}{2\pi}\right)^2}w_{tc} \\ 0 &= e_{xt}\frac{c_pT_x}{n_{tc}\eta_t}\left[1 - \left(\frac{p_t}{p_x}\right)^{\frac{\kappa-1}{\kappa}}\right] - \\ &\quad - e_{ci}\frac{c_pT_a}{n_{tc}\eta_c}\left[\left(\frac{p_c}{p_a}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] - \frac{J}{\left(\frac{60}{2\pi}\right)^2}w_{tc}\end{aligned}\quad (60)$$

where the errors e_{ci} , e_{ie} , e_{xi} , e_{xt} are elements of the correction vector e .

The physical relation of the pressure state equations and mass state equations is in the scaling of the flow terms W by the temperatures T and the $\frac{R\kappa}{V}$ term. If we consider the scaling temperature for the bias w_i as $T_{Si} = (T_{ci} + T_i + T_{xi})/3$ and the scaling temperature for the bias w_x as $T_{Sx} = (T_e + T_x)/2$, one can introduce the same bias term (w) for the mass balance state equation.

The linear least-square problem to estimate the correction vector is formulated as the Euclidean norm minimization problem

$$\hat{e} = \min_e \|Ae + b\|_2^2 \quad (61)$$

for the previously considered sensor and bias alternatives:

Two-sensor setup (p_i, p_x) with biases w_i and w_x (Alt. 1). In this sensor configuration, where pressures p_i, p_x are measured, two biases are placed on the differential equations of the intake manifold pressure w_i and differential equation of the exhaust manifold pressure w_x .

$$A = \begin{bmatrix} T_{ci} & -T_i & T_{xi} & 0 \\ T_{Si} & -T_{Si} & T_{Si} & 0 \\ 0 & T_e & -T_x & T_x \\ 0 & T_{Sx} & -T_{Sx} & -T_{Sx} \end{bmatrix}; e = \begin{bmatrix} e_{ci} \\ e_{ie} \\ e_{xi} \\ e_{xt} \end{bmatrix}; b = \begin{bmatrix} -\frac{V_i}{R\kappa} w_i \\ -\frac{V_i}{R\kappa} w_i \\ -\frac{V_x}{R\kappa} w_x \\ -\frac{V_x}{R\kappa} w_x \end{bmatrix}$$

Two-sensor setup (p_i, p_x) with biases w_i and w_{tc} (Alt. 2). In this sensor configuration where pressures p_i, p_x are measured, two biases are placed on the differential equations of the intake manifold pressure w_i and differential equation of the turbocharger speed w_{tc} . With the bias terms w_i and w_{tc} , the correction vector contains only three elements, since w_{tc} can not be scaled for any other equation.

$$A = \begin{bmatrix} T_{ci} & -T_i & 0 \\ T_{Si} & -T_{Si} & 0 \\ -\frac{c_p T_a}{n_{tc} \eta_c} \left[\left(\frac{p_c}{p_a} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right] & 0 & \frac{c_p T_x}{n_{tc} \eta_t} \left[1 - \left(\frac{p_t}{p_x} \right)^{\frac{\kappa-1}{\kappa}} \right] \end{bmatrix}; e = \begin{bmatrix} e_{ci} \\ e_{ie} \\ e_{xt} \end{bmatrix};$$

$$b = \begin{bmatrix} -\frac{V_i}{R\kappa} w_i \\ -\frac{V_i}{R\kappa} w_i \\ -\frac{J}{\left(\frac{60}{2\pi} \right)^2} w_{tc} \end{bmatrix}$$

Three-sensor setup (p_i, p_x, n_{tc}) with biases w_i, w_x and w_{tc} (Alt. 3). In this sensor configuration, pressures p_i, p_x and the turbocharger speed n_{tc} are measured, where the biases are w_i, w_x and w_{tc} .

$$A = \begin{bmatrix} T_{ci} & -T_i & T_{xi} & 0 \\ T_{Si} & -T_{Si} & T_{Si} & 0 \\ 0 & T_e & -T_x & T_x \\ 0 & T_{Sx} & -T_{Sx} & -T_{Sx} \\ -\frac{c_p T_a}{n_{tc} \eta_c} \left[\left(\frac{p_c}{p_a} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right] & 0 & 0 & \frac{c_p T_x}{n_{tc} \eta_t} \left[1 - \left(\frac{p_t}{p_x} \right)^{\frac{\kappa-1}{\kappa}} \right] \end{bmatrix};$$

$$e = \begin{bmatrix} e_{ci} \\ e_{ie} \\ e_{xi} \\ e_{xt} \end{bmatrix}; b = \begin{bmatrix} -\frac{V_i}{R_i^\kappa} w_i \\ -\frac{V_i}{R_i^\kappa} w_i \\ -\frac{V_x}{R_x^\kappa} w_x \\ -\frac{V_x}{R_x^\kappa} w_x \\ -\frac{J}{\left(\frac{60}{2\pi}\right)^2} w_{tc} \end{bmatrix}$$

The elements of the A -matrix are obtained by applying the state vector estimate (given by the observer $\hat{x}_s$) to the MVEM; for example the intake manifold temperature given by the Eq. (13) is $T_i = \hat{p}_i V_i / \hat{m}_i R$.

The least-square problem is solved by the ordinary least-square closed-form expression

$$\hat{e} = (A^T A)^+ A^T b \quad (62)$$

Since the problem might be ill-conditioned the pseudoinverse of the matrix $A^T A$ is computed via the Moore-Penrose pseudoinverse through the singular value decomposition (SVD) (Golub and Van Loan, 1996) as

$$(A^T A)^+ = V D^+ U^T \quad (63)$$

where V and U are unitary matrices and D is a diagonal matrix with nonnegative real numbers on the diagonal, the singular values, where the tolerance on singular values is δ_{SVD} . Any singular value less than this tolerance, has its inverse set to zero. The A -matrix can be considered as a sensitivity-matrix where further sensitivity study can be performed to inspect the bias placement or inspect the parameter adaptation (if adaptive model is considered).

The mass flow computations are based on the state vector estimate computed by the EKF. After obtaining the estimated state vector from the EKF $\hat{x}_s = [\hat{p}_i, \hat{m}_i, \hat{p}_x, \hat{m}_x, \hat{n}_{tc}]^T$, and computing the correction vector $\hat{e}$ (Eq. 62), the corrected mass flow vector $\hat{W}^{[c]}$ is computed as follows

$$\hat{W}_{ci}^{[c]} = f_{W_{ci}}(\hat{p}_i, \hat{n}_{tc}, \Theta_{W_{ci}}) + \hat{e}_{ci} \quad (64)$$

$$\hat{W}_{ie}^{[c]} = \frac{\hat{m}_i n_e V_d}{V_i 120} \eta_v + \hat{e}_{ie} \quad (65)$$

$$\hat{W}_{xi}^{[c]} = \frac{A_{egr} \hat{p}_x}{\sqrt{R \hat{T}_x}} \hat{\Psi}_{egr} + \hat{e}_{xi} \quad (66)$$

$$\hat{W}_{xt}^{[c]} = f_{W_{xt}}(X_{vgt}, \hat{p}_x, \Theta_{W_{xt}}) + \hat{e}_{xt} \quad (67)$$

	RSE of $\hat{W}_{ci}$	RSE of $\hat{W}_{ci}^{[c]}$
Alt. 1	0.147	0.139
Alt. 2	0.112	0.096
Alt. 3	0.093	0.067

Table 1: Root Square Error (RSE) of uncorrected $\hat{W}_{ci}$ and corrected $\hat{W}_{ci}^{[c]}$ compressor air-flow estimate for different sensor/bias alternatives

5. Experimental evaluation

The presented mass flow estimation and correction strategy is tested off-line on driving cycle engine data introduced in the Appendix B.1.

The quality of the algorithm is evaluated by the Root Square Error (RSE)

$$RSE = \sqrt{\sum_{t=1}^n (MAF_t - \hat{W}_{ci,t})^2} \quad (68)$$

where $n = 800$. The RSE is computed for the compressor air-flow W_{ci} , for which the validation measurement is available from the MAF sensor. The engine's other mass flows can not be easily measured, therefore it is assumed here that if the compressor flow is relatively precise the other mass flows are likely to be precise as well. The RSE index is computed for three considered sensor/bias alternatives, as shown in Table 1. It can be seen in Table 1, that in all three studied cases the correction method improves the estimated air-flows.

Two-sensor setup (p_i, p_x) with biases w_i and w_x (Alt. 1). The estimated biases with the two-sensor setup of the intake and exhaust pressures are shown in Figure 3. The estimated compressor mass flow without the correction $\hat{W}_{ci}$ and the estimated compressor mass flow with the correction $\hat{W}_{ci}^{[c]}$ are compared in Figure 4. In this case, the uncorrected air-flow estimate $\hat{W}_{ci}$ is not very precise mostly during the downspeeding of the engine. Also the placement of biases does not provide sufficient error information toward the true compressor air flow which leads to biased corrected air-flow estimate $\hat{W}_{ci}^{[c]}$ as well.

Two-sensor setup (p_i, p_x) with biases w_i and w_{tc} (Alt. 2). In this alternative, improvement of the estimated compressor air-flow is achieved. The estimated biases with the two-sensor setup of the intake and exhaust

Figure 3: Alt. 1: Estimated biases with two-sensor measurement (p_i, p_x)

Figure 4: Alt. 1: Evaluation measurement and estimated compressor flow with two-sensor measurement (p_i, p_x)

Figure 5: Alt. 2: Estimated biases with two-sensor measurement (p_i, p_x)

pressures are shown in Figure 5. The estimated compressor mass flow without the correction $\hat{W}_{ci}$ and the estimated compressor mass flow with the correction $\hat{W}_{ci}^{[c]}$ are compared in Figure 6. The improved estimates are due to the bias w_{tc} that compensates better for the error of the turbocharger speed differential equation.

Three-sensor setup (p_i, p_x, n_{tc}) with biases w_i, w_x and w_{tc} (Alt. 3). The best performance is obtained for the three-sensor setup alternative. This is expected, since the additional turbocharger speed sensor and additional bias, compared to Alt. 1 and Alt 2., provides additional information which is processed by the observer and the correction algorithm. The estimated biases are shown in Figure 7. The estimated compressor mass flow without the correction $\hat{W}_{ci}$ and the estimated compressor mass flow with the correction $\hat{W}_{ci}^{[c]}$ are compared in Figure 8.

6. Conclusion

A correction method for mass flow quantities of a turbocharged combustion engine is developed. Different combination of the production sensors

Figure 6: Alt. 2: Evaluation measurement and estimated compressor flow with two-sensor measurement (p_i, p_x)

Figure 7: Alt. 3: Estimated biases with three-sensor measurement (p_i, p_x, n_{tc})

Figure 8: Alt. 3: Evaluation measurement and estimated compressor flow with three-sensor measurement (p_i, p_x, n_{tc})

(MAP, EXMP and NTC) can form the measurement vector, which is directly processed by the EKF to compute the augmented state vector with model states and model biases. The mass flows, computed based on the EKF observed states, are further corrected based on the EKF observed biases. In the off-line numerical experiment it is demonstrated, with the production car turbocharged Diesel engine data, that the mass flow correction method significantly improves the estimated air mass flow W_{ci} . The other mass flows: the EGR mass flow W_{xi} , the mass flow from the intake manifold to the engine cylinders W_{ie} and the mass flow from the exhaust manifold to the turbine W_{xt} are estimated as well, however they are not validated in this paper and therefore not discussed. It has been shown that the two-sensor setups with the measured intake manifold pressure and exhaust manifold pressure provide engine's mass flows estimate which precision can be further improved by the additional turbocharger speed measurement in the three-sensor setup. The proposed sensor fusion does not rely on the imprecise (or expensive) production car airflow measurement (MAF). In order to manage effects such as engine ageing, an implementation of the method in a production engine should be accompanied with an online adaptation of slowly varying model parameters. In this way parametric errors do not degrade the mass flow

estimates, since they will not strongly influence the bias estimates.

Acknowledgment. T. P. acknowledges the support for this project from Prof. Luigi del Re, Institute for Design and Control of Mechatronical Systems, Johannes Kepler University, Linz. This work is supported by the Slovak Research and Development Agency under projects APVV-0090-10 and LPP-0118-09. This work is also supported by the Norwegian Research Council and the Norwegian University of Science and Technology.

Appendix A. Extended Kalman Filter

The EKF is applied for estimation of the augmented state of the turbocharged Diesel engine. The following algorithm is in the literature known as the continuous-discrete or hybrid EKF (Brown and Hwang, 2012; Gelb et al., 2001). The dynamic system is given by Eq. (30) and Eq. (31). The process and the measurement noise is

$$z_t \sim N(0, Q_t) \quad v_t \sim N(0, R_t) \quad (\text{A.1})$$

where Q_t is the process noise covariance matrix and R_t is the measurement noise covariance matrix. The initial condition of the state vector is $x_0 \sim N(\hat{x}_0^+, P_0^+)$. The estimate of the state vector at $t = 0$ begins with the initial state vector estimate and with the initial covariance matrix of the initial state vector estimate error

$$\hat{x}_0^+ = E[x_0] \quad (\text{A.2})$$

$$P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \quad (\text{A.3})$$

From time instance $t - 1$, the dynamic system is simulatively propagated one step ahead as

$$\hat{x}_t^- = f(\hat{x}_{t-1}^+, u_{t-1}) \quad (\text{A.4})$$

where $t = 1, 2, \dots$. This one step computation is an a priori state estimate. The time update of the covariance matrix estimate is given by

$$\dot{P} = Z(\hat{x})P + PZ^T(\hat{x}) + Q \quad (\text{A.5})$$

where

$$Z(\hat{x}) = \left. \frac{\partial f_c(x)}{\partial x} \right|_{x=\hat{x}} \quad (\text{A.6})$$

and Q is a spectral density matrix, where $Q = \frac{1}{T}Q_t$ and T is the sampling interval. The covariance matrix estimate of the state vector $\hat{x}_t^-$ estimation error is achieved by simulative propagation of Eq. (A.5)

$$P_t^- = g(P_{t-1}^+, Z(\hat{x}_{t-1}^+)) \quad (\text{A.7})$$

The EKF gain matrix of time instant t is given by

$$K_t = P_t^- L_t^T [L_t P_t^- L_t^T + M_t R_t M_t^T]^{-1} \quad (\text{A.8})$$

and the obtained measurement y_t is used for state vector estimation (a posteriori estimate)

$$\hat{x}_t^+ = \hat{x}_t^- + K_t[y_t - h(\hat{x}_t^-)] \quad (\text{A.9})$$

The covariance matrix of the estimation error of $\hat{x}_t^+$ is computed as

$$P_t^+ = [I - K_t L_t] P_t^- [I - K_t L_t]^T + K_t M_t R_t M_t^T K_t^T \quad (\text{A.10})$$

where

$$L_t(\hat{x}_t^-) = \left. \frac{\partial h(x_t)}{\partial x_t} \right|_{x_t=\hat{x}_t^-} \quad M_t(\hat{x}_t^-) = \left. \frac{\partial h(x_t)}{\partial v_t} \right|_{x_t=\hat{x}_t^-} \quad (\text{A.11})$$

The numerical integration of ordinary differential equations (ODE) is used for the simulative propagation of Eq. (A.4), covariance matrix time update Eq. (A.7) and Jacobian matrix evaluation Eq. (A.6). Since variable-step solvers might be difficult to use for hard real-time applications requiring a deterministic processing time, this study is based on a fixed-step solver. A fixed step solver also tends to give more accurate calculation of finite-difference gradients since numerical errors are more systematic. Any of the fixed-step solvers can simulate the model to any desired level of accuracy, given a small enough step size. Unfortunately, it is generally difficult to decide a priori which combination of solver and step size will yield acceptable results for the continuous states in the shortest processing time. Determining the best solver for a particular model generally requires experimentation. In this study, the Matlab routine `ode2` is used to propagate Eq. (A.4) and Eq. (A.7), which is the fixed step explicit Heun's method (Runge-Kutta method) (Ascher and Petzold, 1998)

$$\tilde{x}_{i+1} = x_i + h f_c(x_i, u_i) \quad (\text{A.12})$$

$$x_{i+1} = x_i + \frac{h}{2}(f_c(x_i, u_i) + f_c(\tilde{x}_i, u_i)) \quad (\text{A.13})$$

where i represents the numerical step index and h is the numerical step size. It can be seen as an extension of the Euler method into a two-stage second-order Runge-Kutta method. Heun's method is a predictor-corrector method with the forward Euler's method, Eq. (A.12), as predictor and the trapezoidal method, (A.13), as corrector (Butcher, 2003). This method was chosen after some experimentation with a set of solvers. The precision of the numerical solution of a given solver as a function of the numerical step size h is evaluated during the run of the EKF. In our case, the solver which gives the best EKF performance (least estimation errors) with other EKF settings unchanged, is chosen. The estimation error is however not the only criterion for how to choose the ODE solver. With regards to a real-time application the computational complexity and processing time need to be considered. The chosen Heun's method is a reasonable compromise between the computational time and solution precision with the given step size h . The Jacobian is approximated via the function's forward finite-difference to evaluate Eq. (A.6),

$$\left. \frac{\partial f_c(x)}{\partial x} \right|_{x=\hat{x}} \approx \left[\frac{f_c(x + d_1\epsilon) - f_c(x)}{\epsilon}, \dots, \frac{f_c(x + d_{n_x}\epsilon) - f_c(x)}{\epsilon} \right]_{x=\hat{x}} \quad (\text{A.14})$$

where d_i is a unit vector in a direction of x_i , $i = 1 \dots n_x$ and ϵ represents a small perturbation.

Implementation of the EKF on a production ECU (Electronic Control Unit) poses some additional challenges, and some tradeoff between computational complexity and numerical accuracy. Computations of the Jacobian should either exploit the polynomial structure of the maps, or structural zeros in the Jacobian if finite differences are used. Robust and efficient numerical implementation of the EKF like Bierman's algorithm (Bierman, 1977) or the square-root/array algorithm (Kailath et al., 2000) must be considered.

Appendix B. Experimental and simulation setup

Appendix B.1. Tested engine data and open loop performance of observer

The driving cycle US FTP75 is performed with a BMW 4-cylinder EU4 common rail direct injection (CRDI), production passenger car, turbocharged Diesel engine to record the data of the intake manifold pressure p_i , exhaust manifold pressure p_x , turbocharger speed n_{tc} and the intake mass flow MAF sensor data. While the sensors have fast dynamics and therefore not modeled

dynamically in our case study, such dynamic models in terms of simple low-pass filters could easily be added to the dynamic estimation model when the sensor dynamic is significant (Guardiola et al., 2013). The sampling interval¹ for all the measurements is $T=0.1$ [s]. The engine is operated under standard laboratory conditions.

In the open loop mode the observer is not using any measured information to compute the estimated states except the model inputs (where n_e is a measurement). In the open loop mode, the states are purely numerically simulated through the engine model from the given initial conditions of states and the given inputs. The inputs for the engine and the model are the percentages of open positions of vanes X_{vgt} and EGR valve X_{egr} , fuel injection W_f and the load which is reflected by the engine speed. The open loop observer gives biased estimates as previously documented in Polóni et al. (2012), where the biases are more significant on intake manifold pressure and turbocharger speed state and less evident on exhaust manifold pressure state.

Appendix B.2. Numerical setup of computation

Good tuning of the EKF depends on precise information about the stochastic properties of noises, which are theoretically assumed to be sequentially uncorrelated Gaussian distributions with zero means. The numerical values can be estimated by the user's careful assumption about the magnitude of measurement and process noises of a given state or a bias of a discrete (sampled) time sequence. After some manual tuning, the numerical values listed in Table B.2 are found to give the acceptable estimation and correction method performance for the engine used for the experiments.

The measurement noise covariance matrix is defined as

$$R_t = \text{diag} [\sigma_{y_1}^2, \sigma_{y_2}^2, \sigma_{y_3}^2] \quad (\text{B.1})$$

The process noise spectral density matrix is

$$Q = \text{diag} [\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7, \Phi_8] \quad (\text{B.2})$$

where the diagonal noise spectral densities are defined and computed as $\Phi_i = (1/T)\sigma_i^2$. The Φ_1 spectral density depends on the intake manifold

¹Note that the sampling interval T differs from the integration step interval h

Sampling interval T	0.1 [s]
Integration step h	0.01 [s]
Finite difference perturbation ϵ	10^{-8}
Singular value threshold δ_{SVD}	$10^{-2} \max(D)$
Initial intake pressure $x_{1,0}$	$9.8 * 10^4$ [kPa]
Initial intake mass $x_{2,0}$	0.01269 [kg]
Initial exhaust pressure $x_{3,0}$	$9.8 * 10^4$ [kPa]
Initial exhaust mass $x_{4,0}$	0.01269 [kg]
Initial turbocharger speed $x_{5,0}$	$3.0 * 10^4$ [rpm]
Initial intake pressure derivative bias $w_{i,0}$	0 [kPa.s ⁻¹]
Initial exhaust pressure derivative bias $w_{x,0}$	0 [kPa.s ⁻¹]
Initial turbocharger speed derivative bias $w_{tc,0}$	0 [rpm.s ⁻¹]
Initial intake pressure p_i variance $\Sigma_{1,0}$	$(0.3 * 10^4)^2$ [kPa ²]
Initial intake mass m_i variance $\Sigma_{2,0}$	$(0.1 * 10^{-2})^2$ [kg ²]
Initial exhaust pressure p_x variance $\Sigma_{3,0}$	$(2 * 10^3)^2$ [kPa ²]
Initial exhaust mass m_x variance $\Sigma_{4,0}$	$(1 * 10^{-4})^2$ [kg ²]
Initial turbocharger speed n_{tc} variance $\Sigma_{5,0}$	$(11 * 10^3)^2$ [rpm ²]
Initial intake pressure derivative bias w_i variance $\Sigma_{6,0}$	$(10^5)^2$ [kPa ² s ⁻²]
Initial exhaust pressure derivative bias w_x variance $\Sigma_{7,0}$	$(10^5)^2$ [kPa ² s ⁻²]
Initial turbocharger speed derivative bias w_{tc} variance $\Sigma_{8,0}$	$(10^5)^2$ [rpm ² s ⁻²]
Measurement noise:	
Standard deviation of the intake pressure σ_{y_1}	$0.02 * 10^5$ [kPa]
Standard deviation of the exhaust pressure σ_{y_2}	$0.08 * 10^5$ [kPa]
Standard deviation of the turbocharger speed σ_{y_3}	$0.1 * 10^4$ [rpm]
Process noise:	
Standard deviation of the intake pressure σ_1	$0.06 * 10^5$ [kPa]
Standard deviation of the intake mass σ_2	$0.5 * 10^{-4}$ [kg]
Standard deviation of the exhaust pressure σ_3	$0.08 * 10^5$ [kPa]
Standard deviation of the intake mass σ_4	$0.5 * 10^{-4}$ [kg]
Standard deviation of the turbocharger speed σ_5	$1.5 * 10^4$ [rpm]
Standard deviation of the intake pressure derivative bias σ_6	10^4 [kPa.s ⁻¹]
Standard deviation of the exhaust pressure derivative bias σ_7	10^4 [kPa.s ⁻¹]
Standard deviation of the turbocharger speed derivative bias σ_8	10^4 [rpm.s ⁻¹]

Table B.2: Settings of numerical computation

pressure process noise variance σ_1^2 and sampling time, and is computed as $\Phi_1 = 10[s^{-1}](0.06*10^5[kPa])^2[kPa^2.s^{-2}.Hz^{-1}]$, where the other spectral densities are similarly computed. The initial state vector estimate is set to $\hat{x}_0^+ = [x_{1,0}, x_{2,0}, x_{3,0}, x_{4,0}, x_{5,0}, w_{i,0}, w_{x,0}, w_{tc,0}]^T$. The initial covariance matrix of the initial state vector estimate error is

$$P_0^+ = \text{diag} [\Sigma_{1,0}, \Sigma_{2,0}, \Sigma_{3,0}, \Sigma_{4,0}, \Sigma_{5,0}, \Sigma_{6,0}, \Sigma_{7,0}, \Sigma_{8,0}], \quad (\text{B.3})$$

where the diagonal elements are in accordance to Eq. (A.3) computed as $\Sigma_i = (x_{i,0} - E[x_{i,0}])^2$ and set by the initial conditions.

References

- Ascher, U. M., Petzold, L. R., 1998. Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. SIAM.
- Bierman, G. J., 1977. Factorization methods for discrete sequential estimation. Academic Press.
- Brown, R. G., Hwang, P. Y. C., 2012. Introduction to Random Signals and Applied Kalman Filtering, 4th Edition. Wiley.
- Butcher, J., 2003. Numerical Methods for Ordinary Differential Equations. John Wiley&Sons, Ltd.
- Desantes, J., Galindo, J., Guardiola, C., Dolz, V., 2010. Air mass flow estimation in turbocharged diesel engines from in-cylinder pressure measurement. Experimental Thermal and Fluid Science 34 (1), 37 – 47.
- Diop, S., Moraal, P., Kolmanovsky, I., Van Nieuwstadt, M., 1999. Intake oxygen concentration estimation for DI diesel engines. In: Control Applications, 1999. Proceedings of the 1999 IEEE International Conference on. Vol. 1. pp. 852 –857 vol. 1.
- Eriksson, L., Wahlström, J., Klein, M., 2010. Physical modeling of turbocharged engines and parameter identification. In: del Re, L., Allgöwer, F., Glielmo, L., Guardiola, C., Kolmanovsky, I. (Eds.), Automotive Model Predictive Control. Vol. 402 of Lecture Notes in Control and Information Sciences. Springer Berlin / Heidelberg, pp. 53–71.

- Gelb, A., Joseph F. Kasper, J., Raymond A. Nash, J., Price, C. F., Arthur A. Sutherland, J., 2001. Applied optimal estimation. The. M.I.T. Press.
- Golub, G. H., Van Loan, C. F., 1996. Matrix computations, 3rd Edition. The Johns Hopkins University Press.
- Guardiola, C., Pla, B., Blanco-Rodriguez, D., Mazer, A., Hayat, O., 2013. A bias correction method for fast fuel-to-air ratio estimation in diesel engines. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering.
URL <http://pid.sagepub.com/content/early/2013/03/28/0954407012473415.abstract>
- Heywood, J. B., 1988. Internal Combustion Engine Fundamentals. McGraw-Hill.
- Höckerdal, E., Frisk, E., Eriksson, L., 2009. Observer design and model augmentation for bias compensation with a truck engine application. Control Engineering Practice 17 (3), 408 – 417.
- Höckerdal, E., Frisk, E., Eriksson, L., 2011. EKF-based adaptation of look-up tables with an air mass-flow sensor application. Control Engineering Practice 19 (5), 442 – 453.
- Jung, M., 2003. Mean-value modelling and robust control of the airpath of a turbocharged diesel engine. Ph.D. thesis, Sidney Sussex College, University of Cambridge.
- Kailath, T., Sayed, A. H., Hassibi, B., 2000. Linear Estimation. Prentice Hall.
- Kang, J.-M., Haskara, I., Chang, C.-F., Wang, Y.-Y., 2011. Adaptive intake oxygen estimation in a diesel engine. Patent Application, US 2011/0184632 A1.
- Kolmanovsky, I., Moraal, P., van Nieuwstadt, M., Stefanopoulou, A., 1998. Issues in modelling and control of intake flow in variable geometry turbocharged engines. In: 18th IFIP Conference of Systems Modelling and Optimization.

- Kolmanovsky, I., Sivergina, I., Sun, J., 2006. Simultaneous input and parameter estimation with input observers and set-membership parameter bounding: theory and an automotive application. *International Journal of Adaptive Control and Signal Processing* 20 (5), 225–246.
- Kolmanovsky, I. V., Jankovic, M. J., Van Nieuwstadt, M. J., Moraal, P. E., 2000. Method of estimating mass airflow in turbocharged engines having exhaust gas recirculation. Patent, US 6035639.
- Polóni, T., Rohaľ-Ilkiv, B., Alberer, D., del Re, L., Johansen, T. A., 2012. Comparison of sensor configurations for mass flow estimation of turbocharged diesel engines. In: Alberer, D., Hjalmarsson, H., del Re, L. (Eds.), *Identification for Automotive Systems*. Vol. 418 of *Lecture Notes in Control and Information Sciences*. Springer, pp. 303–326.
- Stefanopoulou, A. G., Storset, O. F., Smith, R., 2004. Pressure and temperature-based adaptive observer of air charge for turbocharged diesel engines. *International Journal of Robust and Nonlinear Control* 14 (6), 543–560.
- Storset, O. F., Stefanopoulou, A. G., Smith, R., 2004. Adaptive air charge estimation for turbocharged diesel engines without exhaust gas recirculation. *Journal of Dynamic Systems, Measurement, and Control* 126 (3), 633–643.
- Stotsky, A., Kolmanovsky, I., 2002. Application of input estimation techniques to charge estimation and control in automotive engines. *Control Engineering Practice* 10 (12), 1371 – 1383.
- van Nieuwstadt, M., Kolmanovsky, I., Moraal, P., Stefanopoulou, A., Jankovic, M., jun 2000. EGR-VGT control schemes: experimental comparison for a high-speed diesel engine. *Control Systems Magazine, IEEE* 20 (3), 63 –79.
- Wahlström, J., Eriksson, L., 2011. Modelling diesel engines with a variable-geometry turbocharger and exhaust gas recirculation by optimization of model parameters for capturing non-linear system dynamics. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering* 225 (7), 960–986.
URL <http://pid.sagepub.com/content/225/7/960.abstract>

Wang, J., 2008. Air fraction estimation for multiple combustion mode diesel engines with dual-loop EGR systems. *Control Engineering Practice* 16 (12), 1479 – 1486.