

NONLINEAR MOVING HORIZON OBSERVER FOR ESTIMATION OF STATES AND PARAMETERS IN UNDER-BALANCED DRILLING OPERATIONS

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ABSTRACT

It is not possible to directly measure the total mass of gas and liquid in the annulus and geological properties of the reservoir during petroleum exploration and production drilling. Therefore, these parameters and states must be estimated by on-line estimators with proper measurements. This paper describes a nonlinear Moving Horizon Observer to estimate the annular mass of gas and liquid, and production constants of gas and liquid from the reservoir into the well during Under-Balanced Drilling with measuring the choke pressure and the bottom-hole pressure. This observer algorithm based on a low-order lumped model is evaluated against Joint Unscented Kalman filter for two different simulations with low and high measurement noise covariance. The results show that both algorithms are capable of identifying the production constants of gas and liquid from the reservoir into the well, while the nonlinear Moving Horizon Observer achieves better performance than the Unscented Kalman filter.

NOMENCLATURE

UBD Under-balanced drilling.

UKF Unscented Kalman filter

LOL Low-order lumped model.

INTRODUCTION

In recent years there has been increasing interest in Under-Balanced Drilling (UBD). UBD has the potential to both decrease drilling problems and improve hydrocarbon recovery. In conventional (over-balanced) drilling, or Managed Pressure Drilling (MPD), the pressure of the well must be kept greater than pressure of reservoir to prevent influx from entering the well. But in an UBD operation, the hydrostatic pressure in the circulating downhole fluid system must be kept greater than pressure of collapse and less than pressure of reservoir

$$p_{coll}(t,x) < p_{well}(t,x) < p_{res}(t,x) \quad (1)$$

at all times t and positions x . Since hydrostatic pressure in the circulating downhole fluid system is intentionally lower than the reservoir formation pressure, influx fluids (oil, free gas, water) from the reservoir are mixed with rock cuttings and drilling fluid (“mud”) in the annulus. Therefore, modeling of the UBD operation should be considered as multiphase flow. Different aspects of modeling relevant for UBD have been studied in the literature [1–4].

Estimation and control design in MPD has been investigated by several researchers [5–12]. However, due to the complexity of multi-phase flow dynamics in UBD operations, there are few studies on estimation and control of UBD operations [1, 13–16]. Nygaard et al. [13] compared and evaluated the performance of

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the extended Kalman filter, the ensemble Kalman filter and the unscented Kalman filter to estimate the states and production index in UBD operation. Lorentzen et al. [14] designed an ensemble Kalman filter to tune the uncertain parameters of a two-phase flow model in the UBD operation. In Nygaard et al [15], a finite horizon nonlinear model predictive control in combination with an unscented Kalman filter was designed for controlling the bottom-hole pressure based on a low order model developed in [1], and the unscented Kalman filter was used to estimate the states, and the friction and choke coefficients. Nikoo-fard et al. [16] designed Lyapunov-based adaptive observer, recursive least squares and joint unscented Kalman filter based on a low-order lumped model to estimate states and parameters during UBD operations by using the calculated total mass of gas and liquid. The performance of adaptive estimators are compared and evaluated for two cases. This paper estimates states and parameters only by using real-time measurements of the choke and the bottom-hole pressures.

Due to mathematical complexity presented by nonlinearity, state and parameter estimation of nonlinear dynamical systems is one of the challenging topics in the control theory and has been the subject of many studies since its advent. Nonlinear Moving Horizon Observer is one of the powerful method which estimates states and parameters simultaneously [17–19]. At each sampling time, a Moving Horizon Observer estimate the state and parameter by minimizing a cost function over the previous finite horizon, subject to the nonlinear model equations. Robustness to lack of persistent excitation and measurement noise is due to the use of an apriori prediction model as well as some modifications to the basic MHE algorithm [20, 21]. The original Kalman filter based on linear model was developed to estimate both state and parameter of the system usually known as an augmented Kalman filter. Several Kalman filter techniques have been developed to work with non-linear system. The unscented Kalman filter has been shown to have a better performance than other Kalman filter techniques for nonlinear system in same cases [22, 23].

Total mass of gas and liquid in the well could not be measured directly, as well as some geological properties of the reservoir such as production constants of gas and liquid from the reservoir might vary and could be uncertain during drilling operations. Therefore, they must be estimated by adaptive estimators during UBD operations. It is assumed that the bottom-hole pressure (BHP) readings are transmitted continuously to the surface through a wired pipe telemetry with a pressure sensor at the measurement while drilling (MWD) tool [24]. This paper presents the design of a nonlinear Moving Horizon Observer based on a nonlinear two-phase fluid flow model to estimate the total mass of gas and liquid in the annulus and geological properties of the reservoir during UBD operation. The performance of this methods is evaluated against Joint Unscented Kalman Filter for the case of pipe connection operations where the main pump is shut off and the rotation of the drill string and the circulation of fluids

is stopped. The estimators are compared to each other in terms of speed of convergence, sensitivity of noise measurement, and accuracy.

This paper consists of the following sections: The Modeling section presents a low-order lumped model based on mass and momentum balances for UBD operation. The Observer section explains nonlinear Moving Horizon Observer for simultaneously estimating the states and model parameters of a nonlinear system from noisy measurements. In the Simulations section, the simulation results are provided for state and parameter estimation. At the end the conclusion that has been made through this paper are presented.

MODELING

Modeling of Under-balanced Drilling (UBD) in an oil well is a challenging mathematical and industrial research area. Due to existence of multiphase flow (i.e. oil, gas, drilling mud and cuttings) in the system, the modeling of the system is very complex. Multiphase flow can be modeled as a distributed (infinite dimension) model or a lumped (finite dimension) model. A distributed model is capable of describing the gas-liquid behavior along the annulus in the well. In this paper, a low-order lumped (LOL) model is used. The lumped model considers only the gas-liquid behavior at the drill bit and the choke system. This modeling method is very similar to the two-phase flow model found in [1, 25]. The simplifying assumptions of the LOL model are listed as below:

- Ideal gas behavior
- Simplified choke model for gas, mud and liquid leaving the annulus
- No mass transfer between gas and liquid
- Isothermal condition and constant system temperature
- Constant mixture density with respect to pressure and temperature.
- Liquid phase considers the total mass of mud, oil, water, and rock cuttings.

The simplified LOL model equations for mass of gas and liquid in an annulus are derived from mass and momentum balances as follows

$$\dot{m}_g = w_{g,d} + w_{g,res}(m_g, m_l) - \frac{m_g}{m_g + m_l} w_{out}(m_g, m_l) \quad (2)$$

$$\dot{m}_l = w_{l,d} + w_{l,res}(m_g, m_l) - \frac{m_l}{m_g + m_l} w_{out}(m_g, m_l) \quad (3)$$

where m_g and m_l are the total mass of gas and liquid, respectively. The liquid phase is considered incompressible, and ρ_l is the liquid mass density. The gas phase is compressible and

occupies the space left free by the liquid phase. $w_{g,d}$ and $w_{l,d}$ are the mass flow rate of gas and liquid from the drill string, $w_{g,res}$ and $w_{l,res}$ are the mass flow rates of gas and liquid from the reservoir. The total mass outflow rate is denoted by w_{out} .

The total mass outlet flow rate is calculated by the valve equation

$$w_{out} = K_c Z \sqrt{\frac{m_g + m_l}{V_a}} \sqrt{p_c - p_{c0}} \quad (4)$$

where K_c is the choke constant. Z is the control signal to the choke opening, taking its values on the interval $(0, 1]$. The total volume of the annulus is denoted by V_a . p_{c0} is the constant downstream choke pressure (atmospheric). The choke pressure is denoted by p_c , and derived from ideal gas equation

$$p_c = \frac{RT}{M_{gas}} \frac{m_g}{V_a - \frac{m_l}{\rho_l}} \quad (5)$$

where R is the gas constant, T is the average temperature of the gas, and M_{gas} is the molecular weight of the gas. The flow from the reservoir into the well for each phase is commonly modeled by the linear relation with the pressure difference between the reservoir and the well. The mass flow rates of gas and liquid from the reservoir into the well are given by

$$w_{g,res} = K_g (p_{res} - p_{bh}) \quad (6)$$

$$w_{l,res} = K_l (p_{res} - p_{bh}) \quad (7)$$

where p_{res} is the known pore pressure in the reservoir, and K_g and K_l are the production constants of gas and liquid from the reservoir into the well, respectively. Finally, the bottom-hole pressure is given by the following equation

$$p_{bh} = p_c + \frac{(m_g + m_l)g \cos(\Delta\theta)}{A} + \Delta p_f \quad (8)$$

where Δp_f is the friction pressure loss in the well, g is the gravitational constant and $\Delta\theta$ is the average angle between gravity and the positive flow direction of the well. Reservoir parameters could be evaluated by seismic data and other geological data from core sample analysis. But, local variations of reservoir parameters such as the production constants of gas and liquid may be revealed only during drilling. So, it is valuable to estimate the partial variations of some of the reservoir parameters while drilling is performed [13].

TABLE 1. MEASUREMENTS AND INPUTS OF OBSERVER

Variables	Type
Choke pressure (p_c)	Measurement
Bottom-hole pressure (p_{bh})	Measurement
Drill string mass flow rate of gas ($w_{g,d}$)	Input
Drill string mass flow rate of liquid ($w_{l,d}$)	Input
Choke opening (Z)	Input

OBSERVER

In this section, first the Nonlinear Moving Horizon Observer to estimate states and parameters in UBD operation is explained. Then, the joint unscented Kalman filter is presented for same problem. The choke and the bottom-hole pressures are outputs and measurements of the system. The measurements and inputs of the observer are summarized in Table 1. The production constant of gas (K_g) and liquid (K_l) from the reservoir into the well are unknown and must be estimated. Below K_g and K_l are denoted by θ_1 and θ_2 , respectively.

Nonlinear Moving Horizon Observer

The LOL model based on equations (2)-(3) can be represented by a discrete explicit scheme given by

$$x_k = f(x_{k-1}, u_{k-1}) + q_k \quad (9)$$

$$y_k = h(x_k) + r_k \quad (10)$$

$$h(x_k) = [p_c, p_{bh}]^T \quad (11)$$

where q_k and r_k are the process noise and the measurement noise, and u_k is the input. At time t , the information vector of the $N + 1$ last measurements and the N last inputs is defined as

$$I_t = \text{col}(y_{t-N}, \dots, y_t, u_{t-N}, \dots, u_{t-1}) \quad (12)$$

$N + 1$ is the finite horizon or window length. This information can be summarized in a single vector for measurement and input as follow

$$Y_t = \begin{bmatrix} y_{t-N} \\ y_{t-N+1} \\ \vdots \\ y_t \end{bmatrix} \quad U_t = \begin{bmatrix} u_{t-N} \\ u_{t-N+1} \\ \vdots \\ u_{t-1} \end{bmatrix} \quad (13)$$

Nonlinear Moving Horizon Observer cost function can be considered as follows

$$J(\hat{x}_{t-N,t}, \bar{x}_{t-N,t}, I_t) = \|W(Y_t - H_t(\hat{x}_{t-N,t}))\|^2 + \|V(\hat{x}_{t-N,t} - \bar{x}_{t-N,t})\|^2 \quad (14)$$

where V, W are the positive definite weight matrices. The cost function (14) consists of two standard terms: one is a term that penalizes the deviation between the measured and predicted outputs, the other term weights the difference between the estimated state at the start of the horizon from its prediction. The Nonlinear Moving Horizon Observer minimizes the cost function (14) over the window of the current measurement and the historical measurement, subject to the nonlinear model equations. $x_{t-N,t}^o$ is the optimal solution of this problem. The sequence of the state estimates $\hat{x}_{i,t}$ ($i = t - N, \dots, t$) can be yielded from the noise free nonlinear model dynamics. So, a sequence of denoted as estimated output vectors \hat{Y}_t can be formulated as follows

$$\hat{Y}_t = H(\hat{x}_{t-N,t}, U_t) = H_t(\hat{x}_{t-N,t}) = \begin{bmatrix} h(\hat{x}_{t-N,t}) \\ h \circ f^{u_{t-N}}(\hat{x}_{t-N,t}) \\ \vdots \\ h \circ f^{u_{t-1}} \circ \dots \circ f^{u_{t-N}}(\hat{x}_{t-N,t}) \end{bmatrix} \quad (15)$$

A one-step prediction $\bar{x}_{t-N,t}$ is determined from $x_{t-N-1,t-1}^o$ as follow

$$\bar{x}_{t-N,t} = f(x_{t-N-1,t-1}^o, u_{t-N-1}) \quad t = N + 1, N + 2, \dots \quad (16)$$

For parameter estimation, x is augmented with the unknown parameter vector θ with the dynamic model $\theta_{t+1} = \theta_t$.

Joint Unscented Kalman Filter

The Unscented Kalman Filter (UKF) was introduced in [26–28]. The main idea behind the method is that approximation of a Gaussian distribution is easier than approximation on of an arbitrary nonlinear function. The UKF estimates the mean and covariance matrix of estimation error with a minimal set of sample points (called sigma points) around the mean by using a deterministic sampling approach known as the unscented transform. The nonlinear model is applied to sigma points instead of a linearization of the model. So, this method does not need to calculate explicit Jacobian or Hessian. More details can be found in [22, 23, 27, 28].

Two common approaches for estimation of parameters and state variables simultaneously are dual and joint UKF techniques. The dual UKF method uses another UKF for parameter estimation so that two filters run sequentially in every time step. At each

time step, the state estimator updates with new measurements, and then the current estimate of the state is used in the parameter estimator. The joint UKF augments the original state variables with parameters and a single UKF is used to estimate augmented state vector. The joint UKF is easier to implement [23, 28].

Using the joint UKF, the augmented state vector is defined by $x^a = [X, \theta]$. The state-space equations for the the augmented state vector at time instant k is written as:

$$\begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \theta_{1,k} \\ \theta_{2,k} \end{bmatrix} = \begin{bmatrix} f_1(X_{k-1}, \theta_{1,k-1}) \\ f_2(X_{k-1}, \theta_{2,k-1}) \\ \theta_{1,k-1} \\ \theta_{2,k-1} \end{bmatrix} = f^a(X_{k-1}, \theta_{k-1}, u_{k-1}) + q_k \quad (17)$$

m_g and m_l are denoted by x_1 and x_2 , respectively.

SIMULATION RESULTS

The parameter values for the simulated well, LOL model and reservoir are summarized in Table 2. These parameters are used from the offshore test of WeMod simulator [29]. WeMod is a high fidelity drilling simulator developed by the International Research Institute of Stavanger (IRIS). The process noise covariance matrix used in this plant model (LOL model) is

$$Q = \text{diag}[10^{-1}, 10^{-1}, 10^{-4}, 10^{-4}]$$

UKF parameters are determined empirically and used in the series of equations presented in the Appendix. The parameter values for nonlinear Moving Horizon Observer and joint UKF are summarized in Table 3.

The time-step used for discretizing the dynamic model and adaptive estimator was 1 seconds. The initial values for the estimated and real states and parameters are as follows

$$\begin{aligned} x_1 &= 5446.6, \quad x_2 = 54466.5, \quad \theta_1 = 5, \quad \theta_2 = 5 \\ \hat{x}_1 &= 4629.6, \quad \hat{x}_2 = 46296.2, \quad \hat{\theta}_1 = 4.25, \quad \hat{\theta}_2 = 4.25 \end{aligned}$$

The scenario in this simulation is as follows, first the drilling in a steady-state condition is initiated, then at $t = 10$ min the main pump is shut off to perform a connection procedure. The rotation of the drill string and the circulation of fluids are stopped for 10 mins. Next after making the first pipe connection at $t = 20$ min the main pump and rotation of the drill string are restarted. Then at $t = 52$ min the second pipe connection procedure is started, and is completed after 12 mins. Two different simulations with low and high measurement noise covariances are performed. In the

TABLE 2. PARAMETER VALUES FOR WELL AND RESERVOIR

Name	LOL	Unit
Reservoir pressure (p_{res})	270	bar
Collapse pressure (p_{coll})	255	bar
Friction pressure loss (Δp_f)	10	bar
Well total length (L_{tot})	2300	m
Well vertical depth (L)	1720	m
Drill string outer diameter (D_d)	0.1397	m
Annulus volume (V_a)	252.833	m ³
Annulus inner diameter (D_a)	0.2445	m
Liquid flow rate ($w_{l,d}$)	44	Kg/s
Gas flow rate ($w_{g,d}$)	5	Kg/s
Liquid density (ρ_l)	1475	Kg/m ³
Production constant of gas (K_g)	5×10^{-6}	Kg/Pa
Production constant of liquid (K_l)	5×10^{-5}	Kg/Pa
Gas average temperature (T)	25	°C
Average angle ($\Delta\theta$)	0.726	rad
Choke constant (K_c)	0.013	m ²

TABLE 3. PARAMETER VALUES FOR ESTIMATORS

Parameter	Value	Parameter	Value
W	100	V	diag (0.1,0.1, 0.03, 0.05)
$N + 1$	15	κ	0
β	2	α	0.5

first simulation, the choke and the bottom-hole pressures measurements are corrupted by zero mean additive white noise with the following covariance matrix

$$R = \text{diag}[0.5, 0.5]$$

Figures 1 and 2 show the measured and estimated total mass of gas and liquid, respectively. The nonlinear Moving Horizon Observer is more accurate than UKF to estimate the total mass of gas and liquid. The estimation of the production constants of gas and liquid from the reservoir into the well are shown in Figures 3 and 4, respectively. In estimation of the production constants of gas and liquid from the reservoir into the well, nonlinear Mov-

ing Horizon Observer has a very fast convergence rate, about 30 seconds or less. UKF take much longer and it is in the order of minutes. After convergence to the value, the correct nonlinear Moving Horizon Observer has more small fluctuations than UKF for estimation of the production constants of gas and liquid.

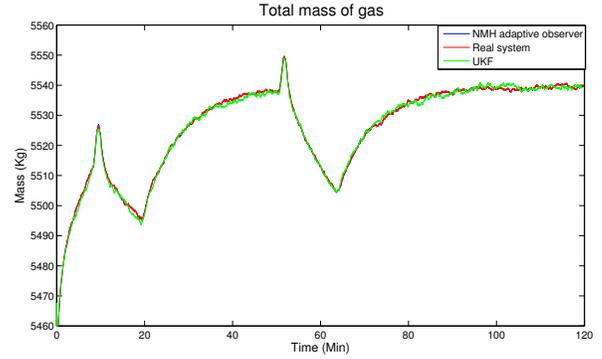


FIGURE 1. TOTAL MASS OF GAS WITH THE LOW MEASUREMENT NOISE COVARIANCE

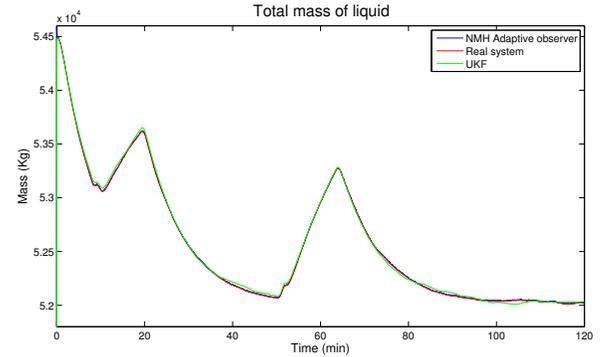


FIGURE 2. TOTAL MASS OF LIQUID WITH THE LOW MEASUREMENT NOISE COVARIANCE

In second simulation, the choke and the bottom-hole pressures are corrupted by zero mean additive white noise with the following covariance matrix

$$R = \text{diag}[50, 50]$$

Figures 5 and 6 show the measured and estimated total mass of gas and liquid, respectively. It is found that the Nonlinear Moving Horizon Observer is more accurate than UKF to estimate the total mass of gas and liquid. The estimation of the production

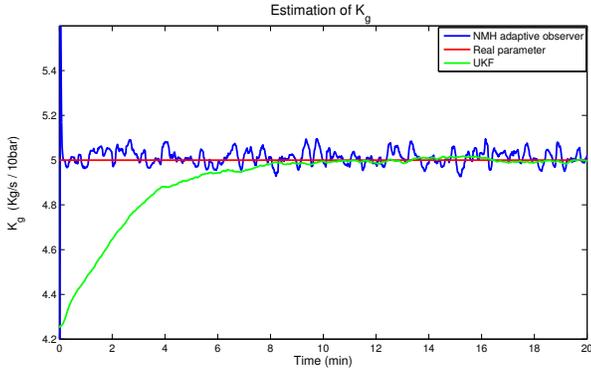


FIGURE 3. PRODUCTION CONSTANT OF GAS WITH THE LOW MEASUREMENT NOISE COVARIANCE

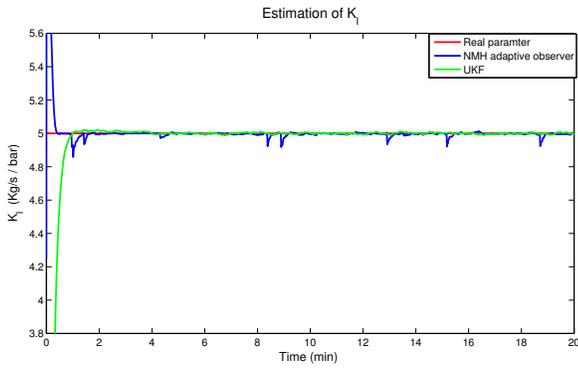


FIGURE 4. PRODUCTION CONSTANT OF LIQUID WITH THE LOW MEASUREMENT NOISE COVARIANCE

constants of gas and liquid from reservoir into the well are illustrated in Figures 7 and 8, respectively. In estimation of the production constants of gas and liquid from the reservoir into the well, the nonlinear Moving Horizon Observer has a very fast convergence rate, about 30 seconds or less, while the UKF takes almost 7 minutes.

In this paper, performance of these adaptive estimators is evaluated through the root mean square error (RMSE) metric. The RMSE metric for nonlinear Moving Horizon Observer and UKF in two cases are summarized in Table 4.

According to the RMSE metric table, nonlinear Moving Horizon Observer has the better performance than joint UKF for state and parameter estimation while it has low and high measurement noise covariances.

CONCLUSIONS

This paper describes nonlinear Moving Horizon Observer to estimate states and parameters during pipe connection procedure in UBD operations. The low-order lumped model presented here

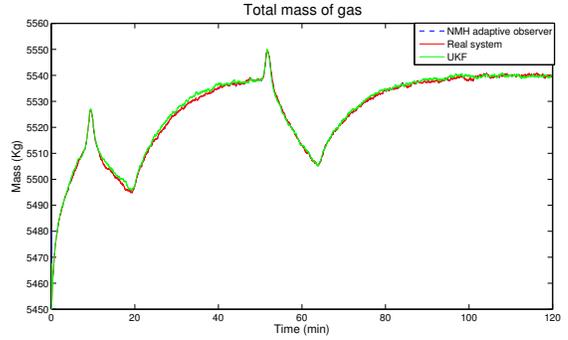


FIGURE 5. TOTAL MASS OF GAS WITH THE HIGH MEASUREMENT NOISE COVARIANCE

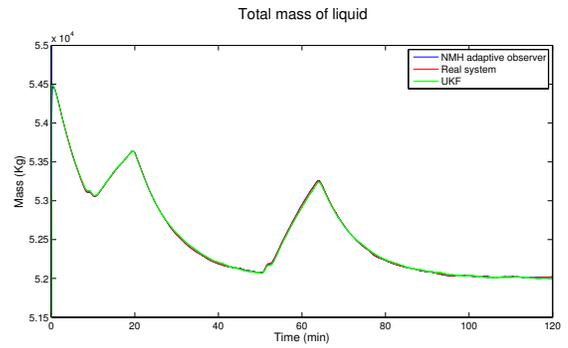


FIGURE 6. TOTAL MASS OF LIQUID WITH THE HIGH MEASUREMENT NOISE COVARIANCE

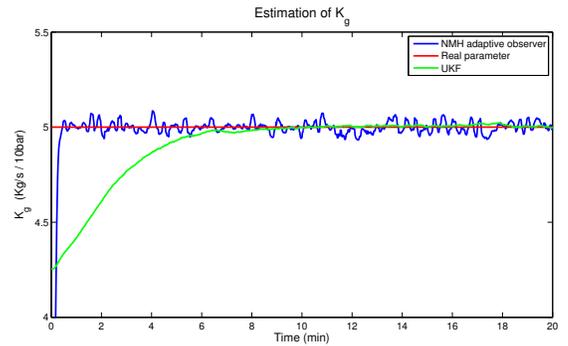


FIGURE 7. PRODUCTION CONSTANT OF GAS WITH THE HIGH MEASUREMENT NOISE COVARIANCE

only captures the major phenomena of the UBD operation. Simulation results demonstrate satisfactory performance of nonlinear Moving Horizon Observer and joint UKF for state and parameter estimation during pipe connection procedure with low and high measurement noise covariances. It is found that the nonlinear Moving Horizon Observer shows better convergence and per-

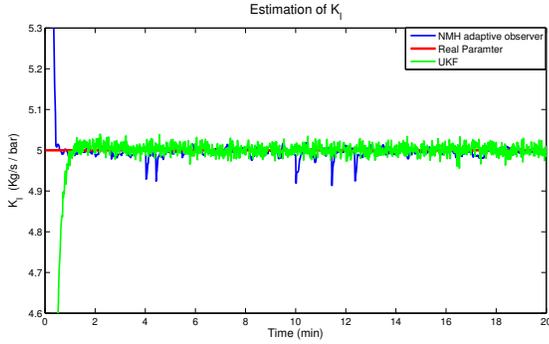


FIGURE 8. PRODUCTION CONSTANT OF LIQUID WITH THE HIGH MEASUREMENT NOISE COVARIANCE

TABLE 4. RMSE METRIC

Method	R	x_1	x_2	θ_1	θ_2
Moving Horizon Observer	0.5	0.315	10.3	0.04	0.100
UKF	0.5	0.328	36.4	0.078	0.152
Moving Horizon Observer	50	0.978	34.9	0.115	0.667
UKF	50	1.631	96.3	0.084	0.177

formance than joint UKF with low and high measurement noise covariances. According to the RMSE metric, nonlinear Moving Horizon Observer can perform better in terms of accuracy and robustness to measurement noise.

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Appendix A: UKF

A set of $2L+1$ sigma points is derived from the augmented state

$$\begin{aligned} (\chi_{k-1})_0 &= \hat{x}_{k-1} \\ (\chi_{k-1})_i &= \hat{x}_{k-1} + (\sqrt{(L+\lambda)P_{K-1}})_i, i = 1, \dots, L \\ (\chi_{k-1})_i &= \hat{x}_{k-1} - (\sqrt{(L+\lambda)P_{K-1}})_{i-L}, i = L+1, \dots, 2L \end{aligned} \quad (18)$$

where L is the dimension of the augmented states. $(\sqrt{(L+\lambda)P_{K-1}})_i$ is column i of the matrix square root of $(L+\lambda)P_{K-1}$. $(\chi_{k-1})_i$ is the i th column of the sigma point matrix χ_{k-1} . The design parameter λ is defined by

$$\lambda = \alpha^2(L + \kappa) - L \quad (19)$$

The spread of the sigma points around the state estimate is denoted by the constant α and usually set to $10^{-4} < \alpha < 1$, κ is a secondary scaling parameter usually set to zero [28]. The UKF has two distinct steps: prediction and correction. In UKF prediction step, compute the predicted state mean \hat{x}_k^- and the predicted error covariance P_k^- as

$$(\chi_k)_i = f^a((\chi_{k-1})_i) \quad , i = 0, \dots, 2L \quad (20)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} (\chi_k)_i \quad (21)$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} [(\chi_k)_i - \hat{x}_k^-][(\chi_k)_i - \hat{x}_k^-]^T + Q_k \quad (22)$$

where Q is the process covariance matrix. $W_i^{(m)}$ and $W_i^{(c)}$ are defined by

$$W_0^{(m)} = \frac{\lambda}{(L+\lambda)} \quad (23)$$

$$W_i^{(m)} = \frac{1}{2(L+\lambda)} \quad , i = 1, \dots, 2L$$

$$W_0^{(c)} = \frac{\lambda}{(L+\lambda)} + (1 - \alpha^2 + \beta) \quad (24)$$

$$W_i^{(c)} = \frac{1}{2(L+\lambda)} \quad , i = 1, \dots, 2L$$

$W^{(m)}$ and $W^{(c)}$ are the weighting matrix for the state mean calculation and the covariance calculation, respectively. The scaling parameter β is used to incorporate part of the prior knowledge of the distribution of state vector. For Gaussian distribution, $\beta = 2$ is optimal [27]. In UKF correction step, the predicted weighted mean measurement can be computed as follows

$$(Y_k)_i = h((\chi_k)_i) \quad , i = 0, \dots, 2L \quad (25)$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} (Y_k)_i \quad (26)$$

In the UKF formulation, the Kalman gain is computed as follows

$$K_k = P_{\hat{x}_k \hat{y}_k} P_{\hat{y}_k \hat{y}_k}^{-1} \quad (27)$$

where

$$P_{\hat{x}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [(\chi_k)_i - \hat{x}_k^-] [(Y_k)_i - \hat{y}_k^-]^T \quad (28)$$

$$P_{\hat{y}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [(Y_k)_i - \hat{y}_k^-] [(Y_k)_i - \hat{y}_k^-]^T + R_k \quad (29)$$

covariance of the measurement and the cross-covariance of the state and measurement are denoted by $P_{\hat{y}_k \hat{y}_k}$ and $P_{\hat{x}_k \hat{y}_k}$, respectively. R_k is the measurement noise covariance matrix. Finally, the last step is to compute the updated state mean \hat{x}_k and the updated error covariance P_k given by

$$\hat{x}_k = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-) \quad (30)$$

$$P_k = P_k^- - K_k P_{\hat{y}_k \hat{y}_k} K_k^T \quad (31)$$