Task assignment for cooperating UAVs under radio propagation path loss constraints

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Abstract— **Unmanned Aerial Vehicles (UAVs) may be used for surveillance of power lines and railways and topological surveying to support the planning of new road routes. Motivated by these applications we will in this paper formulate a Mixed Integer Linear Programming (MILP) problem to be used for coarse offline path planning of such missions. Many of the missions rely on real time transfer of sensor data back to a human operator at the base station, to allow for intervention if the collected data show something of particular interest. In our approach we allow for the use of multiple UAVs where one or more of the vehicles also can be used as relay nodes for the transmitted data. The path obtained by solving the optimization problem is analyzed using a realistic radio propagation path loss simulator. If the radio propagation path loss exceeds the maximum design criterion the optimization problem is solved again with stricter communication constraint, and the procedure is continued in an iterative manner until the criterion is met.**

I. INTRODUCTION

A. Background and contribution

In this paper we describe a complex task assignment scenario for multiple UAVs using MILP. A task is in this paper specified by a sequence of waypoints. During the servicing of a task, that is, while visiting the waypoints of a task, the UAVs are required to communicate either directly or by using other UAVs as relay nodes. In the MILP formulation the ability to communicate at a certain data rate depends on the distance between the nodes. For this distance to give a realistic picture of the communication properties, the paths found by solving the MILP problem are analyzed at every time step using the radio path loss simulator SPLAT! [1]. SPLAT! uses digital elevation data to calculate field strength and path loss based on the Longley-Rice Irregular Terrain Model [2]. It comprises a variety of propagation modes: line-of-sight with single ground reflection for short distances, a double knife-edge diffraction propagation model for medium distances, and a tropospheric scatter model for long distances [3]. These distances will depend on a variety of other parameters provided by the user, and will be introduced later in this article. If the path loss estimate calculated by SPLAT! is too high to maintain communication at the desired rate, the communication constraints, or the maximal distance where communication is assumed feasible, is tightened in the MILP problem. The process is then repeated until paths are found in which communication can be maintained at a predefined criterion during the servicing of the tasks.

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Task assignment for multiple UAVs have been considered in several papers, see for instance [4], [5] and [6]. Path planning of UAVs using MILP has been treated in [7], [8], [9] and [10] to mention a few. In our previous work [10] we showed how paths for multiple UAVs can be planned with the aim of creating a communication chain between two base stations. In this paper we aim at finding the paths which solves a multi-task mission as fast as possible, and where both direct and multi-hop communication may be used. We therefore generalize the results of [10] by allowing for different communication strategies. In addition the scenario considered here is of a much more complex character, composed of multiple tasks and where cooperation between UAVs may be required to solve the overall mission. A main novelty of the approach taken here and in [10] is the use of a radio path loss simulator to achieve a more accurate prediction of the communication properties.

II. MILP PROBLEM FORMULATION

The mission objective is to perform inspection along sequences of waypoints by UAVs, and real time transmission of sensor data back to the base station. The waypoints are not necessarily within communication range for direct transmission, and we will therefore allow for one or more UAVs to function as relay links. We will assume that the tasks consists of segments (e.g. road, railway or power line segments), which can be divided into one or more waypoints. The interest in real time transmission of sensor data while observing these segments, is to allow for a human operator to intervene if something irregular is found in the sensor data.

A. Vehicle model

At the planning problem considered in this paper we assume that the p^{th} UAV is simply described by the discrete time model

$$\boldsymbol{p}_{p(i+1)} = \boldsymbol{p}_{pi} + \Delta t \boldsymbol{v}_{pi}\,, \tag{1}$$

$\forall p \in \{1,\ldots,n_p\}, \forall i \in \{1,\ldots,N\}$, where n_p is the number of UAVs, Δt is the sample time, N is the optimization horizon, i denotes a specific time sample and where $\boldsymbol{p}_{pi} := (p_{1pi}, p_{2pi}, p_{3pi}) := (x_{pi}, y_{pi}, z_{pi})^\top$, and $\boldsymbol{v}_{pi} := (v_{1pi}, v_{2pi}, v_{3pi})^\top$, with x_p, y_p, z_p and v_{1p}, v_{2p}, v_{3p} being the positions and velocities, respectively, along the orthogonal axes of a local East-North-Up coordinate reference frame.

B. Task assignment

We will assume that there are n_l tasks, and that each task $l \in \{1,\ldots,n_l\}$ is comprised of a set of waypoints. Let $\mathcal{W}_l$ denote the set of the serial numbers of the waypoints which belong to task l. A special meaning is given to the final task $\mathcal{W}_{n_l}$. It contains only one waypoint, which is located at the base station. It is assumed that $\mathcal{W}_r \bigcap \mathcal{W}_s = \emptyset$, $\forall r, s \in \{1, \ldots, n_l\}$, i.e. that each waypoint belong to one and only one task. Shortly, we will add constraints such that waypoint w is visited by vehicle p at time step i, if the binary variable b_{piw}^{wp} is *true*, see section II-C. We will require that each waypoint of tasks $\mathcal{W}_l, \forall l \in \{1, \ldots, n_l - 1\}$, is visited once and once only. Mathematically this is formulated by the equality constraint

$$\sum_{p=1}^{n_p} \sum_{i=1}^{N} b_{piw}^{\text{wp}} = 1\,, \tag{2}$$

$\forall w \in \mathcal{W}_1 \cup \ldots \cup \mathcal{W}_{n_l - 1}$, where N is the mission horizon. By assigning all vehicles the final task, all vehicles will return to the base station before the end of the mission. This assignment is ensured by the constraints

$$\sum_{i=1}^{N} b_{piw}^{\text{wp}} \geq 1\,, \tag{3}$$

$\forall p \in \{1, \ldots, n_p\}, \forall w \in \mathcal{W}_{n_l}$. We also add the constraints

$$b_{p(i+1)w}^{\text{wp}} \geq b_{piw}^{\text{wp}}\,, \tag{4}$$

$\forall p \in \{1, \ldots, n_p\}, \forall i \in \{1, \ldots, N-1\}, \forall w \in \mathcal{W}_{n_l}$, which together with the implication to be presented in (14) and (28), means that vehicle p will remain at the base station once it has arrived there. Given that $b_{piw} = 1$ for $w \in \mathcal{W}_{n_l}$, the implication in (14) constrain the position of the vehicles to the final waypoint, where as equation (28) constrain the velocity to zero. These constraints are important, because we then indirectly require that all tasks $\mathcal{W}_1$ to $\mathcal{W}_{n_l - 1}$ are executed before the the last vehicle return to the base station. Since each waypoint $w \in \mathcal{W}_1 \cup \ldots \cup \mathcal{W}_{n_l - 1}$, is visited only once, the time elapsed before a waypoint is visited is given by

$$\theta_w = \sum_{p=1}^{n_p} \sum_{i=1}^{N} i b_{piw}^{\text{wp}}\,, \tag{5}$$

$\forall w \in \mathcal{W}_1 \cup \ldots \cup \mathcal{W}_{n_l - 1}$, where θ_w is an variable we have introduced in our optimization problem. Furthermore, we require that the waypoints within the same task are visited in a specific order. Let $\mathcal{L}$ represent those tasks with more than one waypoint, and let $\tilde{\mathcal{W}}_l$ contain all the waypoints of $\mathcal{W}_l$, except the last one (given that $\mathcal{W}_l$ contains more than one waypoint). Then the visiting order can be achieved by requiring that

$$\theta_{r+1} > \theta_r\,, \tag{6}$$

$\forall r \in \tilde{\mathcal{W}}_l$, $l \in \mathcal{L}$. The time elapsed before vehicle p return to the base station is given by θ_p^{finish}, when we add the additional constraints

$$\theta_p^{\text{finish}} \leq M^{\text{finish}}(1 - b_{piw}^{\text{wp}}) + i b_{piw}^{\text{wp}}\,, \tag{7}$$

$$\theta_p^{\text{finish}} \geq i(1 - b_{piw}^{\text{wp}})\,, \tag{8}$$

$\forall p \in \{1, \ldots, n_p\}, \forall i \in \{1, \ldots, N\}, \forall w \in \mathcal{W}_{n_l}$, where M^{finish} is a constant chosen sufficiently large, for instance as $M^{\text{finish}} := N$. Here, and in the rest of the document, we will by *sufficiently large* (or *sufficiently small*) in this context mean that the constant should be chosen large (small) enough to maintain the original logical implication the constraint is meant to realize. Although M^{finish} in theory could be taken arbitrarily large, this is not recommended for computational efficiency [11]. In YALMIP, logic implications can be expressed instead of big-M formulations such as (7), and YALMIP will will automatically derive big-M coefficients by analyzing the expression.

Since we want to minimize the overall mission time - the time elapsed until the last vehicle arrives at the final waypoint - we introduce the variable η^{finish} and require that

$$\eta^{\text{finish}} \geq \theta_p^{\text{finish}}\,, \tag{9}$$

$\forall p \in \{1, \ldots, n_p\}$, and set our objective to minimizing the cost function

$$J^{\text{finish}} = \gamma^{\text{finish}} \eta^{\text{finish}}\,, \tag{10}$$

where γ^{finish} is a positive scalar. Equation (9) can be satisfied for any sufficiently large η^{finish}. However, by minimizing η^{finish} in the cost function, we achieve the desired effect, which is to minimize the overall mission time. We do not want vehicles to arrive at the final waypoint simultaneously, as this may cause the UAVs to collide. Therefore, we also require a temporal separation between the arrival at the final waypoint, that is,

$$\theta_p^{\text{finish}} \geq \theta_q^{\text{finish}} + t^{\text{separation}} \tag{11}$$

$\forall p \in \{1, \ldots, n_p - 1\}, q \in \{p+1, \ldots, n_p\}$, where $t^{\text{separation}} \in \mathbb{N}$ is the number of time steps separating the UAVs at the arrival of the final waypoint. Thus far, there is nothing restricting multiple vehicles each accomplishing parts of a task. As this is undesirable, we introduce additional binary variables b_{pl}^{task} which are *true* if and only if task l is served by vehicle p. This is achieved by imposing the constraints

$$-\sum_{w \in \mathcal{W}_l} \sum_{i=1}^{N} b_{piw}^{\text{wp}} \leq -n^{\mathcal{W}_l} b_{pl}^{\text{task}} \tag{12}$$

$\forall p \in \{1, \ldots, n_p\}, \forall l \in \{1, \ldots, n_l - 1\}$ and

$$\sum_{p=1}^{N} b_{pl}^{\text{task}} = 1\,, \tag{13}$$

$\forall l \in \{1, \ldots, n_l - 1\}$, where $n^{\mathcal{W}_l}$ is the number of waypoints of task l. Still, there is the possibility that a UAV switch back and forth between different tasks. We will allow this behavior, but as we will see in the simulations, it may be beneficial to accomplish one task at the time, due to more demanding communication constraints during the accomplishment of a task.

C. *Flying over waypoints*

We consider a waypoint characterized by the ENU coordinates $\left(p_{1w}^{\text{wp}}, p_{2w}^{\text{wp}}, p_{3w}^{\text{wp}}\right)$ to be visited if a UAV is flying through a cube containing the waypoint. More precisely, we assume each zone to be a cube with sides of length $2d^{\text{wp}}$, and require that

$$b_{piw}^{\text{wp}} \implies \left\{ p_{spi} \in \left(p_{sw}^{\text{wp}} \pm d^{\text{wp}} \right) \forall s \in \{1,2,3\} \right. , \tag{14}$$

$\forall p \in \{1, \ldots, n_p\}, i \in \{1, \ldots, N\}, w \in \mathcal{W}_1 \cup \ldots \cup \mathcal{W}_{n_l}$, such that if the binary variable $b_{piw}^{\text{wp}} \in \{0, 1\}$ is *true* then vehicle p flies through waypoint w at time step i. For brevity the implementation of the implication is left out.

D. Data gathering for immediate transmission

We assume that the bandwidth required for transmission is substantially larger during the execution of a task, and ignore the possible need for communication during transit between tasks. We introduce the binary variable b_{pi}^{sensor} which is *true* if vehicle p is serving a task at time step i. This implication can be stated as

$$\bigvee_{l=1}^{n_l-1} \left(- \sum_{w \in \mathcal{W}_l} \sum_{k=i}^{N} b_{pkw}^{\text{wp}} < 0 \wedge - \sum_{w \in \mathcal{W}_l} \sum_{k=1}^{i} b_{pkw}^{\text{wp}} < 0 \right) \iff b_{pi}^{\text{sensor}} , \tag{15}$$

and should be implemented $\forall p \in \{1, \ldots, n_p\}$ and $\forall i \in \{1, \ldots, N\}$. The actual implementation of the implication in (15) is left out for brevity.

E. Data flow for immediate transmission

We will in the following sometimes commonly refer to the base station and the UAVs as *nodes*. The communication network will be modeled as a flow network, a directed graph where each edge has a limited capacity. The flow into a node equals the flow out of a node, except if the node is a sink or a source. We assume that while vehicle p is servicing a task - that is for those time steps i the binary variable b_{pi}^{sensor} is *true* - the amount of data needed to be transmitted back to base station, is given by c^{sensor}. In our setup, the base station is the only sink, where as the UAVs act as sources during servicing of a task. Since we want real time transmission during the servicing of a task, no data will be stored in the flow network. We require that the rate at which data is gathered by vehicle p is equals the net outgoing data rate, that is,

$$c^{\text{sensor}} b_{pi}^{\text{sensor}} = \sum_{q=1}^{n_p+1} c_{pqi} , \tag{16}$$

$\forall p \in \{1, \ldots, n_p\}, i \in \{1, \ldots, N\}$. The optimization variable c_{pqi} is the data rate from node p to node q at time step i, and (16) represents in that respect flow conservation. We emphasize that c_{pqi} is negative if the net flow is from node q to node p. Furthermore, all sensor data should arrive at the base station within the time step, which means that

$$\sum_{p=1}^{n_p} c^{\text{sensor}} b_{pi}^{\text{sensor}} = \sum_{p=1}^{n_p} c_{p(n_p+1)i} , \tag{17}$$

$\forall p \in \{1, \ldots, n_p\}, i \in \{1, \ldots, N\}$. We also include the additional constraints

$$c_{ppi} = 0 , \tag{18}$$

$\forall p \in \{1, \ldots, n_p\}, i \in \{1, \ldots, N\}$ to prevent the vehicles to send and receive the same information,

$$c_{pqi} = -c_{qpi} , \tag{19}$$

$\forall p \in \{1, \ldots, n_p + 1\}, q \in \{1, \ldots, n_p + 1\}, i \in \{1, \ldots, N\}$ to ensure that the net flow from p to q is the opposite of the net flow from q to p and

$$c_{(n_p+1)pi} \leq 0 , \tag{20}$$

$\forall p \in \{1, \ldots, n_p\}, i \in \{1, \ldots, N\}$ since the base station is only receiving data. Here, $c_{p(n_p+1)i}$ denote the rate in which data is transmitted from vehicle p to the base station (denoted by subscript $n_p + 1$) at time step i. To reduce complexity we have assumed instantaneous transmission. To reflect the fact that transmission is only possible when the different nodes are within each others communication range, we also add to our optimization problem the constraints

$$c_{pqi} \leq C^{\max} \tilde{b}_{pqi}^{\text{con}} \tag{21}$$

$\forall p \in \{1, \ldots, n_p\}, \forall q \in \{1, \ldots, n_p + 1\}, i \in \{1, \ldots, N\}$, where $C^{\max}$ is the maximum data rate and $\tilde{b}_{pqi}^{\text{con}}$ is a binary variable which is *true* if and only if vehicle q is within communication distance of vehicle p at time step i. The constraints required to give $\tilde{b}_{pqi}^{\text{con}}$ this property, are introduced in the following section. We remark that by using three or more UAVs, a ring topology of communicating nodes may be formed. To avoid these unwanted solutions, we could for instance include a term in the cost function which penalizes the total data rate in the network.

F. Connectivity constraints

When formulating the MILP, we will assume that the ability of node p to successfully transmit data at a specified rate to node q, at some time instance i, depends on whether the relative distance between the two nodes are below a certain threshold, R_{pqi}. This threshold would typically depend on the antenna gains of the receiver and transmitter node, surrounding terrain, data rate, etc. We stress that R_{pqi} is not necessarily equal to R_{qpi}, that is, the threshold depends on the direction of communication. Instead of requiring that node q are within a sphere of radius R_{pqi} of node p, we require that node q is within a polygon that approximates the sphere. The approximation is formed by taking the inner product of the vector $\boldsymbol{\chi}_{pqi} := (x_{pi} - x_{qi}, y_{pi} - y_{qi}, z_{pi} - z_{qi})^\top$ and

$$\boldsymbol{\xi}_{kl} := \begin{bmatrix} \cos(\theta_k)\sin(\phi_l) \\ \sin(\theta_k)\sin(\phi_l) \\ \cos(\phi_l) \end{bmatrix} , \tag{22}$$

where $\theta_k := 2\pi k / D^{\text{con}}$ and $\phi_l := 2\pi l / D^{\text{con}}$ and $k \in \{1, \ldots, D^{\text{con}}/2\}$, $l \in \{1, \ldots, D^{\text{con}}\}$ and the discretization level D^{con} is some constant even integer greater or equal to 4. As already pointed out, we introduce binary indicator variables $\tilde{b}_{pqi}^{\text{con}} \in \{0, 1\}$ such that

$$\tilde{b}_{pqi}^{\text{con}} = 1 \iff \boldsymbol{\chi}_{pqi}^\top \boldsymbol{\xi}_{kl} - R_{pqi} \leq 0 , \tag{23}$$

$\forall p \in \{1, \ldots, n_p + 1\}$, $q \in \{1, \ldots, n_p + 1\}$, $i \in \{1, \ldots, N\}$, $k \in \{1, \ldots, D^{\text{con}}/2\}$, $l \in \{1, \ldots, D^{\text{con}}\}$ that is, the indicator variable $\tilde{b}^{\text{con}}_{pqi}$ is *true* if and only if, node p can directly transmit to vehicle q, where $p, q = n_p + 1$, denote the base station. The logical statement in (23) can be achieved by the introducing a number of additional optimization variables, but out of brevity we will simply refer to [10, Proposition 1] for the implementation of the implication.

G. Velocity constraints

Let the optimization variable V_{pi} be an approximation of the magnitude of the velocity vector $\boldsymbol{v}_{pi}$. We approximate V_{pi} in a similar manner as in [13], here in the three dimensional case as in [10], by introducing the constraints:

$$\boldsymbol{v}_{pi}^{\top}\boldsymbol{\xi}_{kl} \leq V_{pi}\,, \tag{24}$$

$$\alpha^{\text{vel}}\boldsymbol{v}_{pi}^{\top}\boldsymbol{\xi}_{kl} \geq V_{pi} - M^{\text{vel}}_{pkl}(1 - b^{\text{vel}}_{pikl})\,, \tag{25}$$

$\forall p \in \{1, \ldots, n_p\}$, $i \in \{0, \ldots, N-1\}$, $k \in \{1, \ldots, D^{\text{vel}}/2\}$, $l \in \{1, \ldots, D^{\text{vel}}\}$, where $b^{\text{vel}}_{pikl} \in \{0, 1\}$ are binary optimization variables, D^{vel} is some constant even integer greater or equal to four, and the unit vector $\boldsymbol{\xi}_{kl}$ was defined in (22) with $\theta_k := 2\pi k/D^{\text{vel}}$ and $\phi_l := 2\pi l/D^{\text{vel}}$. The accuracy of the approximation depends on α^{vel} which is a constant slightly greater than one. The closer to one α^{vel} is, the better is the approximation, however, taking it too close may have a negative impact on the computation time of the MILP problem [13]. Furthermore, we require that

$$\sum_{k=1}^{D^{\text{vel}}} \sum_{l=1}^{D^{\text{vel}}/2} b^{\text{vel}}_{pikl} = 1\,, \tag{26}$$

$\forall p \in \{1, \ldots, n_p\}$, $i \in \{0, \ldots, N-1\}$. The constant M^{vel}_{pkl} should satisfy

$$M^{\text{vel}}_{pkl} > \max_{\substack{v_{1pi}, v_{2pi}, v_{3pi} \in [-\overline{V}_p, \overline{V}_p] \\ V^{\text{approx}}_{pi} \in [\underline{V}_p, \overline{V}_p]}} \left\{\alpha^{\text{vel}}\boldsymbol{v}_{pi}^{\top}\boldsymbol{\xi}_{kl} - V^{\text{approx}}_{pi}\right\}, \tag{27}$$

where $\overline{V}_p$ is the maximum velocity of vehicle p. As the speed of the vehicles are approximated by (24), (25) and (26), we simply use that

$$\underline{V}_p(1 - b^{\text{wp}}_{piw}) \leq V_{pi} \leq \overline{V}_p(1 - b^{\text{wp}}_{piw})\,, \tag{28}$$

$\forall p \in \{1, \ldots, n_p\}$, $i \in \{0, \ldots, N-1\}$, $w \in \{\mathcal{W}_{n_l}\}$, where $\underline{V}_p$ and $\overline{V}_p$ are the minimum and maximum velocity, respectively, of vehicle p. With these constraints the vehicles are constrained to zero velocity when they have arrived at the final waypoint.

H. Acceleration cost

To avoid fluctuations in the speed, we introduce the following cost function similar to the one proposed in [7],

$$J^{\text{acc}} = \sum_{p=1}^{n_p} \sum_{i=0}^{N-2} \boldsymbol{r}_p^{\top}\boldsymbol{w}^{\text{acc}}_{pi}\,, \tag{29}$$

with the additional constraints

$$(v_{jpk} - v_{jpi}) \leq w^{\text{acc}}_{jpi}\,, \tag{30}$$

$$-(v_{jpk} - v_{jpi}) \leq w^{\text{acc}}_{jpi}\,, \tag{31}$$

TABLE I

TASKS, AND COORDINATES OF THEIR CORRESPONDING WAYPOINTS

Task	Waypoint Coordinates
$\mathcal{W}_1$	{(4100,2150,200),(4000,2250,200),(3900,2350,200)}
$\mathcal{W}_2$	{(2800,1850,300),(2900,1950,300),(3000,2050,300)}
$\mathcal{W}_3$	{(3500,1000,250)}
$\mathcal{W}_4$	{(4000,1200,250),(4000,1300,250),(4000,1400,250)}
$\mathcal{W}_5$	{(3700,1850,250)}

$\forall p \in \{1, \ldots, n_p\}$, $k = i + 1$, $i \in \{0, \ldots, N-2\}$, $j \in \{1, 2, 3\}$ where $\boldsymbol{w}^{\text{acc}}_{pi} := (w^{\text{acc}}_{1pi}, w^{\text{acc}}_{2pi}, w^{\text{acc}}_{3pi})^{\top}$ and $\boldsymbol{r}_p \in \mathbb{R}^3_{\geq 0}$ is a nonnegative weighting vector. The motivation behind (29) is to penalize the absolute value of acceleration in each direction of the ENU frame, and to avoid a piecewise linear cost function, we have introduced slack variables w^{acc}_{jpi}.

I. Position, anti-collision and anti-grounding constraints

In this paper the UAVs will be required to stay within a rectangular box, and the position constraints may be written

$$\underline{x} \leq x_{pi} \leq \overline{x}\,, \tag{32}$$

$$\underline{y} \leq y_{pi} \leq \overline{y}\,, \tag{33}$$

$$\underline{z} \leq z_{pi} \leq \overline{z}\,, \tag{34}$$

$\forall p \in \{1, \ldots, n_p\}$, $i \in \{1, \ldots, N\}$, where $\underline{x}, \underline{y}, \underline{z}$ and $\overline{x}, \overline{y}, \overline{z}$ are the lower and upper bounds, respectively, on the state vector in the east, north and up directions. The constants $\underline{x}, \underline{y}, \underline{z}$ and $\overline{x}, \overline{y}, \overline{z}$ are provided to the optimizer at start-up. Such position constraints may for instance be caused by airspace restriction, but in any MILP formulation constraints on the optimization problem should be provided as this usually reduces the solver time.

The anti-collision constraints and anti-grounding constraints used, were to the author's knowledge originally introduced in [7] and [9], respectively. Due to space limitations, the reader is referred to [10, (31)-(34)] for anti-collision constraints and [10, (35)-(41)] for anti-grounding constraints, where the notation is consistent with the one of this article.

III. SIMULATIONS

In this section we will use two UAVs to solve the task assignment problem. The tasks and their corresponding waypoints are described in Table I. The initial position of the vehicles are $(3700, 1850, 300)^{\top}$ and $(3700, 1850, 350)^{\top}$, where as the base station is located at $(3750, 1850, 75)^{\top}$. We assume that a human operator will bring the UAVs up to the initial position, and down from the final waypoint. All positions are given in the ENU frame which origin is located at longitude $63.4°$, latitude $10.32°$ and ellipsoidal height 0 relative to the reference ellipsoid defined in WGS84.

The cost to be minimized is given by

$$J = J^{\text{finish}} + J^{\text{acc}}\,, \tag{35}$$

subject to the constraints (1)-(9), (11)-(21), (23)-(26), (28), (30)-(34) and [10, (31)-(41)].

Table III shows the parameters used in the MILP problem. The default solver parameters of Gurobi were used, except

TABLE II

SPLAT! PARAMETERS

Parameter	Value
Earth Dielectric Constant (Relative permittivity)	15
Earth Conductivity (Siemens per meter)	0.005
Atmospheric Bending Constant (N-units)	301
Frequency in MHz (20 MHz to 20 GHz)	2400
Radio Climate (5 = Continental Temperate)	6
Polarization (0 = Horizontal, 1 = Vertical)	0
Fraction of situations (50% of locations)	0.5
Fraction of time (50% of the time)	0.5

TABLE III

MILP PARAMETERS

Parameter	Value	Parameter	Value
d^{TIN}	100 m	$\overline{x}$	4200 m
D^{vel}	8	$\underline{x}$	2700 m
D^{con}	8	$\overline{y}$	2500 m
c^{sensor}	$2\,\text{Mbits}\,\text{s}^{-1}$	$\underline{y}$	900 m
C^{max}	$4\,\text{Mbits}\,\text{s}^{-1}$	$\overline{z}$	500 m
$t^{\text{separation}}$	5 s	$\underline{z}$	0 m
$\overline{V}_1, \overline{V}_2$	$25\,\text{m}\,\text{s}^{-1}$	$\underline{V}_1, \underline{V}_2$	$6\,\text{m}\,\text{s}^{-1}$
$\boldsymbol{r}_1^{\text{acc}}, \boldsymbol{r}_2^{\text{acc}}$	$(0.1, 0.1, 0.2)^\top$	γ^{finish}	100
d_x, d_y, d_z	50 m	d^{wp}	10 m

for MIPGap which was set to 1.0×10^{-2}. The algorithm was run on an HP EliteBook 8540w, with Intel Core i7 CPU Q720 @1.6 GHz, 16 GB RAM and a Windows 7, 64-bit operating system. Furthermore, we used MATLAB version R2011b and the modeling language YALMIP [14] version 20111128 together with the Gurobi Optimizer 4.5 and [15] to define and solve the MILP problem. The Windows version of the radio propagation path loss simulator SPLAT! we used was provided by [16], and the parameters used are shown in Table II. SPLAT! is mainly intended for ground based antennas. However, it is possible to adjust the antenna height, and we have therefore set the antenna height equal to the altitude of the vehicles when analyzing the radio path loss.

We initially set $R_{pqi}^{\text{con}} = 950\ \forall p, q \in \{1, 2, 3 | p \neq q\}$, $i \in \{1, \ldots, N\}$. We emphasize that in this example the third node is the base station. Let the maximum allowed path loss between vehicle p and q to maintain the desired bandwidth during transmission be given by L_{pq}^{max}. If, for any time step i, the radio path loss calculated by SPLAT! is below L_{pq}^{max} and $c_{pqi} \neq 0$, then we set $R_{pqi}^{\text{con}} = R_{pqi}^{\text{con}} - r^{\text{con}}$ and solve the MILP problem again. We use $r^{\text{con}} = 150\,\text{m}$ and $L_{pq}^{\text{max}} = 98\,\text{dB}$ for any $p, q \in \{1, 2, 3 | p \neq q\}$, although the maximum radio path loss *can* be different between different nodes, and does not even have to be symmetric between a pair of nodes. The horizon is 200 s, and the discretization step is of 5 s, i.e. $N = 40$.

Figure 1 shows a top view of the planned path. The black diamonds represents the waypoints, and the yellow circles are the initial positions of the UAVs, which East-North location coincides with the location of the base station. In Figure 2 the path loss between the nodes calculated by SPLAT! are depicted in solid blue line, the maximum path loss for communication in dashed blue line, where as the solid green line is the communication data rate. The shaded areas represents the time steps the nodes are communicating. We see from Figure 2(a) that the initial solution to the MILP problem does not satisfy the radio path loss design criteria when analyzed in SPLAT!. However, at the final iteration depicted in Figure 2(b), these criteria are satisfies since the calculated path loss is below the maximum path loss at all time steps of communication.

TABLE IV

SOLVER TIMES IN SECONDS. (TOTAL SOLVER TIME: 16 678 s)

Iteration	Time	Iteration	Time
1	3209	4	2335
2	8153	5	2222
3	759		

The computational complexity of such MILP problem formulated in this article is poorly scalable, and increases drastically with the number of vehicles and the horizon length. The solver times for each iteration of the scenario in this section is summarized in Table IV. It can be seen that already for two vehicles the solver times can grow beyond practical use. However, reducing the computation time has not been placed emphasis on in this work. Means for reducing the computation time could for instance be making the code more efficient by estimating bounds on the optimization variables in advance such that tighter constraints could be provided to the solver; estimating bounds on the costs of the optimization problem and add these as constraints; reducing the required accuracy of the optimal solution by adjusting the solver parameters; providing a non-optimal solution to the problem at start-up; parallelizing the computation over several computers.

To improve robustness towards inaccuracies in the model and uncertainties that become apparent real time, the proposed path planning strategy should be combined with an online re-planning strategy in a real deployment, for instance as described in [17].

IV. CONCLUSIONS

We have in this paper addressed the problem of task assignment and path planning for multiple UAVs. The demands to high bandwidth means that the vehicles will need to cooperate during the execution of tasks, that is, some vehicles are used as relay nodes for transmitting sensor data back to the base station. We have improved the accuracy of the communication properties by using a path loss simulator to analyze the paths generated from the optimization problem.

V. ACKNOWLEDGMENTS

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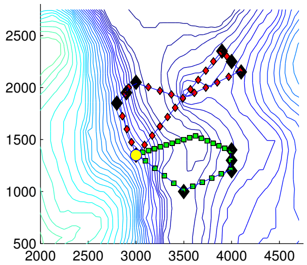
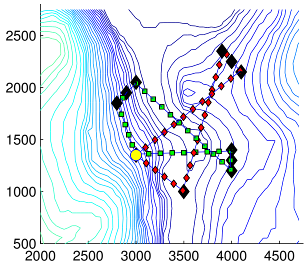
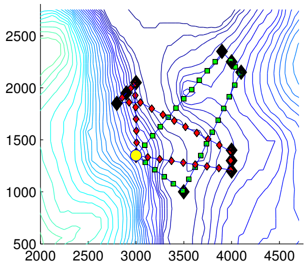


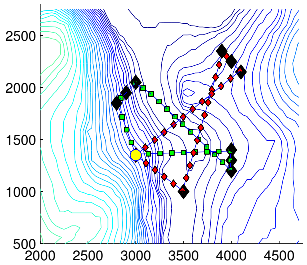
(a) Iteration 1



(b) Iteration 2

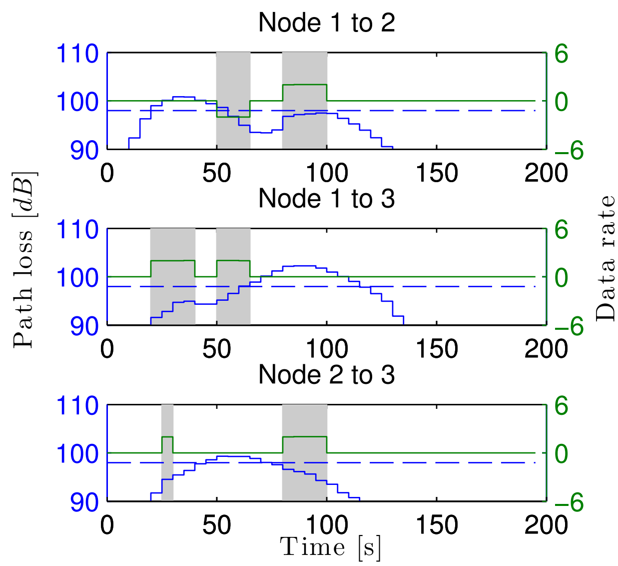


(c) Iteration 4

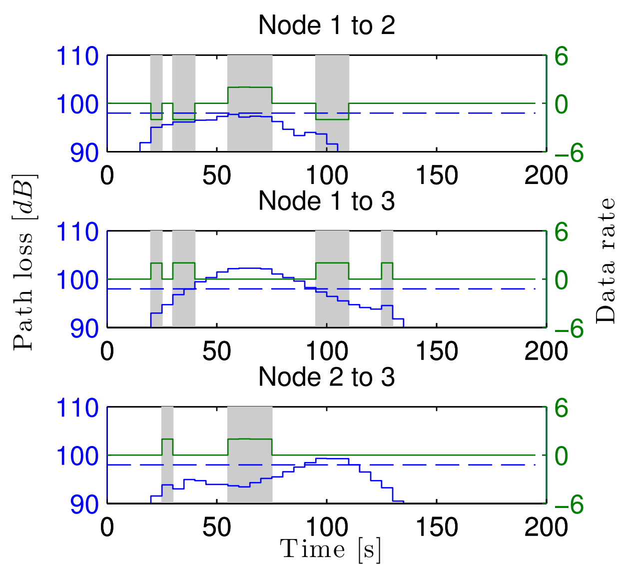


(d) Iteration 5

Fig. 1. Topview of the planned path at (a) 1st iteration, (b) 2nd iteration, (c) 4th iteration and (d) 5th and final iteration. The black diamonds represents the waypoints, and the yellow circles are the initial locations of the UAVs, which coincide with the base station except for the different altitudes. The red diamonds represents UAV 1, and the green squares represents UAV 2.



(a) Iteration 1



(b) Iteration 5

Fig. 2. The calculated path loss using SPLAT! (solid blue), maximum allowed path loss for communication (dashed blue), communication data rate (solid green) and time steps when high bandwidth communication is required (shaded grey area) to accomplish the task at (a) 1st iteration, and (b) 5th and final iteration. Notice that in (a) the calculated path loss exceeds the maximum allowed path loss during time steps for which communication is planned. At the final iteration, shown in (b), this is not the case.

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