# Switched backstepping control of an electropneumatic clutch actuator using on/off valves

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*Abstract*— **In this paper, we consider the problem of controlling an electropneumatic clutch actuator for heavy duty trucks, using on/off valves. These valves are considered to be either open or closed. For a 3rd order model of the system, a controller is designed to govern switching between the available inputs. Local exponential stability of this control is shown, and the performance is investigated by simulations.**

## I. Introduction

Control of pneumatic actuators have been treated in several papers, both with proportional valves [1], [2], and with on/off valves [3]-[6]. In this paper a pneumatic actuator of an electropneumatic clutch system is considered. This pneumatic actuator acts on the clutch plates through the clutch spring, and the state of the clutch is therefore directly dependent on the actuator position. To control this actuator on/off valves are used. These are chosen instead of a proportional valve as they are smaller and cheaper, although their dynamics are harder to model accurately.

On/off valves can be controlled by pulsewidth modulation as in [7]-[9]. This control method has been implemented for the clutch system under consideration here, but as the dynamic and the steady-state relationship between the electrical input to on/off valves and the resulting valve openings is difficult to model accurately, it is desired to find other control methods. One appealing approach is to consider the case when only fully open and closed are possible states of the valves, and design a control law to govern switches between these. This is likely to render a simple control law as it doesn't need exact knowledge of the valve dynamics.

The paper begins with a description of models of the clutch system and some considerations on on/off valves, followed by the design of a controller based on backstepping and Lyapunov theory. The controlled system performance is then verified by simulations.

## II. System description

Figure 1 shows a sketch of the clutch actuator system. To control both supply to and exhaust from the clutch actuator chamber, at least one pair of on/off valves are needed. As we only allow these to be fully open or closed, with two valves and under the assumption of choked flow, we restrict the flow of the clutch actuator to three possible values, maximum flow into the volume, maximum flow out of the volume, or no flow. The electronic control unit (ECU) calculates and sets voltage signals to control the on/off valves. These signals control whether the valve should open or close, and thus also the flow into the actuator. A position sensor measures position and feeds it back to the ECU. To calculate the control signals, knowledge of other states of the system are also needed, and these can be obtained either by sensors or by estimation.

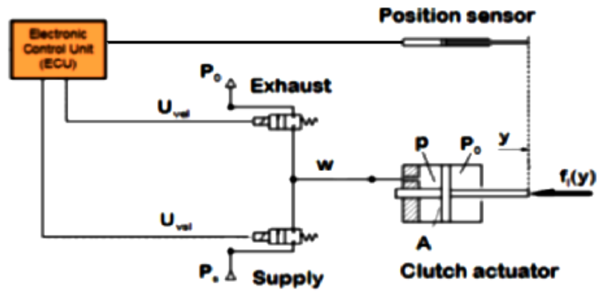


Fig. 1. Drawing of the electropneumatic clutch actuator

Both a 4th and a 6th order model is presented in [10]. A simpler model of the system is the following 3rd order model,

$$\begin{aligned} \dot{y} &= v \\ M\dot{v} &= -f_l(y) - Dv + A\frac{\zeta}{V(y)} - AP_0 \\ \dot{\zeta} &= RT_0 w \end{aligned} \tag{1}$$

where $y$ is position, $v$ is velocity and $\zeta = pV(y)$ is accumulated air, proportional to the amount of air in the actuator valve. $f_l(y)$ is the clutch load which can be described by

$$f_l(y) = K_l(1 - e^{-L_l y}) - M_l y \tag{2}$$

and $V(y)$ is the chamber volume given by

$$V(y) = V_0 + Ay. \tag{3}$$

The parameters are

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| | |
|---|---|
| $A$ | Cross section area of the clutch actuator |
| $P_s$ | Supply pressure |
| $P_0$ | Atmospheric pressure |
| $p$ | Pressure in the clutch actuator |
| $R$ | Gas constant of air |
| $T_0$ | Temperature |
| $L_l$, $M_l$, $K_l$ | Load characteristic terms |
| $V_0$ | Volume at $y = 0$ |
| $D$ | Viscous damping |
| $M$ | Mass of piston |

and the available inputs are

$$w \in \{-U_{\max}, 0, U_{\max}\} \tag{4}$$

Here $U_{\max} = \rho_0 C P_s$ is a simplified expression for the maximum flow capacity through the on/off valves, where $\rho_0$ is the density and $C$ the capacity. This simple model is used in this paper to be able to illustrate our idea without making the steps of the backstepping and the control design too complex.

## III. Control Lyapunov function design

The idea presented in this paper is to use backstepping to find an appropriate control Lyapunov function, and use this to design a valve switching law for our system. We want to control the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{M}(-K_l(1-e^{-L_l x_1}) + M_l x_1 \\ &\quad -D x_2 + A\frac{x_3}{V_0 + A x_1} - A P_0) \\ \dot{x}_3 &= R T_0 w \end{aligned} \tag{5}$$

to a reference equilibrium

$$x^* = [x_1^*, 0, x_3^*]^T$$

where the reference point $x_1^*$ is given and $x_3^*$ can be found from

$$\begin{aligned} 0 &= \frac{1}{M}(-K_l(1-e^{-L_l x_1^*}) + M_l x_1^* \\ &\quad + A\frac{x_3^*}{V_0 + A x_1^*} - A P_0). \end{aligned} \tag{6}$$

or

$$\begin{aligned} x_3^* &= \frac{V_0 + A x_1^*}{A} \\ &\quad (K_l(1-e^{-L_l x_1^*}) - M_l x_1^* + A P_0) \end{aligned} \tag{7}$$

The error system used for control design is then

$$\begin{aligned} \dot{x}_{1e} &= x_2 \\ \dot{x}_{2e} &= \frac{1}{M}(-K_l(1-e^{-L_l x_1}) + M_l x_1 \\ &\quad -D x_2 + A\frac{x_3}{V_0 + A x_1} - A P_0) \\ \dot{x}_{3e} &= R T_0 w \end{aligned} \tag{8}$$

where

$$\begin{aligned} x_{1e} &= x_1 - x_1^* \\ x_{2e} &= x_2 \\ x_{3e} &= x_3 - x_3^* \end{aligned} \tag{9}$$

### A. Backstepping

Assume first no restrictions on the input, such that a backstepping procedure can be used to find a suitable control Lyapunov function for the system.

**Step 1**

First we define

$$z_1 = x_1 - x_1^* \tag{10}$$

which gives

$$\dot{z}_1 = x_2. \tag{11}$$

We choose virtual control $x_2 = \phi_1(z_1) = -k z_1$, where $k$ is a positive constant. From the Lyapunov-like function

$$V_1(z_1) = \frac{\alpha}{2} z_1^2 \tag{12}$$

it is easy to show that this virtual control gives

$$\dot{V}_1 = -\alpha k z_1^2. \tag{13}$$

**Step 2**

The change of variables

$$z_2 = x_2 - \phi_1(z_1) = x_2 + k z_1 \tag{14}$$

transforms the system into

$$\begin{aligned} \dot{z}_1 &= z_2 - k z_1 \\ \dot{z}_2 &= \frac{1}{M}[-K_l(1-e^{-L_l x_1}) - D(z_2 - k z_1) \\ &\quad + M_l x_1 + A\frac{x_3}{V_0 + A x_1} - A P_0] + k z_2 - k^2 z_1 \end{aligned} \tag{15}$$

We now choose $x_3$ as virtual control $\phi_2(x_1, z_1, z_2)$ and take

$$V_2(z_1, z_2) = V_1(z_1) + \frac{\beta}{2} z_2^2. \tag{16}$$

From

$$\begin{aligned} \dot{V}_2 &= \alpha z_2 z_1 - \alpha k z_1^2 + \frac{\beta}{M} z_2(-K_l(1-e^{-L_l x_1}) \\ &\quad + M_l x_1 - D(z_2 - k z_1) \\ &\quad + A\frac{\phi_2(x_1, z_1, z_2)}{V_0 + A x_1} - A P_0) + \beta k z_2^2 - \beta k^2 z_1 z_2 \end{aligned} \tag{17}$$

and by setting

$$\alpha = \beta k^2 \tag{18}$$

we get

$$\begin{aligned} \dot{V}_2 &= -\alpha k z_1^2 + \frac{\beta}{M} z_2(-K_l(1-e^{-L_l x_1}) \\ &\quad + M_l x_1 - D(z_2 - k z_1) + \\ &\quad A\frac{\phi_2(x_1, z_1, z_2)}{V_0 + A x_1} - A P_0) + \beta k z_2^2. \end{aligned} \tag{19}$$

We choose the virtual control as

$$\begin{aligned} \phi_2(x_1, z_1, z_2) &= \frac{V_0 + A x_1}{A}(K_l(1-e^{-L_l x_1}) \\ &\quad - M_l x_1 - D k z_1 + A P_0 - M k z_2) \end{aligned} \tag{20}$$

to get

$$\dot{V}_2 = -\alpha k z_1^2 - \beta \frac{D}{M} z_2^2. \tag{21}$$

**Step 3**

The change of variables

$$\begin{aligned} z_3 &= x_3 - \phi_2(x_1, z_1, z_2) \\ &= x_3 - \frac{V_0 + Ax_1}{A}(K_l(1 - e^{-L_l x_1}) \\ &\quad -M_l x_1 - Dkz_1 + AP_0 - Mkz_2) \end{aligned} \tag{22}$$

transforms the system into

$$\begin{aligned} \dot{z}_1 &= z_2 - kz_1 \\ \dot{z}_2 &= \frac{Az_3}{M(V_0 + Ax_1)} - k^2 z_1 - \frac{D}{M} z_2 \\ \dot{z}_3 &= RT_0 w - (z_2 - kz_1)(K_l(1 - e^{-L_l x_1}) \\ &\quad -M_l x_1 - Dkz_1 + AP_0 - Mkz_2) \\ &\quad -\frac{V_0 + Ax_1}{A}((z_2 - kz_1)K_l L_l e^{-L_l x_1} \\ &\quad -M_l(z_2 - kz_1) + Dk(z_2 - kz_1) - Mk \\ &\quad (\frac{Az_3}{M(V_0 + Ax_1)} - k^2 z_1 - \frac{D}{M} z_2)) \end{aligned} \tag{23}$$

A new Lyapunov function is chosen

$$V_3(z) = V_2(z_1, z_2) + \frac{\lambda}{2} z_3^2 \tag{24}$$

and this leads to

$$\begin{aligned} \dot{V}_3 &= -\alpha k z_1^2 - \beta \frac{D}{M} z_2^2 + \frac{\beta A z_2 z_3}{M(V_0 + Ax_1)} + \\ &\quad \lambda z_3((RT_0 w - (z_2 - kz_1)(K_l(1 - e^{-L_l x_1}) \\ &\quad -M_l x_1 - Dkz_1 + AP_0 - Mkz_2) \\ &\quad -\frac{V_0 + Ax_1}{A}((z_2 - kz_1)K_l L_l e^{-L_l x_1} \\ &\quad -M_l(z_2 - kz_1) + Dk(z_2 - kz_1) \\ &\quad -Mk(\frac{Az_3}{M(V_0 + Ax_1)} - k^2 z_1 - \frac{D}{M} z_2)) \end{aligned} \tag{25}$$

By choosing the input as

$$\begin{aligned} w &= \frac{1}{RT_0}(-\frac{\beta A z_2}{M\lambda(V_0 + Ax_1)} + (z_2 - kz_1) \\ &\quad (K_l(1 - e^{-L_l x_1}) - M_l x_1 + Dkx_1 + AP_0 \\ &\quad -Mkz_2) - \frac{V_0 + Ax_1}{A}((z_2 - kz_1)K_l L_l e^{-L_l x_1} \\ &\quad M_l(z_2 - kz_1) + Dk(z_2 - kz_1) \\ &\quad -Mk(\frac{Az_3}{M(V_0 + Ax_1)} - k^2 z_1 - \frac{D}{M} z_2)) - bz_3) \end{aligned} \tag{26}$$

where $b$ is a positive constant, we get

$$\dot{V}_3 = -\alpha k z_1^2 - \beta \frac{D}{M} z_2^2 - \lambda b z_3^2 \tag{27}$$

which shows exponential stability of the system reference equilibrium with backstepping control.

The backstepping parameters, $k, b, \alpha, \lambda, \beta$, can be decided by considering the error dynamics

$$\begin{aligned} \dot{z}_1 &= z_2 - kz_1 \\ \dot{z}_2 &= \frac{Az_3}{M(V_0 + Ax_1)} - k^2 z_1 - \frac{D}{M} z_2. \\ \dot{z}_3 &= -\frac{\beta A z_2}{M\lambda(V_0 + Ax_1)} - bz_3 \end{aligned} \tag{28}$$

### B. Switched control design

Now we have a Lyapunov function (24) and its time derivative (25) for the system, but as we have restrictions on the input to the system, the input (26) is not directly applicable. Instead we use the Lyapunov function to propose a simple switching law which choose between the available input values (4):

**Control strategy:** At each switching time, choose the available input which provides the smallest value of the Lyapunov function time derivative, $\dot{V}_3$.

Recall the time derivative in equation (25), and notice that the only term dependent on the input is

$$\lambda z_3 RT_0 w \tag{29}$$

such that minimizing $\dot{V}_3$ is achieved by minimizing $\lambda z_3 RT_0 w$. Since $R$, $T_0$ and $\lambda$ are constants, choosing the input which satisfies

$$sgn(w) = -sgn(z_3) \tag{30}$$

will render the smallest $\dot{V}_3$. The control input can then be written as

$$w = \begin{cases} -U_{\max} sgn(z_3) & z_3 \neq 0 \\ 0 & z_3 = 0 \end{cases} \tag{31}$$

Every switch between $w = U_{\max}$ and $w = -U_{\max}$ is done when

$$\begin{aligned} z_3 &= 0 = x_3 - \frac{V_0 + Ax_1}{A}(K_l(1 - e^{-L_l x_1}) \\ &\quad -M_l x_1 - Dkx_{1e} + AP_0 - Mk(x_2 + kx_{1e})) \end{aligned} \tag{32}$$

which will be the switching surface

Such nonlinear switching surfaces are also treated in several papers where sliding mode controllers are designed, as in [11] and [12]. While we design a backstepping controller first and the switching surface is found from this, the sliding mode case define the surface first, and uses this to design a controller that prove stability of the system.

### C. Stability

*Proposition 1:* The equilibrium $x^*$ of the system (5), with the switched control input given by (31) is locally exponentially stable.

*Proof:* First we prove existence, uniqueness and continuity of the solution using Filippov solution theories as in [13]. The discontinuity surface can be described by

$$S := \{\mathbf{z} : z_3 = 0\} \tag{33}$$

and this divides the solution domain $\Omega$ into to regions: $\Omega^- := \{\mathbf{z} : \mathbf{z}_3 < 0\}$ and $\Omega^+ := \{\mathbf{z} : \mathbf{z}_3 > 0\}$. As the right hand side of (23) is defined everywhere in $\Omega$ and are measurable and bounded, the system (23) satisfy condition B of Filippov's solution theory [14]. According to Theorems 4 and 5 in the same reference, we when have local existence and continuity of a solution. Further, since the right hand of (23) is continuous before and after the discontinuity surface, $S$, and this surface is smooth and independent of time, conditions A, B, C of Filippov's solution are satisfied [15]. Following the procedure introduced in [14] the vector functions $f^-$ and $f^+$ are defined as the limiting values of the right-hand sides of the state space equations in $\Omega^-$ and $\Omega^+$ :

$$f^- = \begin{bmatrix} z_2 - kz \\ \frac{Az_3}{M(V_0+Ax_1)} - k^2 z_1 - \frac{D}{M} z_2 \\ RT_0 U_{\max} - \dot{\phi}(x_1, z_1, z_2) \end{bmatrix} \quad (34)$$

$$f^+ = \begin{bmatrix} z_2 - kz \\ \frac{Az_3}{M(V_0+Ax_1)} - k^2 z_1 - \frac{D}{M} z_2 \\ -RT_0 U_{\max} - \dot{\phi}(x_1, z_1, z_2) \end{bmatrix} \quad (35)$$

For all points on the discontinuity surface vector $\mathbf{h}$ is defined as

$$h = f^+ - f^- = \begin{bmatrix} 0 \\ 0 \\ -2RT_0 U_{\max} \end{bmatrix} \quad (36)$$

which is along the normal of the discontinuity surface, $\mathrm{N}_s = (0, 0, 1)^T$. The scalar, $h_N$, defined as the projection of $\mathbf{h}$ on $\mathbf{N}_s$ is

$$h_N = \mathbf{N}_s \mathbf{h} = -\, 2RT_0 U_{\max} < 0 \quad (37)$$

and will always be negative. According to Lemma 7 in [15], the uniqueness of the Filippov solution is then guaranteed.

Second we consider stability of the solution. The Lyapunov time derivative (25) can be rewritten as

$$\dot{V}_3 \leq -\sigma V_3 - \lambda(RT_0 U_{\max}|z_3| + z_3 a(z)) \quad (38)$$

where

$$\begin{aligned} a(z) \;=\; & -bz_3 - \frac{\beta A z_2}{\lambda M(V_0 + Ax_1)} + (z_2 - kz_1) \\ & (K_l(1 - e^{-L_l x_1}) - M_l x_1 - Dkz_1 \\ & + AP_0 - kz_2) + \frac{V_0 + Ax_1}{A}(z_2 - kz_1) \\ & (K_l L_l e^{-L_l x_1} - M_l + Dk) - \frac{V_0 + Ax_1}{A} \\ & (\frac{Az_3}{M(V_0 + Ax_1)} - k^2 z_1 - \frac{D}{M} z_2) \end{aligned} \quad (39)$$

and

$$\sigma = 2\min(k, \frac{D}{M}, b). \quad (40)$$

Since $a(0) = 0$ and $a(z)$ is continuous there must exist an $\epsilon > 0$ such that for $|z| \leq \epsilon$ we have $|a(z)| \leq RT_0 U_{\max}$. It follows that for $|z| \leq \epsilon$ we get

$$\dot{V}_3 \leq -\sigma V_3 \quad (41)$$

and the equilibrium point $x^*$ is locally exponentially stable. ■

*Remark 1:* As the size of $\epsilon$ be will be decided from the area where the input $w$ from (26) fulfills

$$|w| < RT_0 U_{\max}, \quad (42)$$

it will be large in the context of local stability.

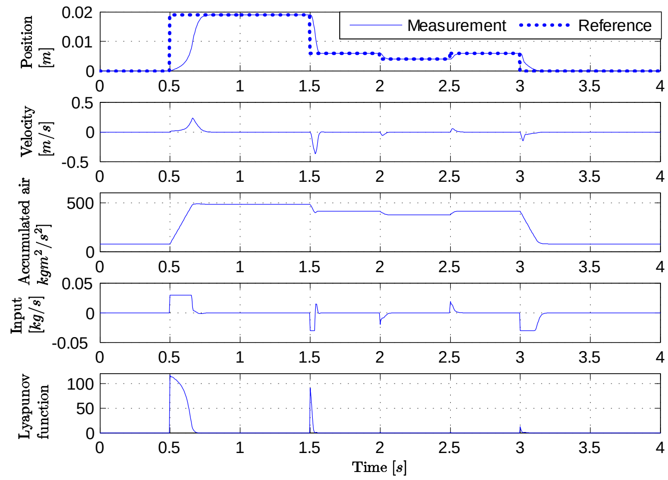


Fig. 2. Saturated backstepping controller

## IV. Simulations

### A. Practical considerations

The on/off valves cannot open/close instantaneously. The sampling time, and hence the minimum switching period, for the simulations is therefore set to 5 ms. This will guarantee that the valves have opened/closed before a new input signal is given.

In order to avoid unnecessary chattering, $w = 0$ have been chosen whenever $V$ is sufficiently close to zero, which is the same as saying that $x$ is close to $x^*$. The system have also been simulated with a pwm controller, which has been submitted as a basis for comparison.

The position reference used in the simulations is a typical clutch sequence, and is shown with dashed lines. It is desired that the controller makes the system reach the reference point within 0.1 s and with a steady state error of less than 0.2 mm in the area where the clutch engages/disengages. Outside this area, the restrictions can be somewhat relaxed.

### B. Backstepping

The control parameters are tuned to obtain fast response of the system with backstepping control. This results in use of inputs which lies high above the maximum capacity of the on/off valves, and a much faster response than for the system with switched control. Figure 2 shows the system with backstepping control, but here saturation have been introduced such that the input never exceed the maximum on/off valve capacity. This is done to be able to compare the responses of the switched and the backstepping control. Note that the backstepping controller will not be directly applicable to the physical clutch actuator systems, as it requires other input than open/closed, and these cannot be calculated accurately due to the complex dynamics of the on/off valves.

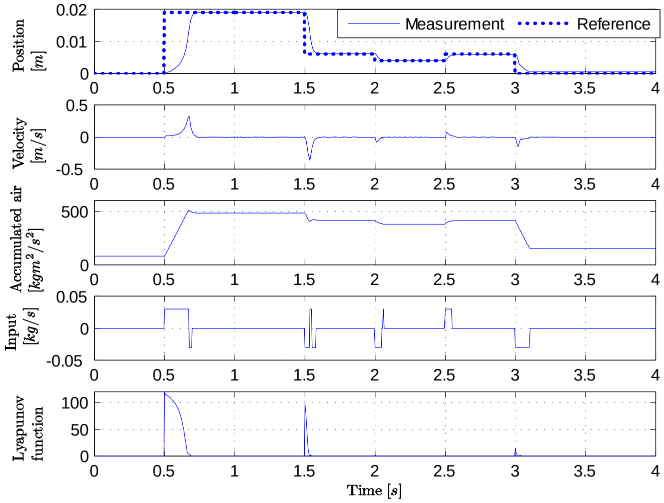


Fig. 3. Switched controller using 2 valves

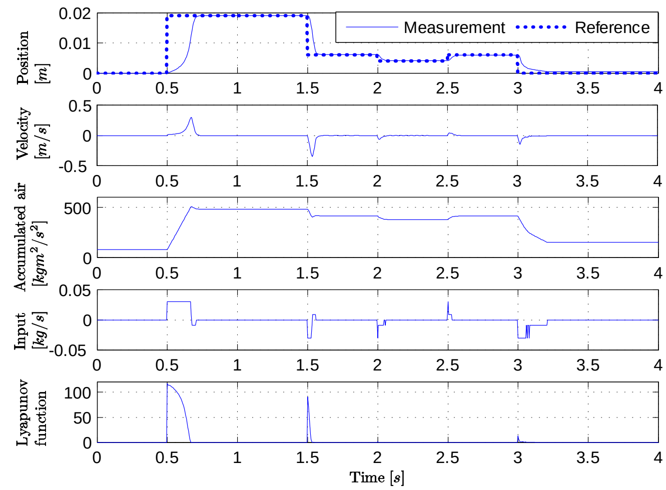


Fig. 4. Switched contol using 4 valves

## C. *Switched control*

Simulation of the system with switched control shows that it behaves as a stable system, see Figure 3, and the requirements to the response and the accuracy are fulfilled. The clutch load acts like a stiff spring, and this is the reason why the position curve make a extra curvature as it approaches the reference point. After this, a larger change of air is needed to make the clutch actuator move towards the desired position.

Figure 4 shows switched control with two sets of on/off valves for control of the flow to the clutch actuator. A set of small and a set of large valves are used, to give more freedom in the controller. The two valves sets have together the same flow capacity as the one consider former. The same switching law is considered, but close to the reference point, only the smallest valve set is used. Again, the value of the Lyapunov function is used as measurement of the distance to the reference point.

The response of the system with backstepping, switched and pwm controller (in Figure 5) are all similar. But the pwm controller needs a much higher number of switches, and the system does not follow the reference points as accurate as with switched and backstepping control. The offsets in the position measurements for the switched and the pwm controller, are mainly due to the fact that we turn off the switching close to the reference point to avoid chattering.

By implementing an extra set of on/off valves we get more freedom in the choice of input. The simulations show that the response time gets slightly longer, but the use of control air is reduced. Hence the choice between using one or two sets of valves, depends on whether the response time or the use of control air is the most important factor.

## D. *Robustness*

To test the robustness of the controller we add white noise with standard deviation $0.0142$ mm to the position

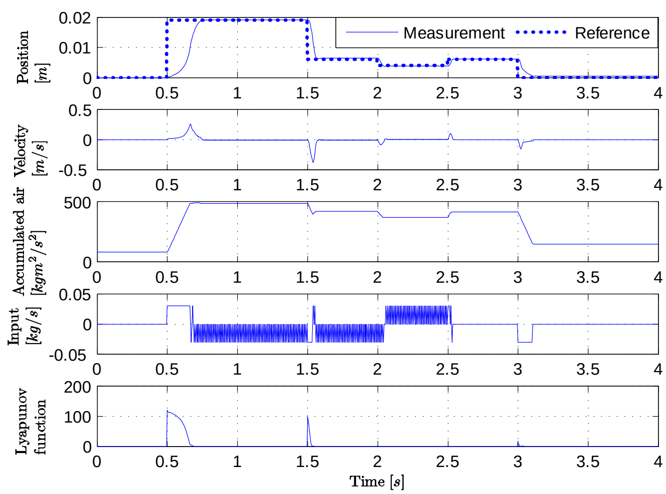


Fig. 5. Pwm-controller

measurements. This noise resembles the actual noise in position measurement from a truck. We also include a more complex friction model in the simulation model, a LuGre model [16]

$$\begin{aligned} f_f(v, y_f) &= Dv + K_f y_f + D_f \dot{y}_f \\ \dot{y}_f &= g_f(v, y_f) = v - \frac{K_f}{F_d}|v| y_f. \end{aligned} \tag{43}$$

This makes the model resemble the actual system better, but can also be seen on as a model error as it is not considered in the design of the controller. The resulting behavior is shown in Figure 6, and we see that the controller still works satisfactory with noise present, but more control air is needed.

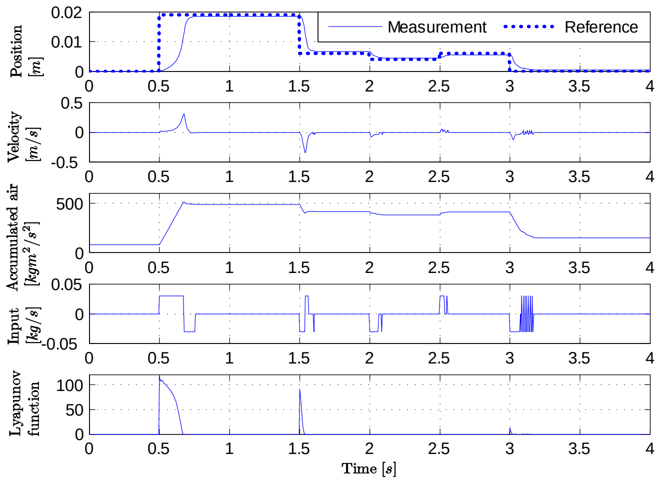


Fig. 6. Switched control when noise is included

## V. Concluding remarks

A control law for a switched pneumatic clutch system using on/off valves was developed. This control law was designed using backstepping theory, and by restricting the on/off valves to be either fully open or fully closed. It was implemented and the simulations show that the switched control is a good alternative for control of the clutch actuator. The controller fulfills the accuracy and response requirements, and seems to be robust to noise. It obtains superior performance compared to the pwm controller for accuracy and less switches are needed.

The design of the switched controller makes it straightforward to include additional valves for allocating flow to the clutch actuator if this is desired. Another advantage of the design is that it does not required exact knowledge of the dynamics of the on/off valves, even though it is useful to have knowledge of valve parameters to calculate opening voltage and appropriate switching period. This is a benefit if valves are to be replaced, since this only will affect the parameters and not the design of the switched controller.

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## Appendix

| **Variable** | **Value** | **Unit** | **Description** |
|---|---|---|---|
| | | | |
| $A$ | $12.3e^{-3}$ | $m^2$ | Actuator area |
| $P_0$ | $1e^5$ | $Pa$ | Ambient pressure |
| $T_0$ | 293 | $K$ | Temperature |
| $R$ | 288 | $\frac{J}{kgK}$ | Gas constant of air |
| $M$ | 10 | $kg$ | Mass of piston |
| $V_0$ | $0.8e^{-3}$ | $m^3$ | Volume at y=0 |
| $D$ | 2000 | $\frac{Ns}{m}$ | Viscous damping |
| $K_l$ | 5000 | $\frac{N}{m}$ | Load characteristic term |
| $L_l$ | 500 | - | Load characteristic term |
| $M_l$ | 25000 | $\frac{N}{m}$ | Load characteristic term |
| $y_{\min}$ | 0 | $m$ | Minimum position |
| $y_{\max}$ | $25e^{-3}$ | $m$ | Maximum position |
| $C$ | $26.7e^{-9}$ | $\frac{m^4 s}{kg}$ | Capacity |
| $\rho_0$ | 1.185 | $\frac{kg}{m^3}$ | Density |
| $p_h$ | $9.5e^5$ | $Pa$ | Supply pressure |
| $k$ | 20 | - | Tuning parameters |
| $b$ | 10 | - | Tuning parameters |
| $\lambda$ | 0.01 | - | Tuning parameters |
| $\beta$ | 1 | - | Tuning parameters |
| $\alpha$ | 400 | - | Tuning parameters |