CONTROL OF THE THREE STATE MOORE-GREITZER COMPRESSOR MODEL USING A CLOSE-COUPLED VALVE

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Abstract

In this paper we propose surge and stall controllers for a close coupled valve in a compression system. A Moore-Greitzer model is presented for the compressor and the valve. The valve modifies the characteristic of the compressor and allows for stable operation beyond the original surge line. The design tool used is backstepping. Under the assumption of no disturbances, global uniform asymptotic stability is proven using feedback from mass flow. In the case of mass flow and pressure disturbances, a damping term is included in the controller, and feedback from mass flow and pressure is employed. Global uniform boundedness is proven. The proposed controllers do not rely on feedback from rotating stall amplitude.

1. Introduction

If the flow through a compressor is throttled to the surge-line, the flow becomes unstable. This instability can take the form of either rotating stall, surge or both. Surge is an axisymmetric oscillation of the flow which reduces the compressor efficiency, and can possibly damage the compressor. Rotating stall is a circumferential variation of the flow which results in a reduced pressure rise. A number of approaches to control of surge and rotating stall have been proposed. A review of the different approaches can be found in [1].

The use of a close-coupled valve (hereafter named CCV) for control of compressor surge was studied in [10]and [11]. Experimental results of compressor surge control using a CCV was reported in [2]. In [11] this strategy was compared, using linear theory, to a number of other possible methods of actuation and sensing. The conclusion was that the most promising methods of surge control is to actuate the system with feedback from the mass flow measurement to a CCV or an injector. Here we will study the use of a CCV as a means of controlling both surge and rotating stall. In order to include the CCV in a model of the compression system, we use the modeling technique of [9]. A three state Moore-Greitzer model including the CCV is presented.

Here we will use backstepping [5] to derive a control law for a CCV which gives a GUAS equilibrium beyond the original surge line. The controller will ensure avoidance of rotating stall as well as surge. When no disturbances is present, an upper bound on the positive slope of the compressor characteristic is the only system parameter required for implementation.

As in [10], disturbances in the pressure rise will be considered and in addition we will also consider disturbances in the plenum outflow. Under mild assumptions on the disturbances, global uniform boundedness will be proven in the presence of both pressure and mass flow disturbances.

Backstepping was used in [6] and [7] to design anti surge and anti stall controllers when the throttle is the control variable. In contrast to this, we use the pressure drop across the CCV as the control variable. In [6] and [7] the controller uses feedback from mass flow and pressure. As will be shown, the application of the backstepping procedure to CCV control, in the case of no disturbances, results in a control law which uses feedback from mass flow only.

As opposed to throttle control, CCV control modifies the compressor characteristic. This allows for, at the cost of a pressure loss over the valve, recovery from rotating stall beyond the surge line. Although the pressure rise achieved in the compression system with a steady pressure drop across the CCV is comparable with the pressure rise achieved when the machine is in rotating stall, the CCV approach is to prefer as blade vibration is avoided. This is due to the fact that when the compressor is in rotating stall, the stall cell(s) are rotating at a fraction of the rotational speed of the rotor, and the blades are moving in and out of the stalled flow [8].

2. Including a CCV in the Moore-Greitzer model

A compressor in series with a CCV will be studied in the following. With close-coupled is understood that the distance between the compressor outlet and the valve is so small that no significant mass storage can take place [10]. The equivalent compressor characteristic is given as

$$\Psi_e(\phi) = \Psi_c(\phi) - \Psi_v(\phi), \tag{1}$$

where $\Psi_c(\phi)$ and $\Psi_v(\phi)$ are the compressor pressure rise and valve pressure drop respectively and ϕ is the axial mass flow coefficient. The CCV has a characteristic given by

$$\Psi_v(\phi) = \frac{1}{\gamma^2}\phi^2, \quad (2)$$

where $\gamma > 0$ is proportional to the valve opening. We now set out to repeat the modeling and Galerkin approximation of [9] with the equivalent characteristic Ψ_e replacing Ψ_c. Equation (5) of [9] which gives the pressure rise across the compressor is modified according to

$$\frac{p_E - p_1}{\rho U^2} = NF(\phi) - \frac{1}{2a}\left(2\frac{\partial\phi}{\partial\xi} + \frac{\partial\phi}{\partial\theta}\right) - \Psi_v(\phi) \quad (3)$$

where p_1 and p_e is the static pressure at the entrance and exit of the compressor, ρ is the density, U is the wheel speed at mean diameter, N is the number of compressor stages, $F(\phi)$ is the pressure rise coefficient in the blade passage, a is the reciprocal time-lag parameter of the blade passage, θ is the angular coordinate around the wheel and ξ is nondimensional time defined as $\xi = Ut/R$ where t is the actual time and R is the mean wheel radius.

Using (3) as a starting point and following [9] the following model is found

$$\dot{\psi} = \frac{W/H}{4B^2}\left(\frac{\phi}{W} - \frac{1}{W}\Phi(\psi)\right)\frac{H}{l_c} \quad (4)$$

$$\dot{\phi} = \frac{H}{l_c}\Bigg(-\frac{\psi-\psi_0}{H} - \frac{1}{2}\left(\frac{\phi}{W}-1\right)^3 + 1 + \frac{3}{2}\left(\frac{\phi}{W}-1\right)\left(1-\frac{J}{2}\right) - \frac{1}{\gamma^2}\left(\frac{W^2 J}{2H} + \frac{\phi^2}{H}\right)\Bigg) \quad (5)$$

$$\dot{J} = J\left(1-\left(\frac{\phi}{W}-1\right)^2 - \frac{J}{4} - \frac{1}{\gamma^2}\frac{4W\phi}{3H}\right)\frac{3aH}{(1+ma)W}, \quad (6)$$

where ψ, ϕ and J is the pressure rise coefficient, axial flow coefficient and square of amplitude of angular disturbance (rotating stall) of the axial flow coefficient respectively, B is Greitzer's B-parameter and

$$\Phi(\psi) = \gamma_T\sqrt{\psi} \quad (7)$$

is the throttle characteristic. For a definition of the parameters in the model, see [9]. Differentiation denoted as $(\dot{\ })$ is wrt ξ. The cubic compressor characteristic

$$\Psi_c(\phi) = \psi_{c0} + H\left(1+\frac{3}{2}\left(\frac{\phi}{W}-1\right) - \frac{1}{2}\left(\frac{\phi}{W}-1\right)^3\right), \quad (8)$$

where the parameters $\psi_{c0} > 0$, $H > 0$ and $W > 0$ are defined in [9] has been used. The nondimensionalization employed causes the usual family of constant speed lines in the compressor map to collapse into the single curve given in equation (8). The surge line, which passes through the local maxima of the constant speed lines, is reduced to the local maximum point of $\Psi_c(\phi)$.

The compressor is in equilibrium when $\dot{\phi} = \dot{\psi} = \dot{J} = 0$. If $J(0) = 0$ then $J \equiv 0$ and the equilibrium values ϕ_0 and ψ_0 are given by the intersection of $\Psi_e(\phi)$ and the throttle characteristic. If $J(0) > 0$, and the throttle characteristic crosses Ψ_e to the left of the local maximum, the compressor may[1] enter rotating stall and the equilibrium values ϕ_0 and ψ_0 are given by the intersection of $\Psi_e(\phi)$ and the stall characteristic $\Psi_{es}(\phi)$ which is found by analyzing (6). It is seen that $\dot{J} = 0$ is satisfied for $J = 0$ or

$$J = J_e = 4\left(1 - \left(\frac{\phi}{W}-1\right)^2 - \frac{1}{\gamma^2}\frac{4W\phi}{3H}\right). \quad (9)$$

Inserting (9) in (5) and setting $\dot{\phi} = 0$ gives the expression for $\Psi_{es}(\phi)$:

$$\Psi_{es}(\phi) = \Psi_s(\phi) + \frac{5}{H}\Psi_v(\phi) - \frac{8W}{H\gamma^2}\left(1 - \frac{W^2}{3H^2\gamma^2}\right)\phi, \quad (10)$$

where

$$\Psi_s(\phi) = \psi_{c0} + H\left(1 - \frac{3}{2}\left(\frac{\phi}{W}-1\right) + \frac{5}{2}\left(\frac{\phi}{W}-1\right)^3\right) \quad (11)$$

is the stall characteristic found when the CCV is not present. In figure 1 the various characteristics are shown. As can be seen, the throttle line crosses Ψ_c in the unstable area, and the compressor would go into rotating stall or surge. By introducing the CCV, the throttle line crosses the equivalent characteristic Ψ_e in an area of negative slope. This new equilibrium is thus stable.

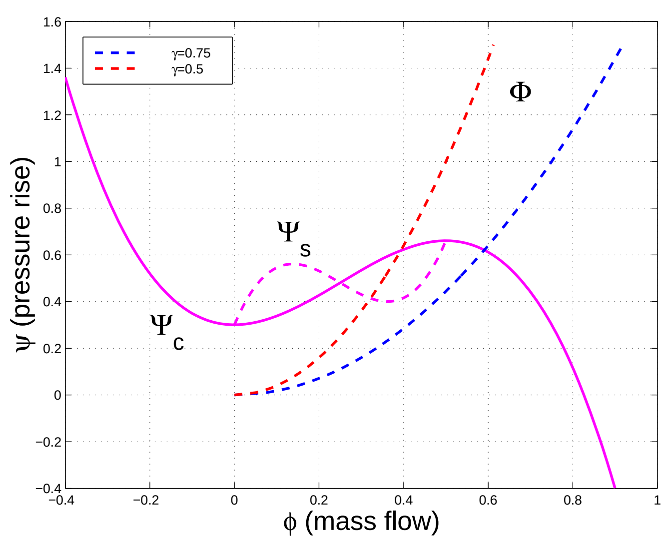

Figure 1: Compressor, CCV and throttle characteristics. The throttle gain is $\gamma_T = 0.61$. The CCV-gain is $\gamma = 1.81$. Other parameters are taken from [9]

[1] This depends on the numerical value of B. Small B gives rotating stall, and large B gives surge [9].

Notice that ignoring the terms including the CCV gain γ in the above equations results in the model of [9]. Notice also in that the CCV introduces additional damping in the $\dot{J}$-equation, see equation (6).

This motivates for designing the CCV-pressure drop such that squared rotating stall amplitude J can be stabilized at $J = 0$. It turns out by examining equation (6) that the required pressure drop to render the bracketed part negative, and thus ensure that $\dot{J} < 0$, is given by

$$\Psi_v(\phi) \geq \frac{3H}{4W}\left(1 - \left(\frac{\phi}{W} - 1\right)^2\right)\phi. \tag{12}$$

In calculating (12) it was assumed that $\phi > 0$, which is not restrictive as rotating stall only occurs for forward flow.

A controller that enables the compressor to operate on both sides of the peak of the characteristic without going into surge or rotating stall is now to be designed.

3. Change of coordinates

In order to simplify the analysis a change of coordinates is performed [10] :

$$\left.\begin{array}{rcl} \hat{\phi} & = & \phi - \phi_0 \\ \hat{\psi} & = & \psi - \psi_0 \\ \hat{\Psi}_e(\hat{\phi}) & = & \Psi_e(\hat{\phi} + \phi_0) - \Psi_e(\phi_0) \\ \hat{\Psi}_c(\hat{\phi}) & = & \Psi_c(\hat{\phi} + \phi_0) - \Psi_c(\phi_0) \\ \hat{\Phi}(\hat{\psi}) & = & \Phi(\hat{\psi} + \psi_0) - \Phi(\psi_0), \end{array}\right\} \tag{13}$$

Using (13) on the model (4)-(6) leads to the transformed equations

$$\dot{\hat{\psi}} = \frac{1}{4B^2 l_c}\left(\hat{\phi} - \hat{\Phi}(\hat{\psi})\right) \tag{14}$$

$$\dot{\hat{\phi}} = \frac{1}{l_c}\left(-\hat{\psi} + \hat{\Psi}_e(\hat{\phi}) - \frac{3H}{4}J\left(\frac{\phi}{W} - 1\right) - \frac{W^2 J}{2\gamma^2}\right) \tag{15}$$

$$\dot{J} = \sigma J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{J}{4}\right) - \frac{4W\sigma}{3H\gamma^2}J\phi, \tag{16}$$

where

$$\hat{\Phi}(\hat{\psi}) = \gamma_T\sqrt{\hat{\psi} + \psi_0} - \gamma_T\sqrt{\psi_0}, \tag{17}$$

$$\hat{\Psi}_c(\hat{\phi}) = -k_3\hat{\phi}^3 - k_2\hat{\phi}^2 - k_1\hat{\phi}, \tag{18}$$

$\sigma = \frac{3aH}{(1+ma)W}$, $k_1 = \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)$, $k_2 = \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)$ and $k_3 = \frac{H}{2W^3}$. Obviously $k_3 > 0$, while $k_1 \leq 0$ if the equilibrium is in the unstable region of the compressor map and $k_1 > 0$ otherwise. The sign of k_2 may vary.

4. Controller design using backstepping

Our aim will be to design a controller $u = u(\phi)$ such that the compressor can be operated in the previous unstable area of the compressor map without going into rotating stall or surge oscillations. It will be a point to keep the sensing requirements as low as possible, that is avoid sensing of stall amplitude. The control variable is chosen as the CCV pressure drop:

$$u = \hat{\Psi}_v(\hat{\phi}). \tag{19}$$

The backstepping methodology of [5] is now employed to design a controller for (14)-(16).

Step 1. The two error variables z_1 and z_2 are defined as

$$z_1 = \hat{\psi} \text{ and } z_2 = \hat{\phi} - \alpha. \tag{20}$$

The control Lyapunov function (clf) for this step is chosen as

$$V_1 = 2B^2 l_c z_1^2 \tag{21}$$

with time derivative

$$\dot{V}_1 = z_1\left(-\hat{\Phi}(z_1) + z_2 + \alpha\right). \tag{22}$$

The load is assumed passive, that is $\hat{\psi}\hat{\Phi}(\hat{\psi}) \geq 0 \ \forall \hat{\phi}$. We have

$$\hat{\psi}\hat{\Phi}(\hat{\psi}) \geq 0 \ \Rightarrow \ -z_1\hat{\Phi}(z_1) \leq 0 \tag{23}$$

As it is desirable to avoid cancelation of useful nonlinearities in (22), the stabilizing function α is not needed and accordingly $\alpha = 0$, which gives

$$\dot{V}_1 = -\hat{\Phi}(z_1)z_1 + z_1 z_2. \tag{24}$$

Although $\alpha = 0$ here, the notation of z_1 and z_2 is kept in the interest of consistency with section 5.

Step 2. The derivative of z_2 is

$$\dot{z}_2 = \frac{1}{l_c}\left(-z_1 + \hat{\Psi}_c(z_2) - \frac{3H}{4}J\left(\frac{\phi}{W} - 1\right) - \frac{W^2 J}{2\gamma^2} - u\right). \tag{25}$$

The clf for this step is

$$V_2 = V_1 + \frac{l_c}{2}z_2^2 + \frac{J}{\sigma}, \tag{26}$$

and $\dot{V}_2$ is calculated as

$$\dot{V}_2 = -z_1\hat{\Phi}(z_1) + z_2(\hat{\Psi}_c(z_2) - u) + J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{J}{4} - \frac{1}{\gamma^2}\frac{4W}{3H}\phi - \frac{3H}{4}\left(\frac{\phi}{W} - 1\right)z_2 - \frac{W^2 J}{2\gamma^2}z_2\right). \tag{27}$$

By choosing u according to

$$u = (c_2 + \delta)z_2, \tag{28}$$

where $c_2 > 0$ and $\delta > 0$ are constants, $\dot{V}_2$ can be written

$$\dot{V}_2 = \sum_{i=1}^{4}\left(\dot{V}_2\right)_i. \tag{29}$$

The four terms in (29) are

$$\left(\dot{V}_2\right)_1 = -z_1\hat{\Phi}(z_1), \tag{30}$$

$$\left(\dot{V}_2\right)_2 = z_2(\hat{\Psi}_c(z_2) - c_2 z_2), \tag{31}$$

$$\left(\dot{V}_2\right)_3 = J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{1}{\gamma^2}\frac{4W}{3H}\phi\right), \tag{32}$$

$$\left(\dot{V}_2\right)_4 = -\left[\begin{array}{cc} J & z_2 \end{array}\right] \boldsymbol{P}(\phi) \left[\begin{array}{c} J \\ z_2 \end{array}\right]. \tag{33}$$

Due to the passivity of the throttle, $\left(\dot{V}_2\right)_1 < 0$. As shown in [3], if c_2 is chosen as

$$c_2 > a_m \geq a > \frac{k_2^2}{4k_3} - k_1, \tag{34}$$

where a_m is the maximum positive slope of the compressor characteristic, then $\left(\dot{V}_2\right)_2 < 0$. Provided (12) is satisfied, $\left(\dot{V}_2\right)_3 < 0$. Finally, δ can always be chosen so that

$$\det \boldsymbol{P} = \frac{\delta}{4} - \left(\frac{3H}{8}\left(\frac{\phi}{W} - 1\right) + \frac{W^2}{4\gamma^2}\right)^2 > 0, \tag{35}$$

and consequently $\left(\dot{V}_2\right)_4 < 0$. The matrix $\boldsymbol{P}(\phi)$ has the form

$$\boldsymbol{P}(\phi) = \left[\begin{array}{cc} \frac{1}{4} & \frac{3H}{8}\left(\frac{\phi}{W} - 1\right) + \frac{W^2}{4\gamma^2} \\ \frac{3H}{8}\left(\frac{\phi}{W} - 1\right) + \frac{W^2}{4\gamma^2} & \delta \end{array}\right].$$

Choosing c_2 and δ so that (34) and (35) are satisfied, $\dot{V}_2$ is upper bounded as

$$\dot{V}_2 \leq -U(z_1, z_2, J), \tag{36}$$

where $U(z_1, z_2, J)$ is a radially unbounded pdf. Consequently the origin of the system system is GUAS. By combining eqs (19) and (28) the control law for the CCV-gain is found:

$$\gamma = \sqrt{\frac{\phi + \phi_0}{(c_2 + \delta)}}. \tag{37}$$

Notice that this control law requires sensing of mass flow ϕ only.

5. Disturbances

In a real compression system there will be disturbances. In [10] the effect of pressure disturbances on the two state Greitzer model [4] was studied. Here we will derive a controller for the three state Moore-Greitzer model in the presence of both time varying flow disturbances $\hat{\Phi}_d(t)$ and pressure disturbances $\hat{\Psi}_d(t)$. When regarding the Moore-Greitzer model as a model of a jet engine, $\hat{\Psi}_d(t)$ would correspond to flow field disturbances caused e.g. by large angle of attack, and $\hat{\Phi}_d(t)$ to combustion induced fluctuations in outlet pressure [1]. In the presence of these disturbances the differential equations describing pressure and mass flow can be written

$$\dot{\hat{\psi}} = \frac{1}{4B^2 l_c}\left(\hat{\phi} - \hat{\Phi}(\hat{\psi}) - \hat{\Phi}_d(t)\right) \tag{38}$$

$$\dot{\hat{\phi}} = \frac{1}{l_c}\left(-\hat{\psi} + \hat{\Psi}_e(\hat{\phi}) + \hat{\Psi}_d(t) - \frac{3HJ}{4}\left(\frac{\phi}{W} - 1\right) - \frac{W^2 J}{2\gamma^2}\right) \tag{39}$$

$$\dot{J} = \sigma J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{J}{4}\right) - \frac{4W\sigma}{3H\gamma^2}J\phi \tag{40}$$

where u is defined according to (19). The disturbances are assumed to be bounded, that is $\|\hat{\Phi}_d\|_\infty$ and $\|\hat{\Psi}_d\|_\infty$ exists.

The backstepping procedure is now used to design a controller that ensures boundedness of the states in the presence of disturbances. To accomplish this, damping terms are included in the controller.

Step 1. As in the previous section, the two error variables z_1 and z_2 are defined as

$$z_1 = \hat{\psi} \text{ and } z_2 = \hat{\phi} - \alpha. \tag{41}$$

The control Lyapunov function (clf) for this step is chosen as

$$V_1 = 2B^2 l_c z_1^2 \tag{42}$$

with time derivative

$$\dot{V}_1 = z_1\left(-\hat{\Phi}(z_1) + z_2 - \hat{\Phi}_d(t) + \alpha\right). \tag{43}$$

The load is again assumed passive, see equation (23). The virtual control α is chosen as

$$\alpha = -d_1 z_1, \tag{44}$$

where $-d_1 z_1$ is a damping term to be used to counteract the disturbance $\hat{\Phi}_d(t)$. $\dot{V}_1$ can now be upper bounded according to

$$\dot{V}_1 \leq -\hat{\Phi}(z_1)z_1 + z_1 z_2 + \frac{\|\hat{\Phi}_d\|_\infty^2}{4d_1}. \tag{45}$$

To obtain the bound in (45) Young's inequality has been used to obtain

$$-\hat{\Phi}_d(t)z_1 \leq d_1 z_1^2 + \frac{\|\hat{\Phi}_d\|_\infty^2}{4d_1}. \tag{46}$$

Step 2. The LFK for this step is chosen as

$$V_2 = V_1 + \frac{l_c}{2}z_2^2 + \frac{J}{\sigma} \tag{47}$$

The derivative of z_2 is

$$\dot{z}_2 = \dot{\hat{\phi}} - \frac{\partial\alpha}{\partial z_1}\dot{z}_1$$

$$\dot{z}_2 = \frac{1}{l_c}\left(-z_1 + \hat{\Psi}_c(\hat{\phi}) + \hat{\Psi}_d(t) - \frac{3HJ}{4}\left(\frac{\phi}{W} - 1\right) - \frac{W^2 J}{2\gamma^2} - u\right) + d_1\frac{1}{4B^2 l_c}\left(\hat{\phi} - \hat{\Phi}(z_1) - \hat{\Phi}_d(t)\right) \tag{48}$$

Using (45) and (48), $\dot{V}_2$ can now be calculated and upper bounded as

$$\begin{aligned}\dot{V}_2 \ \le\ & -\hat{\Phi}(z_1)z_1 + \frac{\|\hat{\Phi}_d\|_\infty^2}{4d_1} + z_2\left(\hat{\Psi}_c(\hat{\phi}) + \hat{\Psi}_d(t) - u\right)\\ & + z_2\frac{d_1}{4B^2}\left(-\hat{\Phi}(z_1) + \hat{\phi} - \hat{\Phi}_d(t)\right)\\ & + J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{1}{\gamma^2}\frac{4W}{3H}\phi\right)\\ & - \frac{J^2}{4} - \frac{3H}{4}\left(\frac{\phi}{W} - 1\right)z_2 - \frac{W^2 J}{2\gamma^2}z_2 \end{aligned} \quad (49)$$

Control law

To counteract the effect of the disturbances, another damping factor d_2 must be included, and u is chosen as

$$\begin{aligned} u \ =\ & (c_2 + \delta)z_2 - k_3\left(\alpha^3 + 3\alpha z_2^2\right) + d_2 z_2\left(1 + \frac{d_1^2}{4B^2}\right)\\ & - k_2\hat{\phi}^2 - k_1\alpha + \frac{d_1}{4B^2}\left(-\hat{\Phi}(z_1) + \hat{\phi}\right), \end{aligned} \quad (50)$$

where c_2 satisfies

$$c_2 > |k_1|. \quad (51)$$

Notice that this control law requires knowledge of the throttle characteristic, the coefficients in the compressor characteristic and the B-parameter. In addition feedback from pressure is needed. Inserting (50) in (49) gives the bound

$$\begin{aligned}\dot{V}_2 \ \le\ & -(c_2 + k_1)z_2^2 - k_3(z_2^4 + 3\alpha^2 z_2^2) - \hat{\Phi}(z_1)z_1\\ & + |z_2|\|\hat{\Psi}_d\|_\infty + \frac{d_1}{4B^2}|z_2|\|\hat{\Phi}_d\|_\infty - \frac{d_1^2}{4B^2}d_2 z_2^2\\ & + \frac{\|\hat{\Phi}_d\|_\infty^2}{4d_1} - d_2 z_2^2 - \left[\begin{array}{cc} J & z_2 \end{array}\right]\boldsymbol{P}(\phi)\left[\begin{array}{c} J \\ z_2 \end{array}\right]\\ & + J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{1}{\gamma^2}\frac{4W}{3H}\phi\right), \end{aligned} \quad (52)$$

Using Young's inequality twice gives

$$|z_2|\|\hat{\Psi}_d\|_\infty \ \le\ d_2 z_2^2 + \frac{\|\hat{\Psi}_d\|_\infty^2}{4d_2} \quad (53)$$

$$\frac{d_1}{4B^2}|z_2|\|\hat{\Phi}_d\|_\infty \ \le\ \frac{1}{4B^2}\left(d_1^2 d_2 z_2^2 + \frac{\|\hat{\Phi}_d\|_\infty^2}{4d_2}\right). \quad (54)$$

The final upper bound for V_2 can now be written as

$$\dot{V}_2 \le -U(z_1, z_2, J) + \frac{1}{\kappa_1}\|\hat{\Phi}_d\|_\infty^2 + \frac{1}{\kappa_2}\|\hat{\Psi}_d\|_\infty^2 \quad (55)$$

where

$$\frac{1}{\kappa_1} = \left(\frac{1}{4d_1} + \frac{1}{16B^2 d_2}\right), \ \frac{1}{\kappa_2} = \frac{1}{4d_2}, \quad (56)$$

and

$$\begin{aligned} U(z_1, z_2, J) \ =\ & (c_2 + k_1)z_2^2 + k_3(z_2^4 + 3\alpha^2 z_2^2) + \hat{\Phi}(z_1)z_1\\ & + \left[\begin{array}{cc} J & z_2 \end{array}\right]\boldsymbol{P}(\phi)\left[\begin{array}{c} J \\ z_2 \end{array}\right]\\ & + J\left(1 - \left(\frac{\phi}{W} - 1\right)^2 - \frac{1}{\gamma^2}\frac{4W}{3H}\phi\right) \end{aligned} \quad (57)$$

is radially unbounded and positive definite, provided δ is chosen according to (35), c_2 is chosen according to (51), and $\Psi_v(\phi)$ satisfies (12). This implies that $\dot{V}_2 < 0$ outside a set $\mathcal{R}_2$. According to [5] the fact that $V_2(z_1, z_2, J)$ and $U(z_1, z_2, J)$ is positive definite and radially unbounded, and $V_2(z_1, z_2, J)$ is smooth implies that there exists class-$\mathcal{K}_\infty$ functions β_1, β_2 and β_3 such that

$$\left.\begin{array}{l}\beta_1(|\boldsymbol{z}|) \le V_2(\boldsymbol{z}) \le \beta_2(|\boldsymbol{z}|)\\ \beta_3(|\boldsymbol{z}|) \le U(\boldsymbol{z})\end{array}\right\} \quad (58)$$

where $\boldsymbol{z} = (z_1 \ \ z_2 \ \ J)^{\mathrm{T}}$. This implies that $\boldsymbol{z}(t)$ is globally uniformly bounded and that $\boldsymbol{z}(t)$ converges to the set

$$\mathcal{R} = \left\{\boldsymbol{z} : |\boldsymbol{z}| \le \beta_1^{-1} \circ \beta_2 \circ \beta_3^{-1}\left(\frac{\|\hat{\Psi}_d\|_\infty^2}{\kappa_1} + \frac{\|\hat{\Phi}_d\|_\infty^2}{\kappa_2}\right)\right\}.$$

It should be noted here that once the bounds on the disturbances $\|\hat{\Phi}_d\|_\infty$ and $\|\hat{\Psi}_d\|_\infty$ are known, the size of the set $\mathcal{R}$ can be made arbitrary small by choosing the damping factors d_1 and d_2 sufficiently large.

Also note that if , as in [10], only pressure disturbances $\hat{\Phi}_d(t)$ are considered, the simple controller (28) is sufficient, provided a damping term $d_2 z_2$ is added to u.

6. Simulations

In this section some simulation results of the proposed controllers are presented. In figure 2 the response of system (4)-(6) with controller (28) is shown. The throttle gain is set at $\gamma_T = 0.61$, resulting in an unstable equilibrium for the unactuated system. The B-parameter is set to $B = 0.5$. This causes the compressor to go into rotating stall, and J increases until J_e is reached. At $\xi = 100$ the controller is switched on, and J decreases until $J = 0$ is reached. The controller parameters were $(c_2 + \delta) = 1$. As can be seen in the lower right plot of figure 2, there is a steady pressure drop across the CCV.

In figure 3, the B-parameter has been set at $B = 1.9$. As can be seen, the compressor is undergoing severe deep surge oscillations. At $\xi = 200$ the controller (28) is switched on, and the compressor is stabilized. The throttle is unchanged, and the controller parameters were $(c_2 + \delta) = 0.75$.

The effect of disturbances in mass flow and pressure is shown in figure 4. The B-parameter is $B = 0.5$ and the compressor stalls. The pressure disturbance and mass flow disturbance are both white noise of amplitude 0.1. The control law (50) with parameters $(c_2 + \delta) = 0.3$, $d_1 = 0.1$ and $d_2 = 0.05$ is switched on at $\xi = 100$, bringing the compressor out of rotating stall, and damping the disturbances.

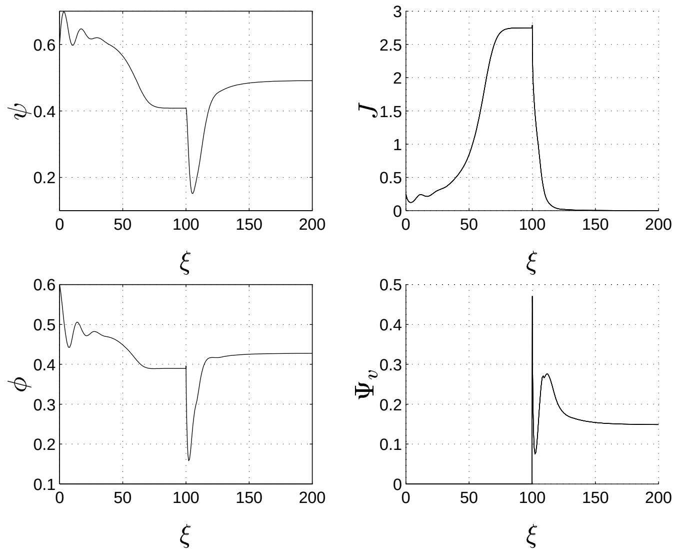

Figure 2: Stabilization of rotating stall

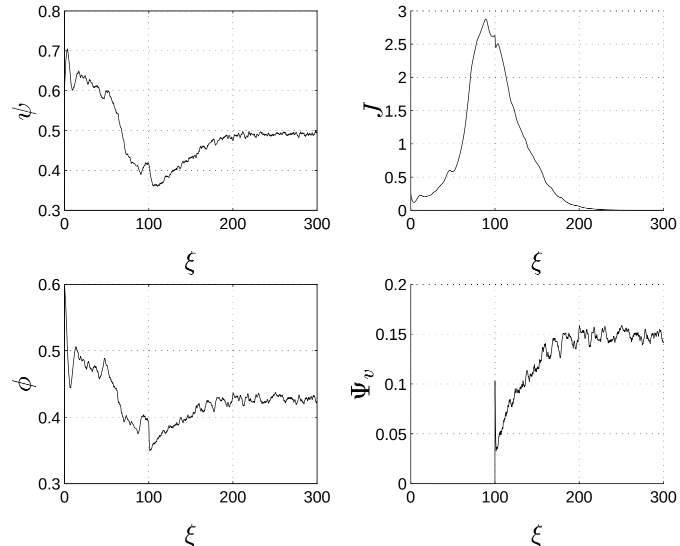

Figure 4: Stabilization of rotating stall with mass flow and pressure disturbances

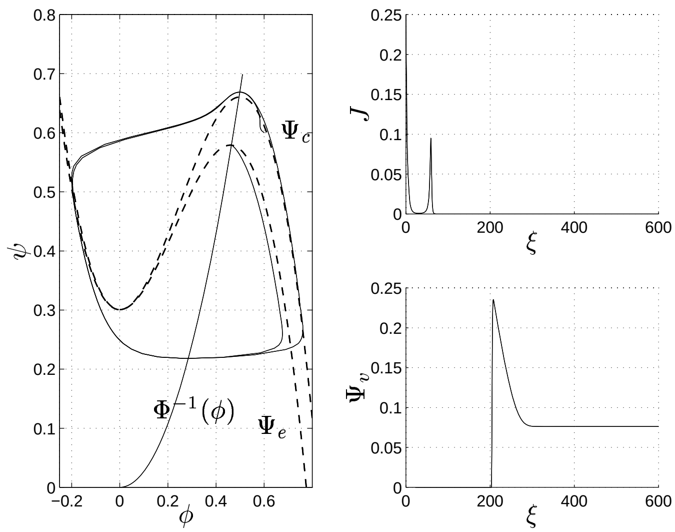

Figure 3: Stabilization of surge

7. Conclusion

Controllers for a close-coupled valve in series with a compressor has been presented. Compressor and valve is described by a three state Moore-Greitzer model. The controllers make it possible to operate the compressor beyond the surge line. Without disturbances a GUAS equilibrium point is ensured, and in the presence of mass flow and pressure disturbances global uniform boundedness is proven.

References

[1] B. de Jager. Rotating stall and surge control : A survey. In *Proceedings of the 35th Conference on Decision and Control*, pages 1857–1862, New Orleans, LA, 1995.

[2] J.L. Dussourd, G.W. Pfannebecker, and S.K. Singhania. An experimental investigation of the control of surge in radial compressors using close coupled resistances. *Journal of Fluids Engineering*, 99:64–76, 1977.

[3] J.T. Gravdahl and O. Egeland. Compressor surge control using a close-coupled valve and backstepping. In *Proceedings of the 1997 American Control Conference*, pages ??–??, Albuquerque, NM, June 1997.

[4] E.M. Greitzer. Surge and Rotating stall in axial flow compressors, Part I: Theoretical compression system model. *Journal of Engineering for Power*, 98:190–198, 1976.

[5] M. Krstić, I. Kanellakopoulos, and P.V. Kokotović. *Nonlinear and Adaptive Control Design*. John Wiley & Sons Inc., 1995.

[6] M. Krstić and P.V. Kokotović. Lean backstepping designs for jet engine compressor model. In *Proceedings of the 4th IEEE Conference on Control Applications*, pages 1047–1052, 1995.

[7] M. Krstić, J.M. Protz, J.D. Paduano, and P.V. Kokotović. Backstepping designs for jet engine stall and surge control. In *Proceedings of the 35th Conference on Decision and Control*, pages 3049–3055, New Orleans, LA, 1995.

[8] F.E. McCaughan. Bifurcation analysis of axial flow compressor stability. *SIAM Journal of Applied Mathematics*, 50(5):1232–1253, 1990.

[9] F.K. Moore and E.M. Greitzer. A theory of post-stall transients in a axial compressor systems: Part I–Development of equations. *Journal of Engineering for Gas Turbines and Power*, 108:68–76, 1986.

[10] J.S. Simon and L. Valavani. A Lyapunov based nonlinear control scheme for stabilizing a basic compression system using a close-coupled control valve. In *Proceedings of the 1991 American Control Conference*, pages 2398–2406, 1991.

[11] J.S. Simon, L. Valavani, A.H. Epstein, and E.M. Greitzer. Evaluation of approaches to active compressor surge stabilization. *Journal of Turbomachinery*, 115:57–67, 1993.