Compressor surge control using a close-coupled valve and backstepping

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Abstract

In this paper we propose anti surge controllers for a close coupled valve in a compression system. The valve modifies the characteristic of the compressor, and allows for stable operation beyond the original surge line. The design tool used is backstepping and global uniform asymptotic stability is proven. Damping terms are included in the controllers, and in the presence of both mass flow and pressure disturbances, global uniform boundedness and convergence to a set is ensured. Under the assumption of decaying disturbances the controller ensures convergence to the origin.

1. Introduction

Compressor surge occurs if the flow is throttled beyond the surge line. Dependent on the system geometry the instability can take the form of either rotating stall, surge or both. In this paper we focus on surge which is an axisymmetric oscillation of the flow. These oscillations severely reduces the compressor efficiency and can possibly damage the compressor. A number of approaches to control of surge (and rotating stall) have been proposed. A review of the different approaches can be found in [1].

The use of a close-coupled valve (CCV) for control of compressor surge was studied in [8], and experimental results of compressor surge control using a CCV was reported in [2]. In [9], this strategy was compared, using linear theory, to a number of other possible methods of actuation and sensing. The conclusion was that the most promising methods of surge control is to actuate the system with feedback from the mass flow measurement to a CCV or an injector. Here we will study the use of a CCV.

In [8] the stability of a compressor with CCV control was studied using a Lyapunov function termed the incremental energy. The control law developed in [8] requires knowledge of the compressor characteristic, and additional adjustments to the controller dictated by the Lyapunov analysis is performed in order to avoid a discontinuous controller.

Here we will use backstepping [4] to derive a control law for a CCV which gives a GUAS equilibrium beyond the original surge line. As in [8], disturbances in the pressure rise will be considered and in addition we will also consider disturbances in the plenum outflow. In the case of only pressure disturbances, we will derive a controller that only requires knowledge of an upper bound on the slope of the compressor characteristic in order to guarantee stability. Discontinuities is not a problem with this controller. Under mild assumptions on the disturbances, global uniform boundedness and convergence will be proven in the presence of both pressure and mass flow disturbances.

Backstepping was used in [5] and [6] to design anti surge and anti stall controllers when the throttle is the control variable. Here, we use the pressure drop across the CCV as the control variable. In [5] and [6] the controller uses feedback from mass flow and pressure. As will be shown, the application of the backstepping procedure to CCV control, in the case of no mass flow disturbances, results in a control law which uses feedback from mass flow only.

2. Compressor and throttle model

The differential equations describing pressure and mass flow oscillations in a compressor-plenum-throttle system is found in [3]. The model is

\begin{align}
\dot{\phi} &= B(\Psi_c(\phi) - \psi) \\
\dot{\psi} &= \frac{1}{B}(\phi - \Phi(\psi)),
\end{align}

where \(\phi\) is the mass flow coefficient (annulus averaged, axial velocity divided by wheel speed, [7]), \(\psi\) is the non dimensional plenum pressure or pressure coefficient (pressure divided by density and the square of wheel speed), \(\Phi(\psi)\) is the throttle characteristic and \(B^1\) is the “B-parameter” defined in [3]. The time variable \(t\) used throughout this text is also nondimensional, and is referred to time for the wheel to rotate one radian.

The compressor characteristic can be modelled as [7],

\[\Psi_c(\phi) = \psi_0 + H \left[1 + \frac{3}{2} \left(\phi - 1\right) - \frac{1}{2} \left(\phi - 1\right)^3\right],\]

where the parameters \(\psi_0 > 0, H > 0\) and \(W > 0\) are defined in [7]. The throttle mass flow \(\Phi(\psi)\) is given by the throttle characteristic

\[\Phi(\psi) = \gamma \sqrt{\psi},\]

where \(\gamma\) is the throttle gain. The compressor is in equilibrium when \(\psi = \phi = 0\). The steady state values of mass flow \(\phi_0\) and plenum pressure \(\psi_0\) are found from the intersection of the throttle characteristic with the compressor characteristic as shown in figure 1. As the compressor is throttled, that is, as \(\gamma\) is decreased, the equilibrium point moves along the compressor characteristic towards lower values of \(\phi\). This is shown in figure 1 for \(\gamma = 0.5\) and \(\gamma = 0.65\).

![Figure 1: Compressor and throttle characteristic.](image)

\[1B = U \frac{a_s}{v_s} \sqrt{\frac{V_p}{A_e L_c}},\]

where \(U\) is compressor speed, \(a_s\) is the speed of sound, \(V_p\) is the plenum volume, \(A_e\) is the flow area and \(L_c\) is the length of ducts and compressor.
The nondimensionalization employed, transforms the usual family of curves in the compressor map, one for each compressor speed, to one single characteristic given by (2). The surge line, which passes through the local maxima of the family of curves is transformed to the local maximum of (2). Equilibria to the right of this local maximum are stable, and equilibria to the left are unstable. That is, if the throttle line crosses the compressor characteristic in an area of positive slope, the compressor will go into surge. The objective of this paper is to design control laws that stabilizes these unstable equilibria.

3. Actuation

A compressor in series with a CCV will be studied in the following. The system is shown in figure 3. With close-coupled throttle and valve the compressor outlet will go into surge. The model of [3] can now be written as

\[ \dot{\Psi} = B(\Psi_c(\phi) - \psi) \]
\[ \dot{\psi} = \frac{1}{B}(\phi - \Phi(\psi)) \]

as the compressor in series with the valve is treated as an equivalent compressor.

4. Change of coordinates

In order to simplify the analysis, the approach of [8] is followed where the system is transformed so that the origin becomes the equilibrium under study. This is done by a change of variables to

\[ \dot{\phi} = \dot{\ Psi} - \phi_0 \]
\[ \dot{\psi} = \psi - \psi_0 \]
\[ \dot{\Psi}_c(\phi) = \Psi_c(\phi + \phi_0) - \Psi_c(\phi_0) \]
\[ \dot{\Psi}_v(\phi) = \Psi_v(\phi + \phi_0) - \Psi_v(\phi_0) \]
\[ \dot{\Phi}(\psi) = \Phi(\psi + \psi_0) - \Phi(\psi_0) \]

where \( \phi_0, \psi_0 \) now is the equilibrium values of \( \phi, \psi \) in (5). Applying the transformations (6) to the model (5), and using (2) and (3) results in the transformed equations

\[ \dot{\Psi} = 1 \frac{1}{B}(\phi - \Psi(\psi)) \]
\[ \dot{\phi} = B(\dot{\Psi}_c(\phi) - \Psi - u) \]

where

\[ \dot{\Psi}_c(\phi) = \gamma \sqrt{\psi + \psi_0 - \gamma \sqrt{\psi_0}} \]
\[ \dot{\Psi}_v(\phi) = -k_3\phi^3 - k_2\phi^2 - k_1\phi, \]

5. Backstepping

The backstepping methodology of [4] will now be employed in designing a control law for the CCV.

**Step 1.** Two error variables are defined as \( z_1 = \dot{\psi} \) and \( z_2 = \phi - \alpha \). The control Lyapunov function (clf) for this step is chosen as

\[ V_1 = \frac{B}{2} z_1^2 \]

with time derivative

\[ \dot{V}_1 = z_1(-\dot{\Psi}(\phi) + z_2 + \alpha) \]

The load is assumed passive, that is \( \dot{\Psi}(\phi) \geq 0 \) \forall \phi \). We have

\[ \ddot{\Psi}(\phi) \geq 0 \Rightarrow -z_1\dot{\Psi}(\phi) \leq 0 \]

As it is desirable to avoid cancellation of useful nonlinearities in (12), the stabilizing function \( \alpha \) is not needed and accordingly \( \alpha = 0 \), which gives

\[ \dot{V}_1 = -\dot{\Psi}(\phi) z_1 + z_2. \]

Although the virtual control \( \alpha \) is not needed here, in the interest of consistency with the following sections this notation is kept.

**Step 2.** The derivative of \( z_2 \) is

\[ \dot{z}_2 = B\dot{\Psi}_c(\phi) - Bz_1 - Bu. \]

The clf for this step is

\[ V_2 = V_1 + \frac{1}{2B} z_2^2 \]

with derivative

\[ \dot{V}_2 = -z_1\dot{\Psi}_c(\phi) + z_2 \left( \dot{\Psi}_v(\phi) - u \right) \]

Notice that \( V_2 \) as defined by (16) is identical to the incremental energy of [8].

**Control law.** The control variable \( u \) will be chosen so that (17) is made negative definite. To this end we define the linear control law

\[ u = c_2 z_2, \]

where the controller gain \( c_2 > 0 \) is chosen so that

\[ z_2 \Psi_c(\phi) - c_2 z_2^2 < 0 \]

Using (10) this implies that \( c_2 \) must satisfy

\[ -k_3 z_2^2 + \frac{k_2}{k_3} z_2 + \frac{k_1 + c_2}{k_3} < 0. \]

Finding the roots of the above bracketed expression, it is seen that (20) is satisfied if \( c_2 \) is chosen according to

\[ c_2 > \frac{k_2^2}{4k_3} - k_1. \]

Although (21) implies that the compressor characteristic must be known in order to determine \( c_2 \), it can be shown that the knowledge of a bound on the positive slope of the characteristic is sufficient. Differentiating (10) twice wrt \( \phi \) reveals that the
maximum positive slope occurs for \( \dot{\phi} = \dot{\phi}_m = -\frac{k_2}{3k_3} \) and is
given by

\[
a = \frac{d\Psi_e(\phi)}{d\phi} \bigg|_{\phi=\phi_m} = \frac{k_2^2}{3k_3} - k_1.
\]  

(22)

Assuming that only an upper bound \( a_m \) on the positive slope
of \( \Psi_e(\phi) \) is known, a conservative condition for \( c_2 \) is

\[ c_2 > a_m \geq a > \frac{k_2}{4k_3}. \]

(23)

Thus the price paid for not knowing the exact coefficients of
the compressor characteristic is a somewhat conservative con-
dition for the controller gain \( c_2 \). Notice also that no knowledge
of Greitzer’s B-parameter or its upper bound is required in for-
mulating the controller. The final expression for \( V_z \) is then

\[ V_z = -z_1 \hat{\phi}(z_1) + \hat{\Psi}_e(z_2)z_2 - c_2z_2^2 = -W(z_1, z_2) \leq 0. \]

(24)

The closed loop system can be written as

\[
\dot{z}_1 = \frac{1}{B}(\hat{\phi}(z_1) + z_2)
\]

(25)

\[
\dot{z}_2 = B(-z_1 + \hat{\Psi}_e(z_2) - c_2z_2).
\]

(26)

It then follows from the LaSalle-Yoshizawa theorem that the equi-
ilibrium point \( z_1 = z_2 = 0 \) is globally uniformly asymptot-
ically (GUAS).

5.1. Sensing requirements

Up to this point the pressure drop \( \hat{\Psi}_v \) across the CCV has been
considered the control variable. It is now assumed that the
CCV has a characteristic of the form

\[
\Psi_v(\phi) = \frac{1}{\gamma_{cc}} \phi^2,
\]

(27)

where \( \gamma_{cc} > 0 \) is proportional to the valve opening. Notice that
the assumption of no mass storage between the CCV and the
compressor (hence close-coupled) implies that the same mass
flow \( \phi \) is seen by both the compressor and the CCV. According
to (6) and (27) \( u \) is given by

\[
u = \Psi_v(\phi) = \Psi_v(\phi + \phi_0) - \Psi_v(\phi_0) \]

\[
u = \frac{1}{\gamma_{cc}} \phi^2 - \Psi_v(\phi_0).
\]

(28)

Inserting

\[
u = c_2 \phi = c_2(\phi - \phi_0)
\]

(29)
in (28) and solving for \( \gamma_{cc} \) gives a control law for \( \gamma_{cc} \):

\[
\gamma_{cc} = \frac{\phi}{\sqrt{c_2(\phi - \phi_0) + \Psi_v(\phi_0)}}.
\]

(30)

This control law requires only sensing of the mass flow \( \phi \).

6. Disturbances

As in [8] the effect of a pressure disturbance \( \hat{\Psi}_d(t) \), and a
flow disturbance \( \hat{\Phi}_d(t) \) is considered. The pressure distur-
bance will accelerate the flow, and the flow disturbance is modelling
unsteady plenum outflow. In the analysis of [8] \( \hat{\Phi}_d(t) \) is set to
zero. Here, both disturbances will be considered. The distur-
bances are time varying, and the only assumption made at this point
is boundedness, that is \( \|\hat{\Phi}_d\|_{\infty} \) and \( \|\hat{\Psi}_d\|_{\infty} \) exists. With
these disturbances the model is:

\[
\dot{\psi} = \frac{1}{B} (\dot{\phi} - \hat{\Phi}_d(t) - \hat{\Psi}_d(t))
\]

\[
\dot{\phi} = B(\hat{\Psi}_{e}(\phi) - \dot{\phi} + \dot{\Psi}_d(t) - u).
\]

(31)

To ensure boundedness of the system states, damping is in-
cluded in the controller design.

6.1. Pressure disturbances

First, pressure disturbances will be considered. That is,
\( \hat{\Psi}_d(t) \) is set to zero as in [8]. The backstepping procedure is as follows:

Step 1. Identical to Step 1 in section 5.

Step 2. The derivative of \( z_1 \) is

\[
z_1 = B\hat{\Psi}_e(\phi) - Bz_1 + B\hat{\Psi}_d(t) - Bu.
\]

(32)

\( \hat{V}_z \) is chosen as

\[
\hat{V}_z = V_1 + \frac{1}{2B}\frac{d}{2} z_1^2
\]

(33)

where \( \hat{V}_z \) can be bounded according to

\[
\hat{V}_z = -\hat{\phi}(z_1) + \hat{\Psi}_e(z_2)z_2 - c_2z_2^2
\]

(34)

Control law. To counteract the effect of the disturbance, a
damping factor \( d_2 > 0 \) is included and \( u \) is chosen as

\[
u = c_2z_2 + d_2z_2,
\]

(35)

\( \dot{V}_z \) is chosen so that (23) is satisfied. Inserting (35) in (34) gives

\[
\hat{V}_z = -z_1 \hat{\phi}(z_1) + \hat{\Psi}_e(z_2)z_2 - c_2z_2^2 + \hat{\Psi}_d(t)z_2 - d_2z_2^2.
\]

(36)

Use of Young’s inequality gives

\[
z_2 \hat{\Psi}_d(t) \leq d_2z_2^2 + \frac{\hat{\Psi}_d(t)^2}{4d_2} \leq d_2z_2^2 + \frac{I_2^2}{4d_2},
\]

(37)

and \( \hat{V}_z \) can be bounded according to

\[
\hat{V}_z \leq -W(z_1, z_2) + \frac{\hat{\Psi}_d(t)^2}{4d_2} \leq -W(z_1, z_2) + \frac{1}{4d_2} \|\hat{\Psi}_d\|_{\infty}^2.
\]

(38)

where

\[
W(z_1, z_2) = z_1 \hat{\phi}(z_1) - (\hat{\Psi}_e(z_2)z_2 - c_2z_2^2)
\]

(39)

is radially unbounded and positive definite. This implies that
\( \hat{V}_z < 0 \) outside a set \( \mathcal{R}_1 \) in the \( z_1 - z_2 \) plane.

According to [4], the fact that \( \hat{V}_z(z_1, z_2) \) and \( W(z_1, z_2) \)
is positive definite and radially unbounded, and \( \hat{V}_z(z_1, z_2) \) is
smooth, implies that there exists class-\( K_{\infty} \) functions \( \beta_1, \beta_2 \)
and \( \beta_3 \) such that

\[
\beta_1(|z_1|) \leq \hat{V}_z(z_1, z_2) \leq \beta_2(|z_1|) \]

\[
\beta_3(|z_1|) \leq W(z_1, z_2).
\]

(40)

where \( z = (z_1, z_2)^T \). It now follows from lemma 2.26 in [4],
that \( z(t) \) is globally uniformly bounded and that \( z(t) \) converges
to the set

\[
\mathcal{R}_1 = \left\{ z : |z| \leq \beta_1^{-1} \circ \beta_2 \circ \beta_3^{-1} \left( \frac{\|\hat{\Psi}_d\|_{\infty}^2}{4d_2} \right) \right\}
\]

(41)

\[
\beta_1(|z_1|) \leq \hat{V}_z(z_1, z_2) \leq \beta_2(|z_1|) \leq W(z_1, z_2).
\]

Since \( \alpha = 0 \Rightarrow z_1 = \hat{\psi} \) and \( z_2 = \hat{\phi} \) the global uniform
boundedness and convergence for \( \psi(t) \) and \( \phi(t) \) and even \( \psi(t) \)
and \( \phi(t) \) follows.

Notice that the controller (35) is essentially the same as
(18), with the only difference being that (35) requires a larger
in order to suppress the disturbance. Consequently, the
same sensing requirements as described in section 5.1 apply.

6.1.1. Convergence to the origin

In this section we show that an additional assumption on the disturbance ensures
that the controller (35) not only makes the states globally uni-
formly bounded, but also guarantees convergence to the origin.

It is now assumed that the disturbance term \( \hat{\Psi}_d(t) \) is upper
bounded by a monotonically decreasing non-negative function
\( \bar{\Psi}_d(t) \) such that

\[
\|\hat{\Psi}_d(t)\|_{\infty} \leq \bar{\Psi}_d(t) \forall t \geq 0
\]

(42)

and

\[
\lim_{t \to \infty} \bar{\Psi}_d(t) = 0.
\]

(43)

Inspired by the calculations starting on p. 75 in [4] for a simple
scalar system, we introduce the signal \( V_2(z(t)e^{\alpha t}) \) for use in
the convergence proof. Notice that the positive constant \( \alpha \) intro-
duced at this point is used for analysis only, and is not included
in the implementation of the control law. It now follows that
\[
\frac{d}{dt} \{V_2(z)e^{ct}\} = (V_2(z)+cV_2(z))e^{ct} \\
\leq (-W(z)+\frac{\hat{\phi}^2(t)}{4d_2} + cV_2(z))e^{ct} \\
\leq (-\beta_2(|z|)+c\beta_2(|z|))e^{ct} + \frac{\hat{\phi}^2(t)}{4d_2}e^{ct}.
\] (44)
By choosing \(c\) according to
\[c \leq \beta_2^{-1} \circ \beta_2(|z|) \leq \beta_2^{-1} \circ \beta_2(||z||_{\infty}),\]
where the existence of \(||z||_{\infty}\) follows from (41), (44) gives
\[
\frac{d}{dt} \{V_2(z)e^{ct}\} \leq \frac{\hat{\phi}^2(t)}{4d_2}e^{ct}.
\] (46)
By integrating (46) and using an argument similar to the one in the proof of lemma 2.24 in [4], it can be shown that
\[
V_2(z(t)) \leq V_2(z(0))e^{-ct} + \frac{1}{4d_2} \left( \Psi_d^2(0)e^{-\frac{ct}{2}} + \Psi_d^2(t/2) \right). \tag{47}
\]
Since \(\lim_{t \to \infty} \Psi_d^2(t/2) = 0\) it follows that
\[
\lim_{t \to \infty} V_2(z(t)) = 0. \tag{48}
\]
As \(V_2\) is positive definite it follows that
\[
\lim_{t \to \infty} z(t) = 0. \tag{49}
\]
Thus we have shown that under the additional assumptions (42) and (43) on the disturbance term, \(z(t)\) converges to the origin. This also implies that \(\phi(t)\) and \(\psi(t)\) converges to the point of intersection of the compressor and throttle characteristic.

### 6.2. Pressure and flow disturbances

At this point we include the flow disturbance \(\hat{d}(t)\) in the analysis. The backstepping procedure is as follows:

**Step 1.** As before two error variables \(z_1\) and \(z_2\) are defined as
\[z_1 = \tilde{z} \quad \text{and} \quad z_2 = \dot{z} - \alpha.\]
Again, \(V_1\) is chosen as
\[
V_1 = \frac{B}{2} z_1^2, \tag{50}
\]
with derivative
\[
\dot{V}_1 = z_1 - (\dot{\hat{\phi}}(z_1) + z_2 - \hat{\phi}_d(t) + \alpha), \tag{51}
\]
where (31) is used. The virtual control \(\alpha\) is chosen as
\[\alpha = -d_1 z_1, \tag{52}\]
where \(-d_1 z_1\) is a damping term to be used to counteract the disturbance \(\hat{\phi}_d(t)\). \(V_1\) can now be written as
\[
\dot{V}_1 = -d_1 z_1^2 + z_1 z_2 - \hat{\phi}_d(t) z_1 - \dot{\hat{\phi}}(z_1) z_1, \tag{53}
\]
and upper bounded according to
\[
\dot{V}_1 \leq -\dot{\hat{\phi}}(z_1) z_1 + z_1 z_2 + \frac{||\hat{\phi}_d||_{\infty}^2}{4d_1}, \tag{54}\]
To obtain the bound in (54), Young's inequality has been used to obtain
\[
-\hat{\phi}_d(t) z_1 \leq d_1 z_1^2 + \frac{\hat{\phi}_d^2(t)}{4d_1} \leq d_1 z_1^2 + \frac{||\hat{\phi}_d||_{\infty}^2}{4d_1}. \tag{55}\]

**Step 2.** The derivative of \(z_2\) is
\[
\dot{z}_2 = B\dot{\hat{\phi}}(\phi) - Bz_1 + B\hat{\phi}_d(t) - \frac{\partial \alpha}{\partial z_1} \frac{1}{B} (\dot{\hat{\phi}}(z_1) + \dot{\phi}),
\]
\[
+ \frac{1}{B} \frac{\partial \alpha}{\partial z_1} \hat{\phi}_d(t) - Bu. \tag{56}\]
From (52) it is seen that
\[
\frac{\partial \alpha}{\partial z_1} = -d_1. \tag{57}\]
\(V_2\) is chosen as
\[
V_2 = V_1 + \frac{1}{2B} z_2^2. \tag{58}\]
Using (54) and (56), an upper bound on \(\dot{V}_2\) is
\[
\dot{V}_2 \leq -\hat{\phi}(z_1) z_1 + \frac{||\hat{\phi}_d||_{\infty}^2}{4d_1} + z_2 \left( \dot{\hat{\phi}}(\phi) + \hat{\phi}_d(t) \right) + \frac{d_1}{B^2} \left( -\hat{\phi}(z_1) + \phi \right) - \frac{d_1}{B} \hat{\phi}_d(t) - u. \tag{59}\]
**Control law.** To counteract the effect of the disturbances, a damping factor \(d_2\) must be included and \(u\) is chosen as
\[
u = c_2 z_2 - k_3 (\alpha^3 + 3\alpha^2 z_2) - k_2 \phi^2 - k_1 \alpha \tag{60a}
\]
\[
+ \frac{d_1}{B^2} \left( -\hat{\phi}(z_1) + \phi \right) + d_2 z_2 \left( 1 + \frac{d_2}{B^2} \right). \tag{60b}\]
The parameter \(c_2\) is now chosen according to
\[c_2 > |k_1|. \tag{61}\]
Notice that this control law requires knowledge of the coefficients in the compressor characteristic, the throttle characteristic and the B-parameter. Inserting (60) in (59) gives
\[
\dot{V}_2 \leq -(c_2 + k_1) z_2^2 - k_3 (z_2^4 + 3z_2^2) - \hat{\phi}(z_1) z_1 + \frac{||\hat{\phi}_d||_{\infty}^2}{4d_1} \tag{62a}
\]
\[
- d_2 z_2^2 + ||\hat{\phi}_d||_{\infty} + \frac{d_1}{B} ||\hat{\phi}_d||_{\infty} - \frac{d_2^3}{B^2} d_2 z_2^2. \tag{62b}\]
Using Young's inequality twice gives
\[
||\hat{\phi}_d||_{\infty} \leq d_2 z_2^2 + \frac{||\hat{\phi}_d||_{\infty}^2}{4d_2} \tag{63a}
\]
\[
\frac{d_1}{B^2} ||\hat{\phi}_d||_{\infty} \leq \frac{1}{B^2} \left( d_2^2 z_2^2 + \frac{||\hat{\phi}_d||_{\infty}^2}{4d_2} \right). \tag{63b}\]
The final upper bound for \(V_2\) can now be written as
\[
\dot{V}_2 \leq -W(z_1, z_2) + \frac{1}{\kappa_1} ||\hat{\phi}_d||_{\infty}^2 + \frac{1}{\kappa_2} ||\hat{\phi}_d||_{\infty}^2 \tag{65a}
\]
where
\[
\frac{1}{\kappa_1} = \left( \frac{1}{4d_1} + \frac{1}{4B^2 d_2} \right), \quad \frac{1}{\kappa_2} = \frac{1}{4d_2} \tag{66a}
\]
and
\[
W(z_1, z_2) = (c_2 + k_1) z_2^2 + k_3 (z_2^4 + 3z_2^2) + \hat{\phi}(z_1) z_1 \tag{67a}
\]
is radially unbounded and positive definite. This implies that \(V_2 < 0\) outside a set \(R_2\) in the \(z_1 - z_2\) plane. As in section 6.1, the functions \(V_2(z)\) and \(W(z)\) exhibits the properties in (40).
Again, it can be shown that this implies that \(z(t)\) is globally uniformly bounded and that \(z(t)\) converges to the set \(R_2\) defined in (41).

### 6.2.1. Convergence to the origin
In order to prove convergence of \(z(t)\) to the origin, we make the following assumption on \(\hat{\phi}_d(t)\):
\[
||\hat{\phi}_d||_{\infty} \leq \bar{\phi}_d(t) \forall t \geq 0 \tag{69}
\]
and
\[
\lim_{t \to \infty} \bar{\phi}_d(t) = 0. \tag{70}\]
\(\bar{\phi}_d(t)\) is a monotonically decreasing non-negative function. The assumptions on \(\hat{\phi}_d(t)\) in equations (69) and (70) are also used. By using the same arguments as in section 6.1.1., but with two disturbance terms, it can be shown that
\[
V_2(z(t)) \leq V_2(z(0))e^{-ct} + \frac{1}{c_1 \kappa_1} \left( \bar{\phi}_d^2(0)e^{-\frac{ct}{2}} + \bar{\phi}_d^2(t/2) \right) \tag{71a}
\]
\[
+ \frac{1}{c_2 \kappa_2} \left( \bar{\phi}_d^2(0)e^{-\frac{ct}{2}} + \bar{\phi}_d^2(t/2) \right). \tag{71b}\]
Now \( \lim_{t \to \infty} \overline{u}_d(t/2) = 0 \) and \( \lim_{t \to \infty} \overline{u}_d(t/2) = 0 \) implies that \( \lim_{t \to \infty} V_2(z(t)) = 0 \) and by the positive definiteness of \( V_2 \) it follows that

\[
\lim_{t \to \infty} z(t) = 0.
\]

Thus we have shown that under the assumptions (42), (43), (69) and (70) on the disturbance terms, \( z(t) \) converges to the origin.

This also implies that \( \phi(t) \) and \( \psi(t) \) converges to the origin and that \( \phi(t) \) and \( \psi(t) \) converges to the point of intersection of the compressor and throttle characteristic.

7. Simulations

The characteristic of [7] and \( B = 1 \) is used in all simulations. In figure 3, an equilibrium located in the unstable area of the compressor map is stabilized. The throttle gain is set to \( \gamma = 0.5 \) so that the intersection of the throttle line and the compressor characteristic is located on the part of the characteristic that has positive slope, and thus the equilibrium is unstable, see figure 3. In the two plots to the left in figure 3, it is shown how the controller (18) with \( c_2 = 6 \) stabilizes the system, while in the two plots to the right noise has been added, and the system is stabilized by the controller (60) with \( d_1 = 1 \) and \( c_2 = d_2 = 3 \).

In figure 4 a step in the throttle gain occurs at \( t = 30 \). The gain changes from \( \gamma = 0.65 \) to \( \gamma = 0.5 \) and, consequently, the compressor goes into surge as shown in the two leftmost plots in figure 4. In this simulation a pressure disturbance is included. In the two rightmost plots, the controller (35) with \( c_2 = d_2 = 3 \) is active. As can bee seen the compressors remains stable after the throttle change. The damping of the disturbance can also be observed.

8. Conclusion

Anti surge controllers for a close-coupled valve in series with a compressor have been developed. By the application of the backstepping methodology, a control law which uses feedback from mass flow only has been derived. Only a upper bound on the slope of the compressor characteristic is required to implement this controller. The controller is used both in the case of no disturbances and in the presence of pressure disturbances.

Figure 3: The throttle gain is set to \( \gamma = 0.5 \), and the compressor is surging. The controllers are switched on at \( t = 20 \). The pressure disturbance is white noise of amplitude 0.15, and the mass flow disturbance is white noise of amplitude 0.1.

Figure 4: The compressor is throttled from \( \gamma = 0.65 \) to \( \gamma = 0.5 \) at \( t = 30 \). The pressure disturbance is white noise of amplitude 0.10.

A more complicated control law is derived for the case of both pressure- and mass flow disturbances. In order to implement this controller, the compressor characteristic and the \( B \)-parameter must be known.

The control laws stabilizes the undisturbed Moore-Greizer model in the previously unstable area of the compressor map. In the presence of disturbances globally uniformly boundedness of both mass flow and pressure is ensured. Under the assumption of decaying disturbances, convergence to the origin is proved.

References


