Stability analysis of 6-DOF force/position control for robot manipulators

Jan Tommy Gravdahl and Olav Egeland
Institutt for Teknisk Kybernetikk
Norges Tekniske Høgskole
7034 Trondheim, Norway

Bruno Siciliano and Stefano Chiaverini
Dipartimento di Informatica e Sistemistica
Università degli Studi di Napoli Federico II
80125 Napoli, Italy

Abstract

In this paper we propose a force/position control scheme in a 6-DOF task space which is a generalization of the parallel control scheme in a 3-DOF task space. The controller consists of a PD action on the position loop, a PI action on the force loop together with gravity compensation and feedforward of the desired contact force. Kinestatic filtering is not used in the scheme. Euler parameters are adopted to describe end-effector orientation. The model of interaction wrench (force and moment) is formulated with a generalized spring. Stability of the proposed scheme is established through the Lyapunov method.

1. Introduction

There are a number of significant applications of robot manipulators where the end effector is in contact with the environment, and contact forces must be handled properly by the robot controller.

If force sensor information is not used for control purposes, one can assign a suitable dynamic behaviour between position and force variables, e.g. using impedance control [6] so that stability is maintained when the end effector is in contact with the environment. Another solution to the problem is to use feedback from a force sensor e.g. in the wrist. This is done in hybrid force/position control [7],[10] where the task space is divided into force controlled and position controlled directions using kinestatic filtering.

A conceptually different approach to force/position control using feedback from a force sensor is the parallel control strategy [2]. In this scheme kinestatic filtering is not used. Instead both force and position variables are controlled along each task space direction using a PI action for the force feedback and a PD action for the position feedback. Thus, at steady state the force control loop dominates the position control loop in the task directions where interaction occurs. This makes the scheme suitable to handle contacts with an unstructured environment and unplanned collisions, which are known to represent a drawback to hybrid controllers. Stability of the parallel control scheme in a 3-DOF task space was analyzed in [3] in the case of point contact.

In this paper the parallel force/position control scheme is generalized to a 6-DOF task space. The end-effector control deviation is formed by a position component and an orientation component, where the latter is formulated in terms of Euler parameters. The end-effector force and moment are grouped into a wrench vector, while the twist vector is formed by the end-effector angular and linear velocities. The controller is shown to guarantee asymptotic stability of the equilibrium configuration for a proper choice of the feedback gains.

2. Preliminaries

In this section we briefly recall some useful results from [8]. The configuration of a rigid body can be described by a homogeneous transformation matrix

$$\boldsymbol{T} \triangleq \begin{pmatrix} \boldsymbol{R} & \boldsymbol{p} \\ \boldsymbol{0}^{\mathrm{T}} & 1 \end{pmatrix} \in \mathbb{R}^{4\times 4} \tag{1}$$

where $\boldsymbol{R} \in \mathbb{R}^{3\times 3}$ is the rotation matrix and $\boldsymbol{p} \in \mathbb{R}^3$ is the position vector of the rigid body in base coordinates. The time derivative of the configuration $\boldsymbol{T}$ is

$$\dot{\boldsymbol{T}} = \begin{pmatrix} \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R} & \boldsymbol{v} \\ \boldsymbol{0}^{\mathrm{T}} & 0 \end{pmatrix} \tag{2}$$

where $\boldsymbol{\omega}$ is the angular velocity in base coordinates, $\boldsymbol{v}$ is the velocity in base coordinates and $\boldsymbol{S}(\boldsymbol{\omega}) \in \mathbb{R}^{3\times 3}$ is the skew-symmetric matrixform of $\boldsymbol{\omega}$. A generalized spring is defined by introducing a potential $V(\boldsymbol{T})$. By using a first order approximation of the spring differential it can be shown that the symmetric stiffness matrix $\boldsymbol{K}$ of the generalized spring is given by $\boldsymbol{K} \triangleq d^2V(\boldsymbol{I})$. The spring wrench is then given by $\boldsymbol{w} = (\boldsymbol{f}^{\mathrm{T}}\ \boldsymbol{m}^{\mathrm{T}})^{\mathrm{T}} = \boldsymbol{K}\boldsymbol{\delta}$ where $\boldsymbol{\delta} \triangleq (\boldsymbol{\theta}^{\mathrm{T}}\ \boldsymbol{d}^{\mathrm{T}})^{\mathrm{T}}$. The screw vector $\boldsymbol{\delta}$ represents a first-order approximation of the elastic displacement $\boldsymbol{T}$. $\boldsymbol{f}$ and $\boldsymbol{m}$ are the force and moment acting on the body.

3. Dynamic model

The equations of motion of the manipulator in a 6-DOF task space can be written as

$$\boldsymbol{D}(\boldsymbol{T})\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{T},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{T}) = \boldsymbol{u} - \boldsymbol{w}. \tag{3}$$

Here $\boldsymbol{T}$ as defined in (1) specifies the configuration of the end effector. $\boldsymbol{\nu}$ is the velocity twist vector of the end effector and is found from $\dot{\boldsymbol{T}}$, and $\boldsymbol{w}$ is the contact wrench between the end effector and the environment. Also $\boldsymbol{D}(\boldsymbol{T})$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{T},\boldsymbol{\nu})\boldsymbol{\nu}$ is the vector of centrifugal and Coriolis terms, $\boldsymbol{g}(\boldsymbol{T})$ is the vector of gravity terms, and $\boldsymbol{u}$ is the vector of input generalized forces. We consider a non-redundant manipulator with 6 joints so that no internal motion occurs. It is assumed that the manipulator is bounded away from singularities.

4. Controller

The constant desired configuration and contact wrench are given respectively by

$$\boldsymbol{T}_d = \begin{pmatrix} \boldsymbol{R}_d & \boldsymbol{p}_d \\ \boldsymbol{0}^{\mathrm{T}} & 1 \end{pmatrix},\ \boldsymbol{w}_d = \begin{pmatrix} \boldsymbol{f}_d \\ \boldsymbol{m}_d \end{pmatrix}, \tag{4}$$

while the control deviations are defined as $\widetilde{\boldsymbol{p}} = \boldsymbol{p}_d - \boldsymbol{p}$, $\widetilde{\boldsymbol{R}} = \boldsymbol{R}\boldsymbol{R}_d^{\mathrm{T}}$ and $\widetilde{\boldsymbol{w}} = \boldsymbol{w}_d - \boldsymbol{w}$. Let $(\widetilde{\eta}, \widetilde{\boldsymbol{\epsilon}})$ be the Euler parameters associated with $\widetilde{\boldsymbol{R}}$. The deviation in configuration is described by $\boldsymbol{d}_T \triangleq (-\widetilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\ \widetilde{\boldsymbol{p}}^{\mathrm{T}})^{\mathrm{T}}$. We propose the following control law which generalizes the control law of [2]:

$$\boldsymbol{u} = -\boldsymbol{K}_D\boldsymbol{\nu} + \boldsymbol{K}_P\boldsymbol{d}_T + \boldsymbol{w}_d + \boldsymbol{g}(\boldsymbol{T}) + \boldsymbol{K}_F\widetilde{\boldsymbol{w}} + \boldsymbol{K}_I\int_0^t \widetilde{\boldsymbol{w}}(\sigma)d\sigma. \tag{5}$$

Here the feedback gain matrices are constant and diagonal and given by

$$\boldsymbol{K}_i = \text{block diag}\{k_{iR}\boldsymbol{I}_3, k_{iP}\boldsymbol{I}_3\} > \boldsymbol{0} \quad \text{for} \quad i \in \{D,P,F,I\}$$

where $\boldsymbol{I}_3$ is the identity matrix $\in \mathbb{R}^{3\times 3}$. We also define $\boldsymbol{K}_{DR} = k_{DR}\boldsymbol{I}_3$, $\boldsymbol{K}_{DP} = k_{DP}\boldsymbol{I}_3$, $\boldsymbol{K}_{PR} = k_{PR}\boldsymbol{I}_3$, $\boldsymbol{K}_{PP} = k_{PP}\boldsymbol{I}_3$. It is assumed that the system has a constant equilibrium configuration $\boldsymbol{T}_\infty = \begin{pmatrix} \boldsymbol{R}_\infty & \boldsymbol{p}_\infty \\ \boldsymbol{0}^{\mathrm{T}} & 1 \end{pmatrix}$. By defining the deviations $\widehat{\boldsymbol{p}} = \boldsymbol{p}_\infty - \boldsymbol{p}$, $\widetilde{\boldsymbol{p}}_\infty = \boldsymbol{p}_d - \boldsymbol{p}_\infty$, $\widehat{\boldsymbol{R}} = \boldsymbol{R}\boldsymbol{R}_\infty^{\mathrm{T}}$ and $\widetilde{\boldsymbol{R}}_\infty = \boldsymbol{R}_\infty \boldsymbol{R}_d^{\mathrm{T}}$, defining the vector

$$\boldsymbol{s} = \int_0^t \widetilde{\boldsymbol{w}}(\sigma)d\sigma + \boldsymbol{K}_I^{-1} \begin{pmatrix} -k_{PR}\widetilde{\eta}\widetilde{\boldsymbol{\epsilon}}_\infty \\ k_{PP}\widetilde{\boldsymbol{p}}_\infty \end{pmatrix} \tag{6}$$

and lettting $(\widehat{\eta}, \widehat{\boldsymbol{\epsilon}})$ be the Euler parameters associated with $\widehat{\boldsymbol{R}}$, it can be shown that the system described by the equations (3) and (6), the kinematic equations and control law (5) can be written in the form

$$\dot{\boldsymbol{z}} = \boldsymbol{F}\boldsymbol{z} \tag{7}$$

where

$$\boldsymbol{z} = (\boldsymbol{\nu}^{\mathrm{T}}\ \widehat{\boldsymbol{\epsilon}}^{\mathrm{T}}\ \widehat{\boldsymbol{p}}^{\mathrm{T}}\ \boldsymbol{s}^{\mathrm{T}}\ \widetilde{\boldsymbol{w}}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{24},$$

$$\boldsymbol{F} = \begin{pmatrix} -\boldsymbol{D}^{-1}(\boldsymbol{C}+\boldsymbol{K}_D) & \begin{pmatrix} -\boldsymbol{D}^{-1}\boldsymbol{K}_{PR}\boldsymbol{E} \\ \boldsymbol{O} \end{pmatrix} & \begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{D}^{-1}\boldsymbol{K}_{PP} \end{pmatrix} & \boldsymbol{D}^{-1}\boldsymbol{K}_I & \boldsymbol{D}^{-1}(\boldsymbol{K}_F+\boldsymbol{I}) \\ \frac{1}{2}\begin{pmatrix} \widehat{\eta}\boldsymbol{I} - \boldsymbol{S}(\widehat{\boldsymbol{\epsilon}}) & \boldsymbol{O} \end{pmatrix} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \begin{pmatrix} \boldsymbol{O} & -\boldsymbol{I} \end{pmatrix} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \frac{k_{PR}}{2k_{IR}}\boldsymbol{H}(\widehat{\boldsymbol{\epsilon}}) & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{I} \\ -\boldsymbol{K}(\boldsymbol{I}-\boldsymbol{P}_K) & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \end{pmatrix}, \tag{8}$$

$\boldsymbol{E} \triangleq \widetilde{\eta}_\infty \boldsymbol{I} - \boldsymbol{S}(\widetilde{\boldsymbol{\epsilon}}_\infty)$ and $\boldsymbol{H}(\widehat{\boldsymbol{\epsilon}}) = \text{block diag}\{\widetilde{\boldsymbol{\epsilon}}_\infty \widehat{\boldsymbol{\epsilon}}^{\mathrm{T}},\ \boldsymbol{O}\}$. Here $\boldsymbol{P}_K$ is a projection matrix or twist filter as described in [4]; notice that $\boldsymbol{P}_K$ is used for analysis purposes only.

5. Stability

In this section Lyapunov stability of the proposed control scheme is established. The stability proof is a generalization of the results of [2] where point contact in a 3-DOF task space was discussed, while here general contacts in a 6-DOF task space are treated. In the proof we have taken advantage of the results of [1] and [9].

Theorem 1 Assume that there is an equilibrium configuration $\boldsymbol{T}_\infty$ so that assumption $\widetilde{\eta}_\infty > 0$ is satisfied. Then there exists a choice of constant feedback gain matrices $\boldsymbol{K}_D$, $\boldsymbol{K}_P$, $\boldsymbol{K}_F$ and $\boldsymbol{K}_I$ that makes the origin of the system $\dot{\boldsymbol{z}} = \boldsymbol{F}\boldsymbol{z}$ asymptotically stable.

Sketch of proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z} + (2k_{PR} + \beta k_{DR})(\widehat{\eta} - 1)^2 \tag{9}$$

where

$$\boldsymbol{P} = \begin{pmatrix} 2\boldsymbol{D} & \beta\boldsymbol{D}\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{O} \end{pmatrix} & -\rho\boldsymbol{D}\begin{pmatrix} \boldsymbol{O} \\ \boldsymbol{I} \end{pmatrix} & -\delta\boldsymbol{D}\boldsymbol{K}_I & \boldsymbol{O} \\ \beta\begin{pmatrix} \boldsymbol{E}^{\mathrm{T}} & \boldsymbol{O} \end{pmatrix}\boldsymbol{D} & 2(2\boldsymbol{K}_{PR} + \beta\boldsymbol{K}_{DR}) & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ -\rho\begin{pmatrix} \boldsymbol{O} & \boldsymbol{I} \end{pmatrix}\boldsymbol{D} & \boldsymbol{O} & 2\boldsymbol{K}_{PP} + \rho\boldsymbol{K}_{DP} & \boldsymbol{O} & \boldsymbol{O} \\ -\delta\boldsymbol{K}_I\boldsymbol{D} & \boldsymbol{O} & \boldsymbol{O} & 2\boldsymbol{\Lambda}\boldsymbol{K}_I & -\gamma\boldsymbol{K}_I \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & -\gamma\boldsymbol{K}_I & \gamma^2\boldsymbol{K}_F \end{pmatrix}, \tag{10}$$

β, γ, δ and ρ are positive constants and $\boldsymbol{\Lambda} = \text{block diag}\{\rho\boldsymbol{I}_3, \beta\boldsymbol{I}_3\}$. By examining (9) it is seen that V is positive definite whenever

$$\max\{k_{IP}, k_{IR}\} < \min\left\{\frac{\min\{\rho,\beta\}}{\delta^2\lambda_M}, \min\{\rho k_{FR}, \beta k_{FP}\}\right\},$$

$$k_{DP} > \rho\lambda_M,\ k_{DR} > 2\beta\lambda_M, \tag{11}$$

where λ_M is the greatest eigenvalue of $\boldsymbol{D}$. By computing the time derivative of V along system trajectories it can be shown [5] that $-\dot{V}$ is positive definite whenever the following conditions are met:

$$\boldsymbol{K}_P > c_1\boldsymbol{I} \tag{12}$$

$$\overline{c}_2(\boldsymbol{K}_P, \boldsymbol{K}_F)\boldsymbol{I} > \boldsymbol{K}_I > \underline{c}_2(\boldsymbol{K}_P, \boldsymbol{K}_F)\boldsymbol{I} \tag{13}$$

$$\boldsymbol{K}_D > c_3(\boldsymbol{K}_P, \boldsymbol{K}_F, \boldsymbol{K}_I, \boldsymbol{K})\boldsymbol{I}, \tag{14}$$

where c_1, $\underline{c}_2$, $\overline{c}_2$ and c_3 are positive constants. At this point a proper choice of the constants β, γ, δ and ρ can be found that satisfies conditions (11) and (12)–(14). This in turn implies that the equilibrium $\boldsymbol{z} = \boldsymbol{0}$ is asymptotically stable, in view of Lyapunov's theorem.

6. Acknowledgments

The research reported in this work was supported partly by *Center of Maritime Control Systems at NTH/SINTEF* and partly by *Consiglio Nazionale delle Ricerche* under contract n. 93.00905.PF67 .

7. References

[1] S. Arimoto and F. Miyazaki, "Stability and robustness of PID feedback control for robot manipulators of sensory capability," *Robotics Research: 1st Int. Symp.*, M. Brady and R.P. Paul (Eds.), MIT Press, Boston, MA, pp. 783–799, 1984.

[2] S. Chiaverini and L. Sciavicco, "The parallel approach to force/position control of robotic manipulators," *IEEE Trans. on Robotics and Automation*, vol. 9, pp. 361–373, 1993.

[3] S. Chiaverini, B. Siciliano, and L. Villani, "Force/position regulation of compliant robot manipulators," *IEEE Trans. on Automatic Control*, vol. 39, pp. 647–652, 1994.

[4] K.L. Doty, C. Melchiorri, and C. Bonivento, "A theory of generalized inverses applied to robotics," *Int. J. of Robotics Research*, vol. 12, pp. 1–19, 1993.

[5] J.T. Gravdahl, *Stabilitetsanalyse av parallell kraft/posisjonsstyring for manipulatorer*, siv.ing. thesis, Department of Engineering Cybernetics, The Norwegian Institute of Technology, 1994.

[6] N. Hogan, "Impedance control: An approach to manipulation, Parts I–III," *ASME J. of Dynamic Systems, Measurement and Control*, vol. 107, pp. 1–24, 1985.

[7] M.H. Raibert and J.J. Craig, "Hybrid position/force control of manipulators," *ASME J. of Dyn. Systems, Measurement, and Control*, vol. 103, pp. 126–133, 1987.

[8] C. Samson, M. Le Borgne, and B. Espiau, *Robot Control: The Task Function Approach*, Oxford Engineering Science Series, vol. 22, Oxford University Press, Oxford, UK, 1991.

[9] J.T. Wen and K. Kreutz-Delgado, "The attitude control problem," *IEEE Trans. on Automatic Control*, vol. 36, pp. 1148–1162, 1991.

[10] T. Yoshikawa, "Dynamic hybrid position/force control of robotic manipulators—Description of hand constraints and calculation of joint driving force," *IEEE J. of Robotics and Automation*, vol. 3, pp. 386–392, 1987.