Singularity-Free Dynamic Equations of AUV-Manipulator Systems

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Abstract: In this paper we derive the singularity-free dynamic equations of AUV-manipulator systems using a minimal representation. Autonomous underwater vehicles (AUVs) are normally modeled using the singularity-prone Euler angles, but introducing quasi-coordinates allows us to derive the dynamics using minimal and globally valid non-Euclidean configuration coordinates. This is a great advantage as the configuration space of an AUV is non-Euclidean. We thus obtain a computationally efficient and singularity-free formulation of the dynamic equations with the same complexity as the conventional Lagrangian approach. The closed form formulation makes the proposed approach well suited for system analysis and model-based control. This paper focuses on the kinematic and dynamic properties of AUV-manipulator systems and we present the explicit matrices needed for implementation together with several mathematical relations that can be used to speed up the algorithms. The hydrodynamic and damping forces are also included in the equations. By presenting the explicit equations needed for implementation, the approach presented becomes more accessible and engineers and programmers can implement the results without extensive knowledge of the mathematical background.

Keywords: AUV-manipulator dynamics, robot modeling, singularities, quasi-coordinates.

1. INTRODUCTION

A good understanding of the dynamics of robotic manipulators mounted on autonomous underwater vehicles (AUVs) is important in a wide range of applications. Especially, the use of robots in harsh and remote areas has increased the need for AUV-robot solutions. A robotic manipulator mounted on a moving vehicle is a flexible and versatile solution and thus an efficient way to perform challenging tasks over a large sub-sea area. Operations at deeper water where humans cannot or do not want to operate, require more advanced and robust underwater systems and thus the need for continuously operating robots for surveillance, maintenance, and operation emerges (Love et al., 2004; Antonelli, 2006; McMillan et al., 1995). Recreating realistic models of deep-sea conditions is thus important. Both for simulation and for model-based control the explicit dynamic equations of AUV-manipulator systems need to be implemented in a robust and computationally efficient way to guarantee safe testing and operation of these systems.

In this paper we study in detail how to model AUVs with robotic arms, or underwater robotic vehicles (URVs). For the first time we derive the minimal, singularity free dynamic equations of AUV-manipulator systems using the proposed framework. We also show how to add the hydrodynamic effects such as added mass and damping forces. The dynamic equations have approximately the same complexity as the conventional Lagrangian approach and are better suited for simulation and easier to implement.

It is a well known fact that the kinematics of a rigid body contains singularities if the Euler angles are used to represent the orientation and the joint topology is not taken into account. One solution to this problem is to use a non-minimal representation such as the unit quaternion to represent the orientation. These are not generalized coordinates and can thus not be used in Lagrange's equations. This is a major drawback when it comes to modeling vehicle-manipulator systems as most methods used for robot modeling are based on the Lagrangian approach. It is thus a great advantage if also the vehicle dynamics can be derived from the Lagrange equations.

The use of Lie groups and algebras as a mathematical basis for the derivation of the dynamics of multibody systems can be used to overcome this problem (Selig, 2000; Park et al., 1995). We then choose the coordinates generated by the Lie algebra as local Euclidean coordinates which allows us to describe the dynamics locally. For this approach to be valid globally the total configuration space needs to be covered by an atlas of local exponential coordinate patches. The appropriate equations must then be chosen for the current configuration. The geometric approach presented in Bullo and Lewis (2004) can then be used to obtain a globally valid set of dynamic equations on a single Lie group, such as an AUV with no robot attached.

Even though combinations of Lie groups can be used to represent multibody systems, the formulation is very complex and not suited for implementation in a simulation environment. In Kwatny and Blankenship (2000) quasi-coordinates was used to derive the dynamic equations of fixed-base robotic manipulators using Poincaré's formulation of the Lagrange equations. In Kozlowski and Herman (2008) several control laws using a quasi-coordinate approach were presented, but only robots with conventional 1-DoF joints were considered. Common for all these methods is, however, that the configuration space of the system is described as $q \in \mathbb{R}^n$. This is not a problem when dealing with 1-DoF revolute or prismatic joints but more complicated joints such as ball-joints or free-floating joints then need to be modeled as compound kinematic joints (Kwatny and Blankenship, 2000), i.e., a combination of 1-DoF simple kinematic joints. For joints that use the Euler angles to represent the orientation this leads to singularities in the representation.

In this paper we follow the generalized Lagrangian approach presented in Duindam and Stramigioli (2008) which allows us to combine the Euclidean joints and more general joints, i.e. joints that can be described by the Lie group $SE(3)$ or one of its ten subgroups, and we extend these ideas to AUV-manipulator systems. There are several advantages in following this approach. The use of quasi-coordinates, i.e., velocity coordinates that are not simply the time derivative of the position coordinates, allows us to include joints (or transformations) with a different topology than that of $\mathbb{R}^n$. For example, for an AUV we can represent the transformation from the inertial frame to the AUV body frame as a free-floating joint with configuration space $SE(3)$ and we avoid the singularity-prone kinematic relations between the inertial frame and the body frame velocities that normally arise in deriving the AUV dynamics (Fossen, 2002).

This approach differs from previous work in that it allows us to derive the dynamic equations of vehicle-manipulator systems for vehicles with a configuration space different from $\mathbb{R}^n$. The dynamics are expressed (locally) in exponential coordinates ϕ, but the final equations are evaluated at $\phi = 0$. This has two main advantages. Firstly, the dynamics do not depend on the local coordinates as these are eliminated from the equations and the global position and velocity coordinates are the only state variables. This makes the equations valid globally. Secondly, evaluating the equations at $\phi = 0$ greatly simplifies the dynamics and make the equations suited for implementation in simulation software. We also note that the approach is well suited for model-based control as the equations are explicit and without constraints. The fact that the configuration space of the AUV is a Lie group also simplifies the implementation. Even though the expressions in the derivation of the dynamics are somewhat complex, we have several tools from the Lie theory that allows us to write the final expressions in a very simple form.

The paper is organized as follows. Section 2 gives the detailed mathematical background for the proposed approach. This section can be skipped and practitioners mainly interested in implementation can go straight to Section 4. Section 3 presents the state of the art in AUV-manipulator modeling and Section 4 gives the explicit dynamic equations for the AUV-manipulator dynamics along with some comments on implementing these in a simulation environment. To the authors' best knowledge AUV-manipulator systems have not been studied in detail in literature using the framework presented here. We also include hydrodynamic and damping forces, the added mass and Coriolis matrices and other considerations that are not encountered in fixed-base robot dynamics.

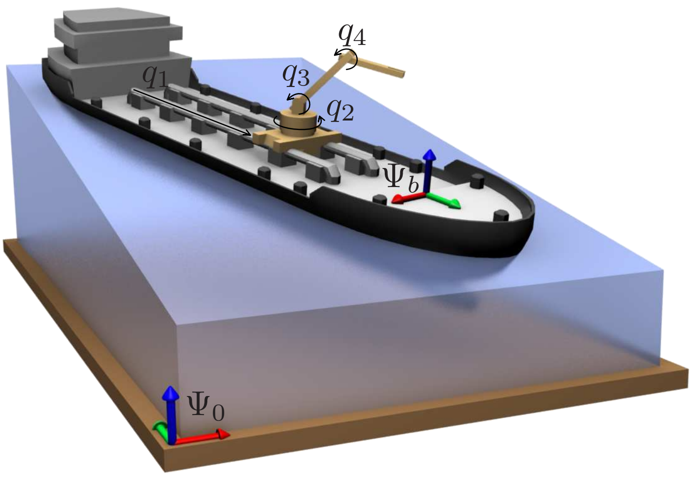

Fig. 1. Model setup for a robot attached to a vehicle with coordinate frame Ψ_b and inertial reference frame Ψ_0.

2. DYNAMIC EQUATIONS OF AUV-MANIPULATOR SYSTEMS

We extend the classical dynamic equations for a serial manipulator arm with 1-DoF joints to include the motion of the AUV on which the manipulator is mounted.

2.1 *AUV-Manipulator Kinematics*

Consider the setup of Figure 1 describing a general n-link robot manipulator arm attached to a vehicle. Choose an inertial coordinate frame Ψ_0, a frame Ψ_b rigidly attached to the vehicle, and n frames Ψ_i (not shown) attached to each link i at the center of mass with axes aligned with the principal directions of inertia. Finally, choose a vector $q \in \mathbb{R}^n$ that describes the configuration of the n joints. Using standard notation (Murray et al., 1994), we describe the pose of each frame Ψ_i relative to Ψ_0 as a homogeneous transformation matrix $g_{0i} \in SE(3)$ of the form

$$g_{0i} = \begin{bmatrix} R_{0i} & p_{0i} \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4} \tag{1}$$

with rotation matrix $R_{0i} \in SO(3)$ and translation vector $p_{0i} \in \mathbb{R}^3$. This pose can also be described using the vector of joint coordinates q as

$$g_{0i} = g_{0b} g_{bi} = g_{0b} g_{bi}(q). \tag{2}$$

The vehicle pose g_{0b} and the joint positions q thus fully determine the configuration state of the robot. The spatial velocity of each link can be expressed using twists:

$$V_{0i}^0 = \begin{bmatrix} v_{0i}^0 \\ \omega_{0i}^0 \end{bmatrix} = V_{0b}^0 + V_{bi}^0 = \mathrm{Ad}_{g_{0b}} \left(V_{0b}^b + J_i(q)\dot{q} \right) \tag{3}$$

where v_{0i}^0 and ω_{0i}^0 are the linear and angular velocities, respectively, of link i relative to the inertial frame, $J_i(q) \in \mathbb{R}^{6\times n}$ is the geometric Jacobian of link i relative to Ψ_b, the adjoint is defined as $\mathrm{Ad}_g := \left[\begin{smallmatrix} R & \hat{p}R \\ 0 & R \end{smallmatrix}\right] \in \mathbb{R}^{6\times 6}$, and $\hat{p} \in \mathbb{R}^{3\times 3}$ is the skew symmetric matrix such that $\hat{p}x = p \times x$ for all $p, x \in \mathbb{R}^3$. The velocity state is thus fully determined given the twist V_{0b}^b of the vehicle and the joint velocities $\dot{q}$.

2.2 *AUV-Manipulator Dynamics*

The previous section shows how the kinematics of the system can be described in terms of the (global) state variables g_{0b}, q, V_{0b}^b, and $\dot{q}$. To derive the dynamics of the complete mechanism (including the 6-DoF between Ψ_0 and Ψ_b) in terms of these state variables, we follow the generalized Lagrangian method introduced by Duindam and Stramigioli (2008). This method gives the dynamic equations for a general mechanism described by a set $Q = \{Q_i\}$ of configuration states Q_i (not necessarily Euclidean), a vector v of velocity states $v_i \in \mathbb{R}^{n_i}$, and several mappings that describe the local Euclidean structure of the configuration states and their relation to the velocity states. More precisely, the neighborhood of every state $\bar{Q}_i$ is locally described by a set of Euclidean coordinates $\phi_i \in \mathbb{R}^{n_i}$ as $Q_i = \Phi_i(\bar{Q}_i, \phi_i)$ with $\Phi_i(\bar{Q}_i, 0) = \bar{Q}_i$. $\Phi_i(\bar{Q}_i, \phi_i)$ defines a local diffeomorphism between a neighborhood of $0 \in \mathbb{R}^{n_i}$ and a neighborhood of $\bar{Q}_i$.

We start by deriving an expression for the kinetic coenergy of a mechanism, expressed in coordinates Q, v, but locally parameterized by the coordinate mappings for each joint. For joints that can be described by a matrix Lie group, this mapping can be given by the exponential map (Murray et al., 1994). Let $\phi \in se(n, \mathbb{R})$ be the Lie algebra of $SE(3)$, then the exponential map $\exp(\phi)$ is given by

$$e^{\hat{\phi}} = I + \hat{\phi} + \frac{\hat{\phi}^2}{2} \cdots = \sum_{n=0}^{\infty} \frac{\hat{\phi}^n}{n!} \tag{4}$$

where I (no subscript) is the identity matrix. The dynamics are thus expressed in local coordinates ϕ for configuration and v for velocity, and we consider Q a parameter. After taking partial derivatives of the Lagrangian function, we evaluate the results at $\phi = 0$ (i.e. at configuration Q) to obtain the dynamics expressed in global coordinates Q and v as desired. We note that even though local coordinates ϕ appear in the derivations of the various equations, the final equations are all evaluated at $\phi = 0$ and hence these final equations do not depend on local coordinates. The global coordinates Q and v are the only dynamic state variables and the equations are valid globally, without the need for coordinate transitions between various areas of the configuration space. Note also that taking the partial derivatives of the Lagrangian and evaluating at $\phi = 0$ greatly simplifies (4) and the closed form expressions of the exponential map is not needed. This fact greatly simplifies the final equations.

In general, the topology of a Lie group is not Euclidean. When deriving the dynamic equations for AUVs, this is normally dealt with by introducing a transformation matrix that relates the local and global velocity variables. However, forcing the dynamics into a vector representation in this way, without taking the topology of the configuration space into account, leads to singularities in the representation or other deficiencies. To preserve the topology of the configuration space we will use quasi-coordinates, i.e. velocity coordinates that are not simply the time-derivative of position coordinates, but given by a linear relation. Thus, there exist differentiable matrices S_i such that we can write $v_i = S_i(Q_i, \phi_i)\dot{\phi}_i$ for every Q_i. For Euclidean joints this relation is given simply by the identity map while for joints with a Lie group topology we can use the exponential map to derive this relation.

Given a mechanism with coordinates formulated in this generalized form, we can write its kinetic energy as $\mathcal{T}(Q, v) = \frac{1}{2} v^\mathsf{T} M(Q) v$ with $M(Q)$ the inertia matrix in coordinates Q and v the stacked velocities of the vehicle and the robot joints. The dynamics then satisfy

$$M(Q)\dot{v} + C(Q, v)v = \tau \tag{5}$$

with τ the vector of external and control wrenches (collocated with v), and $C(Q, v)$ the matrix describing Coriolis and centrifugal forces given by

$$C_{ij}(Q, v) := \sum_{k,l} \left(\frac{\partial M_{ij}}{\partial \phi_k} S_{kl}^{-1} - \frac{1}{2} S_{ki}^{-1} \frac{\partial M_{jl}}{\partial \phi_k} \right) \Bigg|_{\phi=0} v_l \tag{6}$$

$$+ \sum_{k,l,m,s} \left(S_{mi}^{-1} \left(\frac{\partial S_{mj}}{\partial \phi_s} - \frac{\partial S_{ms}}{\partial \phi_j} \right) S_{sk}^{-1} M_{kl} \right) \Bigg|_{\phi=0} v_l .$$

See Duindam and Stramigioli (2008) for details.

To apply this general result to systems of the form of Figure 1, we write $Q = \{g_{0b}, q\}$ as the set of configuration states where g_{0b} is the Lie group $SE(3)$, and $v = \left[(V_{0b}^b)^\mathsf{T} \ \dot{q}^\mathsf{T} \right]^\mathsf{T}$ as the vector of velocity states. The local Euclidean structure for the state g_{0b} is given by exponential coordinates, while the state q is itself globally Euclidean. Mathematically, we can express configurations (g_{0b}, q) around a fixed state $(\bar{g}_{0b}, \bar{q})$ as

$$g_{0b} = \bar{g}_{0b} \exp \left(\sum_{j=1}^{6} b_j (\phi_b)_j \right), \quad q_i = \bar{q}_i + \phi_i \ \forall \ i \in \{1 \dots n\}$$

with b_j the standard basis elements of the Lie algebra $se(3)$.

From expression (3) for the twist of each link in the mechanism, we can derive an expression for the total kinetic energy. Let $I_b \in \mathbb{R}^{6\times6}$ and $I_i \in \mathbb{R}^{6\times6}$ denote the constant positive-definite diagonal inertia tensors of the base and link i (expressed in Ψ_i), respectively. The kinetic energy $\mathcal{T}_i$ of link i then follows as

$$\begin{aligned} \mathcal{T}_i &= \frac{1}{2} \left(V_{0i}^i \right)^\mathsf{T} I_i V_{0i}^i \\ &= \frac{1}{2} \left((V_{0b}^b)^\mathsf{T} + \dot{q}^\mathsf{T} J_i(q)^\mathsf{T} \right) \mathrm{Ad}_{g_{ib}}^\mathsf{T} I_i \, \mathrm{Ad}_{g_{ib}} \left(V_{0b}^b + J_i(q)\dot{q} \right) \\ &= \frac{1}{2} \left[(V_{0b}^b)^\mathsf{T} \ \dot{q}^\mathsf{T} \right] M_i(q) \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix} = \frac{1}{2} v^\mathsf{T} M_i(q) v \end{aligned} \tag{7}$$

with $M_b = \left[\begin{smallmatrix} I_b & 0 \\ 0 & 0 \end{smallmatrix} \right] \in \mathbb{R}^{(6+n)\times(6+n)}$ for the vehicle and

$$M_i(q) := \begin{bmatrix} \mathrm{Ad}_{g_{ib}}^\mathsf{T} I_i \, \mathrm{Ad}_{g_{ib}} & \mathrm{Ad}_{g_{ib}}^\mathsf{T} I_i \, \mathrm{Ad}_{g_{ib}} J_i \\ J_i^\mathsf{T} \mathrm{Ad}_{g_{ib}}^\mathsf{T} I_i \, \mathrm{Ad}_{g_{ib}} & J_i^\mathsf{T} \mathrm{Ad}_{g_{ib}}^\mathsf{T} I_i \, \mathrm{Ad}_{g_{ib}} J_i \end{bmatrix} \tag{8}$$

for the links. The total kinetic energy of the mechanism is given by the sum of the kinetic energies of the mechanism links and the vehicle, that is,

$$\mathcal{T}(q, v) = \frac{1}{2} v^\mathsf{T} \underbrace{\left(\begin{bmatrix} I_b & 0 \\ 0 & 0 \end{bmatrix} + \sum_{i=1}^{n} M_i(q) \right)}_{\text{inertia matrix } M(q)} v \tag{9}$$

with $M(q)$ the inertia matrix of the total system. Note that neither $\mathcal{T}(q, v)$ nor $M(q)$ depend on the pose g_{0b} nor the choice of inertial reference frame Ψ_0.

Finally we include the gravitational forces. Let the wrench associated with the gravitational force of link i with respect to coordinate frame Ψ_i be given by

$$F_g^i = \begin{bmatrix} f_g \\ \hat{r}_g^i f_g \end{bmatrix} = -m_i g \begin{bmatrix} R_{0i} e_z \\ \hat{r}_g^i R_{0i} e_z \end{bmatrix} \tag{10}$$

where $e_z = [0\ 0\ 1]^\mathsf{T}$ and r_g^i is the center of mass of link i expressed in frame Ψ_i. In our case Ψ_i is chosen so that r_g^i is in the origin of Ψ_i so we have $r_g^i = 0$. The equivalent joint torque associated with link i is given by

$$\tau_g^i = J_i(q)\,\mathrm{Ad}_{g_{0i}}^\mathsf{T}(Q) F_g^i(Q) \tag{11}$$

where J_i is the geometric Jacobian and $\mathrm{Ad}_{g_{0i}} = \mathrm{Ad}_{g_{0b}}\,\mathrm{Ad}_{g_{bi}}$ is the transformation from the inertial frame to frame i. The total effect of the gravity is then given by $N(Q) = \sum_{i=b}^{n} \tau_g^i$ which enters (12) in the same way as τ.

Note that we can write (5) in block-form as follows

$$\begin{bmatrix} M_{VV} & M_{qV}^\mathsf{T} \\ M_{qV} & M_{qq} \end{bmatrix} \begin{bmatrix} \dot{V}_{0b}^b \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} C_{VV} & C_{Vq} \\ C_{qV} & C_{qq} \end{bmatrix} \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \tau_V \\ \tau_q \end{bmatrix} \tag{12}$$

Here the subscript V refers to the first 6 entries and q the remaining n entries.

2.3 Vehicles with Configurations Space $SE(3)$

The configuration space of a free-floating vehicle, such as an AUV, can be described by the matrix Lie group $SE(3)$. In this case we have the mapping (Duindam, 2006)

$$V_{0b}^b = \left(I - \frac{1}{2}\mathrm{ad}_{\phi_V} + \frac{1}{6}\mathrm{ad}_{\phi_V}^2 - \ldots\right)\dot{\phi}_V \tag{13}$$

with $\mathrm{ad}_p = \begin{bmatrix} \hat{p}_{4\ldots6} & \hat{p}_{1\ldots3} \\ 0 & \hat{p}_{4\ldots6} \end{bmatrix} \in \mathbb{R}^{6\times6}$ for $p \in \mathbb{R}^6$ relating the local and global velocity variables. The corresponding matrices S_i can be collected in one block-diagonal matrix $S \in \mathbb{R}^{(6+n)\times(6+n)}$ given by

$$S(Q,\phi) = \begin{bmatrix} \left(I - \frac{1}{2}\mathrm{ad}_{\phi_V} + \frac{1}{6}\mathrm{ad}_{\phi_V}^2 - \ldots\right) & 0 \\ 0 & I \end{bmatrix}. \tag{14}$$

This shows that the choice of coordinates (Q, v) has the required form. We note that when differentiating with respect to ϕ and substituting $\phi = 0$ this simplifies the expression substantially.

To compute the matrix $C(Q, v)$ for our system, we can use the observations that $M(q)$ is independent of g_{0b}, that $S(Q,\phi)$ is independent of q, and that $S(Q,0) \equiv I$. Furthermore, the partial derivative of M with respect to ϕ_V is zero since M is independent of g_{0b}, and the second term of (6) is only non-zero for the C_{VV} block of $C(Q, v)$. Firstly, C_{VV} depends on both the first and the second term in (6). We have $i, j = 1\ldots6$. Note that $\frac{\partial M_{ij}}{\partial \phi_k} = 0$ for $k < 7$ and $\frac{\partial S_{ij}}{\partial \phi_k} = 0$ for $i, j, k > 6$. This simplifies C_{VV} to

$$C_{ij}(Q,v) = \sum_{k=7}^{6+n} \left(\frac{\partial M_{ij}}{\partial \phi_k} - \underbrace{\frac{1}{2}\frac{\partial M_{jk}}{\partial \phi_i}}_{=0} \right)\Bigg|_{\phi=0} v_k \tag{15}$$

$$+ \sum_{k=1}^{6} \left(\frac{\partial S_{ij}}{\partial \phi_k} - \frac{\partial S_{ik}}{\partial \phi_j} \right)\Bigg|_{\phi=0} (M(q)v)_k.$$

Furthermore, if we write $S = (I - \frac{1}{2}\mathrm{ad}_{\phi_V} + \frac{1}{6}\mathrm{ad}_{\phi_V}^2 - \ldots)$ we note that after differentiating and evaluating at $\phi = 0$, $\sum \frac{\partial S_{ij}}{\partial \phi_k}$ is equal to $-\frac{1}{2}\mathrm{ad}_{e_k}$ where e_k is a 6-vector with 1 in the k^{th} entry and zeros elsewhere. Similarly, $\sum \frac{\partial S_{ik}}{\partial \phi_j}$ is equal to $\frac{1}{2}\mathrm{ad}_{e_k}$. This is then multiplied by the k^{th} element of $M(q)v$ when differentiating with respect to ϕ_k so that

$$C_{VV}(Q,v) = \sum_{k=1}^{6} \frac{\partial M_{VV}}{\partial q_k}\dot{q}_k - \mathrm{ad}_{(M(q)v)_V} \tag{16}$$

where $(M(q)v)_V$ is the vector of the first 6 entries (corresponding to V_{0b}^b) of the vector $M(q)v$.

$C_{Vq}(Q, v)$, i.e., $i = 1\ldots6$ and $j = 7\ldots(6+n)$, is found in a similar manner. First we note that $\frac{\partial M_{jk}}{\partial \phi_i} = 0$ for $i = 1\ldots6$ and that $\frac{\partial S_{ij}}{\partial \phi_k} = \frac{\partial S_{ik}}{\partial \phi_j} = 0$ for $j = 7\ldots(6+n)$, so only the first part is non-zero and we get

$$C_{Vq}(Q,v) = \sum_{k=1}^{6} \frac{\partial M_{Vq}}{\partial q_k}\dot{q}_k. \tag{17}$$

Finally, the terms C_{qV} and C_{qq} depend only on the first part of (6) and can be written as (From et al., 2009)

$$C_{qV} = \sum_{k=1}^{n} \frac{\partial M_{qV}}{\partial q_k}\dot{q}_k - \frac{1}{2}\frac{\partial^\mathsf{T}}{\partial q}\left(\begin{bmatrix} M_{VV} & M_{qV}^\mathsf{T} \end{bmatrix} \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix} \right) \tag{18}$$

$$C_{qq} = \sum_{k=1}^{n} \frac{\partial M_{qq}}{\partial q_k}\dot{q}_k - \frac{1}{2}\frac{\partial^\mathsf{T}}{\partial q}\left(\begin{bmatrix} M_{qV} & M_{qq}^\mathsf{T} \end{bmatrix} \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix} \right) \tag{19}$$

The C-matrix is thus given by

$$C(Q,v) = \sum_{k=1}^{n} \frac{\partial M}{\partial q_k}\dot{q}_k \tag{20}$$

$$-\frac{1}{2}\begin{bmatrix} 2\,\mathrm{ad}_{(M(q)v)_V} & 0 \\ \frac{\partial^\mathsf{T}}{\partial q}\left(\begin{bmatrix} M_{VV} & M_{qV}^\mathsf{T} \end{bmatrix} \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix} \right) & \frac{\partial^\mathsf{T}}{\partial q}\left(\begin{bmatrix} M_{qV} & M_{qq}^\mathsf{T} \end{bmatrix} \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix} \right) \end{bmatrix}$$

3. STATE OF THE ART AUV-MANIPULATOR DYNAMICS

3.1 State of the Art AUV Dynamics

A wide range of dynamical systems can be described by the Lagrange equations (Goldstein et al., 2001)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}(x,\dot{x})\right) - \frac{\partial \mathcal{L}}{\partial x}(x,\dot{x}) = \tau \tag{21}$$

where $x \in \mathbb{R}^n$ is a vector of generalized coordinates, $\tau \in \mathbb{R}^n$ are the generalized forces and

$$\mathcal{L}(x,\dot{x}) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R} \triangleq \mathcal{T}(x,\dot{x}) - \mathcal{V}(x). \tag{22}$$

Here, $\mathcal{T}(x,\dot{x})$ is the kinetic and $\mathcal{V}(x)$ the potential energy functions. We assume that the kinetic energy function is positive definite and in the form $\mathcal{T}(x,\dot{x}) \triangleq \frac{1}{2}\dot{x}^\mathsf{T} M(x)\dot{x}$ where $M(x)$ is the inertia matrix. For a kinetic energy function on this form we can recast the Lagrange equations (21) into the equivalent form

$$M_{RB}(x)\ddot{x} + C_{RB}(x,\dot{x})\dot{x} + N(x) = \tau \tag{23}$$

where $C_{RB}(x,\dot{x})$ is the Coriolis and centripetal matrix and $N(x)$ is the potential forces vector defined as $N(x) \triangleq \frac{\partial \mathcal{V}(x)}{\partial x}$. The Coriolis and centripetal matrix is normally obtained by the Christoffel symbols of the first kind (Egeland and Gravdahl, 2003).

In addition, for floating or submerged vehicles we need to add the hydrodynamic forces and moments. The damping forces are collected in the damping matrix D and the restoring forces (weight and buoyancy) are normally included in N. Furthermore, the motion of the AUV will generate a flow in the surrounding fluid. This is known as added mass. For completely submerged vehicles operating at low velocities the added mass is given by a constant matrix $M_A = M_A^\mathsf{T} > 0$. The corresponding Coriolis matrix is given by $C_A = -C_A^\mathsf{T}$ and is found in the same way as C_{RB} by replacing M_{RB} with M_A (Fossen, 2009).

The dynamics of AUVs are usually given as (Fossen, 2002)

$$\dot{\eta} = J(\eta)\nu, \tag{24}$$

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + N(\eta) = \tau \tag{25}$$

where $\eta = [x\ y\ z\ \phi\ \theta\ \psi]^\mathsf{T}$ is the position and orientation of the vessel given in the inertial frame and $\nu = [u\ v\ w\ p\ q\ r]^\mathsf{T}$ is the linear and angular velocities given in the body frame. $D(\nu)$ is the damping and friction matrix, $M = M_{RB} + M_A$ and $C(\nu) = C_{RB}(\nu) + C_A(\nu)$. The ocean current ν_c, expressed in the inertial frame, are added by substituting $\nu_r = \nu - R_{0b}\nu_c$ into the dynamics. The velocity transformation matrix in (24) is given by $J(\eta) = \begin{bmatrix} R_{0b}(\Theta) & 0 \\ 0 & T_\Theta(\Theta) \end{bmatrix}$ where $R_{0b}(\Theta)$ is the rotation matrix and depends only on the orientations of the vessel given by the Euler angles $\Theta = [\phi\ \theta\ \psi]^\mathsf{T}$, represented in the reference frame. $T_\Theta(\Theta)$ is given by (zyx-sequence)

$$T_\Theta(\Theta) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix}. \tag{26}$$

We note that $T_\Theta(\Theta)$, and thus also $J(\eta)$, are not defined for $\theta = \pm\frac{\pi}{2}$. This is the well known Euler angle singularity for the zyx-sequence. The inverse mappings $T_\Theta^{-1}(\Theta)$ and $J^{-1}(\eta)$ are defined for all $\theta \in \mathbb{R}$ but singular for $\theta = \pm\frac{\pi}{2}$.

We can also rewrite the dynamics using general coordinates η, eliminating the body frame coordinates ν by

$$\tilde{M}(\eta)\ddot{\eta} + \tilde{C}(\eta,\dot{\eta})\dot{\eta} + \tilde{D}(\eta,\dot{\eta})\dot{\eta} + \tilde{g}(\eta) = \tilde{\tau} \tag{27}$$

where

$$\begin{aligned} \tilde{M}(\eta) &= J^{-\mathsf{T}}(\eta)MJ^{-1}(\eta), \\ \tilde{g}(\eta) &= J^{-\mathsf{T}}(\eta)g(\eta), \\ \tilde{\tau} &= J^{-\mathsf{T}}(\eta)\tau, \\ \tilde{D}(\eta,\dot{\eta}) &= J^{-\mathsf{T}}(\eta)D(J^{-1}(\eta)\dot{\eta})J^{-1}(\eta), \\ \tilde{C}(\eta,\dot{\eta}) &= J^{-\mathsf{T}}(\eta)\left[C(J^{-1}(\eta)\eta) - MJ^{-1}(\eta)\dot{J}(\eta)\right]J^{-1}(\eta). \end{aligned}$$

Note that (27) is only valid when $J^{-1}(\eta)$ is non-singular.

3.2 State of the Art AUV-Manipulator Dynamics

Write the AUV-manipulator dynamics as (Antonelli, 2006)

$$\dot{\xi} = J(\xi)\zeta, \tag{28}$$

$$M(q)\dot{\zeta} + C(q,\zeta)\zeta + D(q,\zeta)\zeta + N(q,R_{0b}) = \tau \tag{29}$$

where $\xi = \left[\eta^\mathsf{T}\ q^\mathsf{T}\right]^\mathsf{T}$, $\zeta = \left[\nu^\mathsf{T}\ \dot{q}^\mathsf{T}\right]^\mathsf{T}$, $M(q) \in \mathbb{R}^{(6+n)\times(6+n)}$ is the inertia matrix including added mass, $C(q,\zeta) \in \mathbb{R}^{(6+n)\times(6+n)}$ is the Coriolis and centripetal matrix and $D(q,\zeta) \in \mathbb{R}^{(6+n)\times(6+n)}$ is the damping matrix. The velocity transformation matrix is given by

$$J(\xi) = \begin{bmatrix} R_{0b}(\Theta) & 0 & 0 \\ 0 & T_\Theta(\Theta) & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{30}$$

4. THE PROPOSED APPROACH

In this section we show how to derive the AUV-manipulator dynamics without the presence of singularities based on Section 2. The inertia matrix of the AUV is derived in two steps. First, M_{RB} is found from (9). Then the added mass $M_A = M_A^\mathsf{T} > 0$ is found from the hydrodynamic properties and we get $M = M_{RB} + M_A$. We can now use M instead of M_{RB} to derive the Coriolis and centripetal matrix (Fossen, 2002) which gives us $C = C_{RB} + C_A$. As the configuration space of an AUV can be described by the matrix Lie group $SE(3)$ the Coriolis matrix is given by (20) by using M instead of M_{RB}. The dynamic equations can now be written as

$$M(Q)\dot{v} + C(Q,v)v + D(v)v + N(Q) = \tau. \tag{31}$$

Here, $v = \left[(V_{0b}^b)^\mathsf{T}\ \dot{q}^\mathsf{T}\right]^\mathsf{T}$ where V_{0b}^b is the velocity state of the AUV and $\dot{q}$ the velocity state of the manipulator, and $Q = \{g_{0b}, q\}$ where $g_{0b} \in SE(3)$ determines the configuration space of the AUV (non-Euclidean) and q the configuration space of the manipulator (Euclidean). We note that the singularity in (28) is eliminated and the state space (Q, v) is valid globally. $D(v)$ and $N(Q)$ are found in the same way as for the conventional approach (Antonelli, 2006). In the following we make some remarks on implementing the dynamic equations in a software environment.

Computing the Partial derivatives of $M(q_1,\ldots,q_n)$ The partial derivatives of the inertia matrix with respect to $q_1,\ldots,q_n$ are computed by

$$\begin{aligned} &\frac{\partial M(q_1,\ldots,q_n)}{\partial q_k} = \\ &\sum_{i=k}^{n}\left(\begin{bmatrix} I \\ J_i^\mathsf{T} \end{bmatrix}\left[\frac{\partial^\mathsf{T}\operatorname{Ad}_{g_{ib}}}{\partial q_k}I_i\operatorname{Ad}_{g_{ib}} + \operatorname{Ad}_{g_{ib}}^\mathsf{T} I_i\frac{\partial\operatorname{Ad}_{g_{ib}}}{\partial q_k}\right][I\ \ J_i]\right) \\ &+\sum_{i=k+1}^{n}\left(\begin{bmatrix} 0 & \operatorname{Ad}_{g_{ib}}^\mathsf{T} I_i\operatorname{Ad}_{g_{ib}}\dfrac{\partial J_i}{\partial q_k} \\ \dfrac{\partial^\mathsf{T}J_i}{\partial q_k}\operatorname{Ad}_{g_{ib}}^\mathsf{T} I_i\operatorname{Ad}_{g_{ib}} \cdots & \cdots \dfrac{\partial^\mathsf{T}J_i}{\partial q_k}\operatorname{Ad}_{g_{ib}}^\mathsf{T} I_i\operatorname{Ad}_{g_{ib}}J_i + J_i^\mathsf{T}\operatorname{Ad}_{g_{ib}}^\mathsf{T} I_i\operatorname{Ad}_{g_{ib}}\dfrac{\partial J_i}{\partial q_k} \end{bmatrix}\right) \end{aligned} \tag{32}$$

Proposition 1. Express the velocity of joint k as $V_{(k-1)k}^{(k-1)} = X_k\dot{q}_k$ for constant X_k. The partial derivatives of the adjoint matrix is given by

$$\frac{\partial\operatorname{Ad}_{g_{ij}}}{\partial q_k} = \begin{cases} \operatorname{Ad}_{g_{i(k-1)}}\operatorname{ad}_{X_k}\operatorname{Ad}_{g_{(k-1)j}} & \text{for } i < k \le j \\ -\operatorname{Ad}_{g_{i(k-1)}}\operatorname{ad}_{X_k}\operatorname{Ad}_{g_{(k-1)j}} & \text{for } j < k \le i \\ 0 & \text{otherwise} \end{cases}$$

Proof: To prove this, we write out the spatial velocity of frame Ψ_k with respect to $\Psi_{(k-1)}$ when $i < k \le j$:

$$\hat{X}_k\dot{q}_k = \hat{V}_{(k-1)k}^{(k-1)} = \dot{g}_{(k-1)k}g_{(k-1)k}^{-1} = \frac{\partial g_{(k-1)k}}{\partial q_k}g_{k(k-1)}\dot{q}_k$$

where $\hat{X} := \begin{bmatrix} \hat{X}_\omega & X_v \\ 0 & 0 \end{bmatrix}$. Comparing the first and last terms, we get

$$\frac{\partial R_{(k-1)k}}{\partial q_k} = \hat{X}_\omega R_{(k-1)k}, \qquad \frac{\partial p_{(k-1)k}}{\partial q_k} = \hat{X}_\omega p_{(k-1)k} + X_v.$$

We can use this relation in the expression for the partial derivative of $\operatorname{Ad}_{g_{(k-1)k}}$:

$$\frac{\partial \operatorname{Ad}_{g_{(k-1)k}}}{\partial q} = \begin{bmatrix} \frac{\partial R_{(k-1)k}}{\partial q_k} & \frac{\hat{p}_{(k-1)k}}{\partial q_k} R_{(k-1)k} + \hat{p}_{(k-1)k} \frac{\partial R_{(k-1)k}}{\partial q_k} \\ 0 & \frac{\partial R_{(k-1)k}}{\partial q_k} \end{bmatrix}$$
$$= \begin{bmatrix} \hat{X}_\omega & \hat{X}_v \\ 0 & \hat{X}_\omega \end{bmatrix} \begin{bmatrix} R_{(k-1)k} & \hat{p}_{(k-1)k} R_{(k-1)k} \\ 0 & R_{(k-1)k} \end{bmatrix}$$
$$= \operatorname{ad}_{X_k} \operatorname{Ad}_{g_{(k-1)k}} \tag{33}$$

It is now straight forward to show that

$$\frac{\partial \operatorname{Ad}_{g_{ij}}}{\partial q_k} = \operatorname{Ad}_{g_{i(k-1)}} \frac{\partial \operatorname{Ad}_{g_{(k-1)k}}}{\partial q_k} \operatorname{Ad}_{g_{kj}}$$
$$= \operatorname{Ad}_{g_{i(k-1)}} \operatorname{ad}_{X_k} \operatorname{Ad}_{g_{(k-1)k}} \operatorname{Ad}_{g_{kj}}$$
$$= \operatorname{Ad}_{g_{i(k-1)}} \operatorname{ad}_{X_k} \operatorname{Ad}_{g_{(k-1)j}} . \tag{34}$$

The proof is similar for $j < k \leq i$.

Implementation We first define the vector

$$\xi = (M(q)v)_V = \begin{bmatrix} (M(q)v)_1 \\ (M(q)v)_2 \\ \vdots \\ (M(q)v)_6 \end{bmatrix} = \begin{bmatrix} M_{VV} & M_{qV}^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} V_{0b}^b \\ \dot{q} \end{bmatrix}.$$

This gives the adjoint part of the second part of (20) as

$$\operatorname{ad}_\xi = \begin{bmatrix} 0 & -\xi_6 & \xi_5 & 0 & -\xi_3 & \xi_2 \\ \xi_6 & 0 & -\xi_4 & \xi_3 & 0 & -\xi_1 \\ -\xi_5 & \xi_4 & 0 & -\xi_2 & \xi_1 & 0 \\ 0 & 0 & 0 & 0 & -\xi_6 & \xi_5 \\ 0 & 0 & 0 & \xi_6 & 0 & -\xi_4 \\ 0 & 0 & 0 & -\xi_5 & \xi_4 & 0 \end{bmatrix}. \tag{35}$$

The lower part of the matrix in the second term in (20) is calculated in the following way

$$\frac{\partial^{\mathsf{T}}}{\partial q}(M(q)v)_V = \begin{bmatrix} \frac{\partial (Mv)_1}{\partial q_1} & \frac{\partial (Mv)_2}{\partial q_1} & \cdots & \frac{\partial (Mv)_6}{\partial q_1} \\ \frac{\partial (Mv)_1}{\partial q_2} & \frac{\partial (Mv)_2}{\partial q_2} & \cdots & \frac{\partial (Mv)_6}{\partial q_2} \\ \vdots & & \ddots & \vdots \\ \frac{\partial (Mv)_1}{\partial q_n} & \frac{\partial (Mv)_2}{\partial q_n} & \cdots & \frac{\partial (Mv)_6}{\partial q_n} \end{bmatrix} \tag{36}$$

where $\frac{\partial (Mv)_j}{\partial q_k}$ is calculated as

$$\frac{\partial (Mv)_j}{\partial q_k} = \sum_{i=1}^{6+n} \frac{\partial M_{ji}}{\partial q_k} v_i. \tag{37}$$

The second part of (19) is computed in the same way. We thus only need to compute the partial derivative $\frac{\partial M(q)}{\partial q_i}$ once and use the result in the both in the first and second part of (20).

5. CONCLUSIONS

In this paper the dynamic equations of AUV-manipulator systems are derived based on Lagrange's equations. The main contribution is to close the gap between manipulator and AUV dynamics which allows us to derive the AUV-manipulator dynamics using the Lagrange framework and without singularities. We derive the dynamics of the AUV based on Lagrange's equations which naturally extends to include also the manipulator dynamics. The globally valid AUV-manipulator dynamics are thus derived for the first time using the proposed framework.

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