A Geometric Approach to Actuator Failure in Robotic Manipulators

Pål Johan From[1][*], **Cong Dung Pham**[1], and **Jan Tommy Gravdahl**[2]

[1] Department of of Mathematical Sciences and Technology, Norwegian University of Life Sciences, Ås, Norway
[2] Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway

The use of robotic manipulators in remote and sensitive areas calls for more robust solutions when handling joint failure, and the industry demands mathematically robust approaches to handle even the worst case scenarios. Thus, a systematic analysis of the effects of external forces on manipulators with passive joints is presented. In parallel manipulators passive joints can appear as a design choice or as a result of torque failure. In both cases a good understanding of the effects that passive joints have on the mobility and motion of the parallel manipulator is crucial. We first look at the effect that passive joints have on the mobility of the mechanism. Then, if the mobility, considering passive joints only, is not zero we find a condition for which the parallel manipulator is conditionally equilibrated with respect to a specific external force.

1 Introduction

The need for a rigorous theory on what happens when joint failure occurs is important to be able to cope with unforeseen events such as actuation failure. We are interested in the undesired motion generated by the passive joints that cannot be compensated for by the active joints. For parallel manipulators, joint failure may or may not allow a motion generated by the passive joints. If the manipulator does not allow such a passive motion, we will denote it equilibrated. In this case it can resist a wrench in an arbitrary direction either through kinematic constraints or through actuator torques. We obtain this if the manipulator, considering the passive joints only, has mobility equal to zero, i.e. we do not want the passive joints to allow any motion when the active joints are locked. If this property is satisfied the manipulator does not have an unstable singularity, following the classification in [1].

2 Robustness to external forces for parallel manipulators

We find all configurations for which the mechanism is conditionally equilibrated with respect to an external force. Without loss of generality assume that all the passive joints are at the end of the sub-chains, i.e., $\mathcal{M} = \mathcal{M}_A \cdot \mathcal{M}_P$ where $\mathcal{M}_A$ represents the active and $\mathcal{M}_P$ the passive joints. We need to verify if the mechanism, considering the passive joints only, is equilibrated with respect to an external force F_g. We denote the transformation of $\mathcal{M}_P$ by the active joints $\mathcal{M}_A$ as g_A and find [2]

$$\boldsymbol{G}_A = \{g_A \mid R_{g_A^{-1}*} T_{g_A} C_{\mathcal{M}_P} \in R_{g^{-1}*} T_g Q_{S0}\} \tag{1}$$

i.e. the joint positions of the active joints that generate only equilibrated motion $Q_S = R_{g^{-1}*} T_g Q_{S0}$. More specifically,

$$R_{g_A^{-1}*} T_{g_A} C_{\mathcal{M}_P} = R_{g_A^{-1}*} T_{g_A} C_{\mathcal{M}_{P1}} \cap \cdots \cap R_{g_A^{-1}*} T_{g_A} C_{\mathcal{M}_{Pk}} \tag{2}$$

is the attainable spatial velocities of $\mathcal{M}_P$ at g_A and $R_{g^{-1}*} T_g Q_{S0}$ is the equilibrated motion with respect to F_g in a specific reference frame g and $R_{g^{-1}*}$ is the differential of the inverse of the right translation defined as $R_g h = hg$. See [3] for details.

The main observation here is that the infinitesimal motions attainable by $\mathcal{M}_P$, when $\mathcal{M}_P$ is at the end of the chains, are transformed by a rigid transformation g_A which depends on the active joints only. Thus, we denote the Adjoint map of g_A by Ad_{g_A} and write $\overline{\mathcal{M}}_P' = \mathrm{Ad}_{g_A} \overline{\mathcal{M}}_P$.

We will divide the motion of the mechanism into two motions. First, $C_{\mathcal{M}_P}$ is the motion due to the passive joints. This motion is affected by the external disturbances. The other motion is $C_{\mathcal{M}_A}$ which is due to the active joints. This is not affected by the external disturbance. The aim of this paper is to find the configurations of the active joints so that $C_{\mathcal{M}_P} \in Q_S$. We will write $\overline{\mathcal{G}}_{ji}' = \mathrm{Ad}_{g_{j,bi}} \overline{\mathcal{G}}_{ji}$, where $g_{j,bi}$ is the transformation from the base to joint i of chain j. Thus the direction of the twists of the passive joints will depend on the position of the active joints, i.e. $g_{j,bi}$ depends on the position of the active joints.

We need to verify if $\overline{\mathcal{M}}_P \in Q_S$ where $\overline{\mathcal{M}}_P = \overline{\mathcal{M}}_{P1} \cap \overline{\mathcal{M}}_{P2} \cap \cdots \cap \overline{\mathcal{M}}_{Pk}$ and $\overline{\mathcal{M}}_{Pj} = \{\overline{\mathcal{G}}_{j1}', \overline{\mathcal{G}}_{j2}', \cdots, \overline{\mathcal{G}}_{jn_j}'\}$ are the twists $\overline{\mathcal{G}}_{ji}'$ of link j. We can now find the set of conditionally equilibrated configurations as

$$\boldsymbol{G}_A = \{g_A \mid \overline{\mathcal{M}}_P(g(\theta)) \in Q_S\} \tag{3}$$

which is found by $\boldsymbol{G}_A = \{g_A \mid \langle \mathrm{Ad}_{g(\theta)}(\overline{\mathcal{M}}_P), F_g \rangle = 0\}$ which is the set of all equilibrated configurations for $\mathcal{M}$, i.e., the joint positions for which the mechanism is not affected by F_g.

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[*] Corresponding author: e-mail pafr@nmbu.no

3   Example

Free swinging joint faults affect parallel manipulators differently than serial manipulators. For serial manipulators joint faults is extremely serious while this is not always the case for closed chain manipulators due to the kinematic constraints. In this section we present a simple example illustrating the effects of torque failure in parallel mechanisms.

Consider the parallel manipulator in Fig. 1. Assume that the actuated joints are chosen as in Fig. 1 and joint failure occurs in $\mathcal{M}_{12}$. We choose a reference configuration as in Fig. 1 and the twists of each chain is given by [4]

$$\overline{\mathcal{M}}_{P1} = \left\{ \begin{bmatrix} v_x \\ 0 \end{bmatrix}, \begin{bmatrix} p_{12} \times w_x \\ 0 \end{bmatrix}, \begin{bmatrix} p_{13} \times w_x \\ 0 \end{bmatrix}, \begin{bmatrix} p_{14} \times w_z \\ w_z \end{bmatrix} \right\},$$

$$\overline{\mathcal{M}}_{P2} = \left\{ \begin{bmatrix} v_z \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ w_z \end{bmatrix} \right\},$$

$$\overline{\mathcal{M}}_{P3} = \left\{ \begin{bmatrix} v_z \\ 0 \end{bmatrix}, \begin{bmatrix} p_{32} \times w_z \\ w_z \end{bmatrix}, \begin{bmatrix} p_{35} \times w_z \\ w_z \end{bmatrix} \right\},$$

and we get

$$\overline{\mathcal{M}}_P = \overline{\mathcal{M}}_{P1} \cap \overline{\mathcal{M}}_{P2} \cap \overline{\mathcal{M}}_{P3} = \begin{bmatrix} v_z \\ 0 \end{bmatrix}.$$
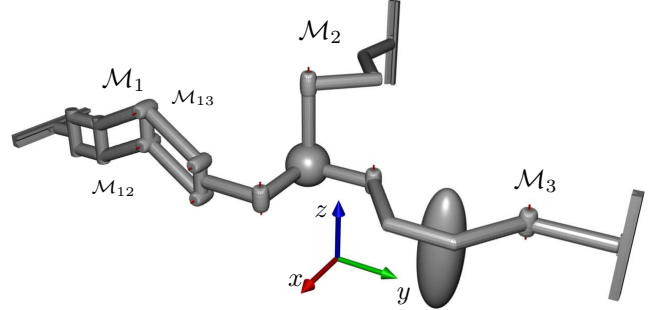

Fig. 1: If joint failure occurs in joint $\mathcal{M}_{12}$, the mechanism is no longer equilibrated with respect to forces in the direction of the z-axis, such as gravitational forces. Joints $\mathcal{M}_{12}$ and $\mathcal{M}_{13}$ are parallelogram joints that generate motion in S^1. Actuated joints are shown as rigid.

Thus for the chosen reference configuration, $\mathcal{M}_P$ is not conditionally equilibrated with respect to the gravitational forces. It is, however, conditionally equilibrated with respect to all forces in the xy-plane, e.g.

$$\langle \overline{\mathcal{M}}_P, \overline{F}_x \rangle = 0, \qquad \langle \overline{\mathcal{M}}_P, \overline{F}_y \rangle = 0, \qquad \langle \overline{\mathcal{M}}_P, \overline{F}_z \rangle \neq 0.$$

We now look into for what configurations this is true. This is straight forward due to the observation that q_A does not appear in neither $\overline{\mathcal{M}}_{P1}$, $\overline{\mathcal{M}}_{P2}$, nor $\overline{\mathcal{M}}_{P3}$, so

$$\overline{\mathcal{M}}'_{Pij} = \mathrm{Ad}_{g_{j,bi}(\theta)} \overline{\mathcal{M}}_{Pij} = \overline{\mathcal{M}}_{Pij}, \qquad \forall i, j, \theta \tag{4}$$

and thus the twists of the passive joints are independent of positions of the active joints. The set of joint positions for which the manipulator is conditionally equilibrated with respect to F_y (a linear force in the direction of the y-axis) is thus given by

$$\Theta_{F_y} = \{\theta \mid \langle \mathrm{Ad}_{g(\theta)}(\overline{\mathcal{M}}_P), F_y \rangle = 0\} = \{\forall \theta\} \tag{5}$$

which means that it can always resist a force in this direction. Similarly, the set of joint positions for which the manipulator is conditionally equilibrated with respect to F_z is thus given by

$$\Theta_{F_z} = \{\theta \mid \langle \mathrm{Ad}_{g(\theta)}(\overline{\mathcal{M}}_P), F_z \rangle = 0\} = \{\emptyset\} \tag{6}$$

which means that it can never resist a force in this direction.

4   Conclusion

A mathematically rigorous framework for analysing the effects of joint failure in parallel manipulators is presented. For parallel manipulators, we can find a set of active joints for which the design itself is fault tolerant. In this sense, the parallel manipulators are more robust than their serial counterparts. On the other hand, when actuator failure occurs and this allows for a motion in the passive joints, we have less flexibility to deal with this in the control algorithms than for serial manipulators. In general we find that the parallel manipulator is either conditionally equilibrated for all configurations, or it is never conditionally equilibrated. Fault tolerance of parallel manipulators should thus be addressed in the design of the mechanism.

References

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