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Combined state and parameter estimation for not fully observable dynamic systems[☆]

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ABSTRACT

In this paper, a simple, yet novel method for state estimation and parameter identification for dynamic systems is presented. Apart from providing estimates of non-measurable state variables, the algorithm is also capable of estimating (constant) system parameters. The estimation algorithm is split in two parts. Firstly, an extended Kalman filter, whose state-space-model is augmented with quasi-linear expressions for parameter values, providing estimates for the state variables and the augmented parameter values. Secondly, a Monte-Carlo-fashioned approach, which identifies the rest of the parameter values that were not included in the augmentation of the state-space model. The Monte-Carlo-approach minimizes an objective function (the error between the measured and the estimated state variable). It is shown that the algorithm is capable of estimating the state- and parameter-values in a satisfying manner. The method is best applied offline and the theoretical developments will be demonstrated in case studies.

1. Introduction

The problem of combined estimation of state variables and model parameters for dynamic systems can be attacked by many methodologies. Some of these are for example (extended) Kalman filters, for which the state-space is augmented by the parameters in a quasi-stationary way. In addition, particle filters are often used in order to estimate state variables and parameter values of linear and nonlinear systems, but with rather high computational cost. This work should be seen in the context of the wide field of system identification, may it be for empirical or first principle models, static or dynamic relationships, inferential measurements, etc. For a standard book in system identification, we refer to Ljung (1999).

The results presented in this paper depict an extension to Backi, Gravdahl, and Skogestad (2018).

1.1. Application-oriented aspects

Even though for some applications, models with good approximations of the real behavior of a plant can be found, often the determination of suitable values for parameters of these models is not an easy task. This is the case for models that are identified by empirical analysis such as system identification, but also for models, which are derived from first principles. In addition, the values of system parameters can change with progressing time in operation, may it be caused by fouling, reduction in friction, and so on. This makes the determination of these parameters a crucial task in order to approach an as precise representation of the system as possible to be used in model-based controllers and estimators.

1.2. Related works

The topic of combined state and parameter estimation and identification is covered well in the literature. Sole state estimation for linear dynamic systems can often be solved by utilizing Luenberger-style observers and Kalman filters. These two principles can also be used to design estimators for nonlinear systems, but require steady linearization of the dynamic system around current state trajectories and operating points.

In addition to the Luenberger and Kalman methods, several nonlinear state observer designs have been developed over the years. These, however, only hold for specific classes of nonlinear

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systems (Arcak & Kokotovic, 2001; Esfandiari & Khalil, 1992; Raghavan & Hedrick, 1994; Rajamani, 1998) and there exist no generalized methods that can be applied for all kinds of systems. In practice, one would often fall back to Kalman filter solutions, since it provides some robustness properties.

All of the abovementioned methods require the principles of observability or at least detectability to be ensured, either for the case of linear systems as described by Kalman or nonlinear systems, as e.g. discussed in Hermann and Krener (1977), Marino and Tomei (1995) and Nijmeijer and van der Schaft (1990) utilizing Lie-derivatives.

The authors in Yu and Bernstein (2016) established initial results towards the development of necessary and sufficient conditions for combined state and parameter estimation. Their investigation holds for classes of linear systems, which inherently become nonlinear for combined state and parameter estimation. The original problem is thereby reformulated as an identifiability problem.

Several works tackle the problem of state and parameter estimation by utilizing Kalman filters and their derivatives. Ma, Ding, Xiong, and Yang (2016) use several Kalman filtering techniques for combined state and parameter estimation for Hammerstein systems in the presence of unknown time delays. Evensen (2009) utilizes an ensemble Kalman filter strategy for combined state and parameter estimation based on Monte Carlo techniques, especially for large systems. In Ding (2014a), the problem of scarcely available measurements in the context of state and parameter estimation is solved by state filtering and parameter identification utilizing Kalman and least squares techniques. In Ding (2014b), an algorithm is introduced that solves the state and parameter estimation problem for classes of linear systems in observer canonical form. Thereby, the parameters represent entries in the system matrix A and the input matrix B of the linear system, a somewhat similar problem to the linear examples presented in this paper. Ma and Ding (2015) presents iterative least squares parameter estimation algorithms, among them a Kalman filter technique, for linear systems, also in observer canonical form, while in Ding, Liu, and Ma (2016) a similar problem is addressed by decomposition methods. As can be seen, the above listed estimation techniques solve special estimation problems, may it be large systems, systems in specific forms and under special circumstances.

Cox (1964) presents a real-time approach for discrete-time linear systems in the context of combined state and parameter estimation in an early work on this topic.

Schön (2006) presents combined state and parameter estimation for differential–algebraic equations, where marginal particle filters and sequential Monte-Carlo-simulations are utilized to attack the problem.

The authors in Xu (2016) introduce an iterative method, in this case for sine signal excitation in order to estimate parameters of linear systems in the form of transfer functions. Guo and Ding (2015) applies yet another iterative method to pseudo-linear autoregressive moving average (ARMA) models.

Generally, the problem of parameter identification and estimation for nonlinear systems is approached by comparing measured data and model candidates, often in a least-squares manner. However, other methods exist, such as utilizing conditional Lyapunov exponents (CLE) as a measure to determine the coupling between data and model, see Abarbanel, Creveling, Farsian, and Kostuk (2009) and Creveling (2008). In addition to the CLE, a dynamical parameter estimation technique must be implemented, which acts as an observer in the classical sense. A real-time algorithm for simultaneous estimation of parameters and identification of anomaly patterns using escort probabilities is presented in Rao, Mukherjee, Sarkar, and Ray (2008). Mehrkanoon, Falck, and Suykens (2012) introduces parameter identification of dynamical systems incorporating least-squares support vector machines. The authors claim that their method can be applied to both time-varying and time invariant dynamics.

1.3. Contributions

An accepted and standard way for combined estimation of states and parameters is the utilization of (extended) Kalman filters by augmenting the original dynamic system with quasi-stationary parameter dynamics. In most cases, this leads for originally linear systems to become nonlinear, since the parameters are now treated as new state variables and typically enter the equations in a nonlinear fashion after augmenting the state space. As for the original system, the augmented system description must still provide full observability (detectability) in order to be able to estimate all state variables, including the parameters. In the context of this work, observability is defined as introduced by Kalman ('linear observability'), meaning that the observability test including the Jacobian A and the output matrix C must provide full rank for all feasible values for the state variables. Hence, the observability matrix should not lose rank for any feasible state combination.

In this work, a simple and practical method for combined state and parameter estimation for (non)linear dynamic systems is presented, where the original system description loses the property of observability after augmentation of the state-space with the parameters. A drawback of the method is the rather high computational effort, since for each set of parameter combinations, one simulation has to be conducted and evaluated. A big advantage, however, is the fact that the method can be applied offline, provided that measurements are available that satisfy properties of persistence of excitation. A lack of this property might imply that a reconstruction of the state variables and parameters from measurement data is not possible at all.

1.4. Remark on identifiability

Although the notion of identifiability naturally arises in the context of parameter estimation, its application is left out in this application-oriented paper. Nonetheless, we refer to Godfrey and DiStefano (1985) and Grewal and Glover (1976) for an overview and insight into this broad topic, latter including many hands-on examples.

1.5. Structure of the paper

Sections 2 and 5 introduce the methodology for estimation of parameters in not fully observable, dynamic systems. In Sections 3, 4 and 6, nonlinear and linear case studies are presented. Comments on the proposed methodology and potential future work follow in Section 7.

2. Methodology

Combined state and parameter estimation applying (extended) Kalman filters (EKF) is a mature technology and can be viewed as a standard way to attack and solve problems of this kind. Nevertheless, these methods solely work for dynamic systems with the property of full observability, which also must hold after augmentation of the state space with the parameters as new state variables for a given output function. Relying on detectability might be enough in some cases, which means that all unobservable modes are stable. Subsequently, only full observability will be regarded as a criterion.

We regard state space models in the form

$$\dot{x} = f(x, u) + w(t),$$
$$y = h(x) + v(t),$$

where $h(x)$ is assumed to be a linear function Cx with constant output matrix C. In addition, $w(t)$ and $v(t)$ denote process and measurement noises with covariance matrices $Q = \text{cov}\{w(t)w^T(t)\}$ and $R = \text{cov}\{v(t)v^T(t)\}$. No covariance is assumed between $w(t)$ and $v(t)$.

2.1. Extended Kalman filter

Designing an EKF implies that the nonlinear state equations $\dot{\hat{x}} = f(\hat{x}, u)$ are linearized around current state trajectories/ operating points $\hat{x}_s$ to obtain the Jacobian

$$A = \left.\frac{\partial f_i}{\partial \hat{x}_j}\right|_{\hat{x}_s}. \tag{1}$$

The Jacobian has to be updated subject to the current operating point of the process and hence the linearization points $\hat{x}_s$ must be held variable, which implies that the system matrix A will be time-dependent, hence $A = A(t)$. The state- and therefore time-varying Jacobian is then utilized to solve the differential matrix Riccati equation and ultimately obtain the time-varying Kalman feedback gain $K(t)$

$$\dot{P}(t) = A(t)P(t) + P(t)A^T(t) - K(t)CP(t) + Q,$$
$$K(t) = P(t)C^T R^{-1}$$

in order for the estimated output $\hat{y} = C\hat{x}$ to track the measured output y and ultimately estimate the unmeasurable state variables.

In case of additional parameter estimation, the quasi-stationary dynamics of the parameters $\dot{\hat{p}} = 0$ are included as new state variables in the state space in the following way

$$\dot{\hat{x}} = f(\hat{x}, \hat{p}, u),$$
$$\dot{\hat{p}} = 0.$$

This involves that the Jacobian (1) increases from primarily $A_{(n\times n)}$ to $A_{((n+\rho)\times(n+\rho))}$, where ρ is the number of parameters and n represents the dimension of the system. Furthermore, the output matrix C must be changed accordingly. Usually, this is performed by adding zeros to the respective entries in C relating to the new state variables $\hat{p}$, since these cannot be measured, and hence $C_{ext} = \begin{bmatrix} C & 0_{(\dim(y)\times\rho)} \end{bmatrix}$. Finally, the dynamics of the observer for the augmented system with $\hat{x}_{ext} = \begin{bmatrix} \hat{x} & \hat{p} \end{bmatrix}^T$ are formulated as

$$\dot{\hat{x}}_{ext} = f_{ext}(\hat{x}_{ext}, u) + K(t)\left(y - C_{ext}\hat{x}_{ext}\right). \tag{2}$$

2.2. Observability

The concept of observability defines a measure that shows if reconstructing unmeasurable, internal states of a system is possible by solely using external outputs. Thereby, it must hold that the well-known observability matrix, which links the system matrix $A(t)$ with the output matrix C has rank n. At this point, it should be noted that observability measures particularly for nonlinear systems exist, as mentioned in the introduction. In the context of the above observer design relying on steady update of the Jacobian around the current state trajectories, the observability matrix must have full rank for all feasible operating points/time instances. Now, if the original system description is augmented by the new state variables representing the full set of parameters $\mathscr{P}$, it might happen that the initially fully observable system might lose this property. However, often it is the case that by only adding a subset $\mathscr{P}_s \in \mathscr{P}$, full observability still holds for the system at hand. It is necessary to test for a maximum allowable subset $\overline{\mathscr{P}_s}$, which ensures the full observability property. It is assumed that also for the augmented system, full observability must hold for all admissible and feasible operating points $\hat{x}_s$ in order to estimate the state variables and the subset of parameters. This implies that also in the case of a state- and time-varying Jacobian the observability matrix must have full rank, ergo not lose rank after augmentation.

2.3. Determination of unobservable parameters

Determination of the values for the unobservable parameters $\hat{p}_{u_i} \in \mathscr{P}_u = \mathscr{P} \setminus \overline{\mathscr{P}_s}$ is performed in a Monte-Carlo-fashioned approach. First, lower and upper bounds for feasible parameter values $\underline{\hat{p}_{u_i}}$ and $\overline{\hat{p}_{u_i}}$ have to be determined and increments between these bounds must be defined. Then, one simulation for each combination of the parameters must be conducted. Ultimately, by defining an objective function to be minimized, the best values for the unobservable parameters can be found. A feasible objective function can thereby be the difference between the measured output y and their respective estimates, since the measurements present the only information available from the real plant

$$\underset{\mathscr{P}_u}{\text{minimize}} \qquad J = \left\| y - C_{ext}\hat{x}_{ext} \right\|_1 = \sum_{t=0}^{t_{sim}} \left| y - C_{ext}\hat{x}_{ext} \right| \tag{3}$$
$$\text{subject to} \qquad (2),$$
$$\underline{\hat{p}_{u_i}} \le \hat{p}_{u_i} \le \overline{\hat{p}_{u_i}},$$

where, here, the 1-norm is chosen as objective function. The reason for this is that for small estimation errors, this norm provides steeper gradients compared to e.g. the 2-norm. Nevertheless, other norms, functions of norms or combinations of norms present potential objective function as well, depending on the system at hand.

In this context it must be mentioned that some system models require persistence of excitation, at least until the parameters have converged. This can be accomplished by e.g. sinusoidal reference signals, but some systems naturally provide persistent excitation since some operating points might lead to limit cycle oscillations of state variables. In this work, only systems with cyclic and oscillatory behavior are considered, caused either by sinusoidal excitation or the dynamics.

2.4. Algorithm

Algorithm 1 generally describes the individual steps to be performed to run simulations.

3. Case study 1: Nonlinear system

This case study demonstrates the application of the methodology from Section 2 to a highly nonlinear system. It is shown in numerical simulations that the method also delivers satisfactory result in the presence of added measurement noise. Parameter estimation for these kind of systems is important if controllers based on feedback linearization are designed for surge control, see e.g. Backi, Gravdahl, and Skogestad (2016).

3.1. Mathematical model

We use a Greitzer compressor model in combination with a close-coupled valve (CCV) formulated in two states. This model has been previously used in e.g. Gravdahl and Egeland (1997). A nonlinear state-estimation approach using the design developed in Arcak and Kokotovic (2001) has been applied to the

Algorithm 1 Identification Algorithm

1: Initialization

- Design the extended Kalman filter (EKF)
- Find the set of parameters $\overline{\mathscr{P}_s}$, for which the augmentation of the original observer model is still fully observable
- Determine lower as well as upper bounds for all $\hat{p}_{u_i} \in \mathscr{P}_u$ and define increments between these bounds
- Choose tuning matrices Q and R for the EKF
- If necessary: Pre-filter noisy measurement

2: Loop

- Define one *for-loop* for each $\hat{p}_{u_i} \in \mathscr{P}_u$
- Run one simulation for each feasible combination within $\mathscr{P}_u$
- Compute the value of the objective function (3) and stack values

3: Evaluation

- Find parameter combination that gives the smallest norm → candidates for real parameter values
- Re-run simulation with minimizing parameter combination

4: Verification

- (Heavy) Oscillations in the error between true value and estimate? → Parameter combination most likely not optimal
- Change bounds and / or increments for $\hat{p}_{u_i}$
- Re-tune EKF and / or potentially change the objective function
- Continue at step 2

model in Backi, Gravdahl, and Grøtli (2013), which, just like Backi et al. (2016), relies on good knowledge of the model parameters. The equations are transformed to the origin and formulated in dimensionless variables

$$\begin{aligned}\dot{\psi} &= \frac{1}{B}\left(\phi - \Phi\left(\psi\right)\right),\\ \dot{\phi} &= B\left(\Psi_c\left(\phi\right) - \psi - u\right).\end{aligned} \tag{4}$$

The input u (manipulated variable) represents the pressure drop across the CCV, the function

$$\Phi\left(\psi\right) = \gamma\left(\operatorname{sgn}(\psi + \psi_0)\sqrt{|\psi + \psi_0|} - \sqrt{\psi_0}\right)$$

indicates the throttle characteristics and the equation

$$\Psi_c\left(\phi\right) = -k_3\phi^3 - k_2\phi^2 - k_1\phi,$$

describes the compressor characteristics.

The variable ϕ is the dimensionless mass flow $\left(\phi = \frac{\dot{m}}{\rho\, UA_c}\right)$, whereas ψ represents the dimensionless pressure $\left(\psi = \frac{p}{0.5\,\rho\, U^2}\right)$. The parameter γ defines the throttle gain. Note furthermore that $\operatorname{sgn}(0) = 0$, that time has been normalized by the Helmholtz frequency, thus $\tau = t\,\omega_H$, and that $u \geq 0$.

The parameters are defined as $B = \frac{U}{2a_s}\sqrt{\frac{V_p}{A_c L_c}} > 0$, where U defines the compressor blade tip speed, a_s represents the speed of sound, V_p is the plenum volume, A_c stands for the flow area and L_c indicates the length of ducts and compressor. It holds furthermore that $k_1 = \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)$, $k_2 = \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)$ and $k_3 = \frac{H}{2W^3}$ with $H > 0$, $W > 0$ and $\phi_0 > 0$.

Using the compressor characteristics in initial (non-transformed) coordinates

$$\psi_0(\phi_0) = \psi_{0_c} + H\left[1 + \frac{3}{2}\left(\frac{\phi_0}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi_0}{W} - 1\right)^3\right] \tag{5}$$

the operating point ψ_0 can be calculated. In addition, the throttle gain can be determined via the relation $\gamma = \frac{\phi_0}{\sqrt{\psi_0}}$ (see Gravdahl & Egeland, 1997). It should be noted that specific operating points (ψ_0, ϕ_0) cause a limit cycle behavior known as surge.

For convenience, the state variables are defined as $\psi = x_1$ and $\phi = x_2$ hereafter. Furthermore, the parameters k_1, k_2 and k_3 are rewritten in terms of H and W, which leads to a reduction of the number of parameters by one. Eventually, the original system (4) can be expressed as

$$\begin{aligned}\dot{x}_1 &= \frac{1}{B}\left[x_2 - \gamma\left(\operatorname{sgn}(x_1 + \psi_0)\sqrt{|x_1 + \psi_0|} - \sqrt{\psi_0}\right)\right],\\ \dot{x}_2 &= B\left[-\frac{H}{2W^3}x_2^3 - \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)x_2^2\right.\\ &\qquad\left. - \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)x_2 - x_1 - u\right],\end{aligned} \tag{6}$$

where ψ_0 is defined in (5). The only state that is assumed measurable is the pressure, since mass flow generally is more challenging to measure in real time, and hence $y = x_1$ and hence $C = \begin{bmatrix}1 & 0\end{bmatrix}$.

3.2. Observer design

The model formulated in (6) is now the basis for the observer design. The parameters H, W and B are assumed uncertain or unknown and ultimately the dynamics of the observer including the quasi-stationary dynamics for the three parameters $\hat{H}$, $\hat{W}$ and $\hat{B}$ are

$$\dot{\hat{x}}_1 = \frac{1}{\hat{B}}\left[\hat{x}_2 - \gamma\left(\operatorname{sgn}(\hat{x}_1 + \hat{\psi}_0)\sqrt{\left|\hat{x}_1 + \hat{\psi}_0\right|} - \sqrt{\hat{\psi}_0}\right)\right], \tag{7}$$

$$\begin{aligned}\dot{\hat{x}}_2 &= \hat{B}\left[-\frac{\hat{H}}{2\hat{W}^3}\hat{x}_2^3 - \frac{3\hat{H}}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}} - 1\right)\hat{x}_2^2\right.\\ &\qquad\left. - \frac{3\hat{H}\phi_0}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}} - 2\right)\hat{x}_2 - \hat{x}_1 - u\right],\end{aligned} \tag{8}$$

$$\dot{\hat{H}} = 0, \quad \dot{\hat{W}} = 0, \quad \dot{\hat{B}} = 0, \tag{9}$$

where $\hat{\psi}_0$ is defined in (5), but now with $\hat{H}$ and $\hat{W}$. The Jacobian for the observer dynamics (7)–(9) is calculated to

$$A = \left.\begin{bmatrix} -\dfrac{\gamma\,\operatorname{sgn}(\hat{x}_1 + \hat{\psi}_0)}{2\hat{B}\sqrt{\left|\hat{x}_1 + \hat{\psi}_0\right|}} & \dfrac{1}{\hat{B}} & A_{13} & A_{14} & A_{15}\\ -\hat{B} & A_{22} & A_{23} & A_{24} & A_{25}\\ & & \mathbf{0}_{(3\times5)} & & \end{bmatrix}\right|_{\hat{x}_s}, \tag{10}$$

where the entries A_{ij} are specified in Appendix.

The observability condition introduced in Section 2.2 states that the above system is not observable for the full set of parameters $\mathscr{P} = \{\hat{H}, \hat{W}, \hat{B}\}$. However, by defining $\overline{\mathscr{P}_s} = \{\hat{H}\}$ and hence $\mathscr{P}_u = \{\hat{W}, \hat{B}\}$, full observability can be obtained with $C_{ext} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$. The Jacobian (10) eventually reduces to the upper left 3–by–3 matrix.

A pre-investigation of the parameters $\hat{W}$ and $\hat{B}$ is possible when investigating the Jacobian (10). As mentioned above, $\hat{W}$ can only be positive, and the same holds for $\hat{B}$. This can also be deduced from (17)–(18). By investigating the square-roots in the first two equations in (17), we see that the radicand

$$\psi_{0_c} + \hat{H}\hat{\alpha} \tag{11}$$

Table 1
Simulation parameters.

A_c	Flow area	0.01 m^2
B	B-Parameter	≈0.832
H	Coefficient	0.18
L_c	Length of ducts and compressor	3 m
U	Compressor blade tip speed	80 m s^{-1}
V_p	Plenum volume	1.5 m^3
W	Coefficient	0.25
a_s	Speed of sound	340 m s^{-1}
ψ_0	Operating point for x_1	0.533
ϕ_0	Operating point for x_2	0.3
ψ_{0_c}	Constant	0.3

$Q_O = \text{diag}\left(10^{-3}, \quad 10^{-1}, \quad 10^{-1}\right), \quad R_O = 10^{-1}$

must be positive. Solving this equation for $\hat{W}$, a lower bound for $\hat{W}$ can be determined as a function of ψ_{0_c} and ϕ_0, which are assumed to be known. Ultimately, $\hat{H}$ is estimated by the EKF.

The other square-root's radicand $\left|\hat{x}_1 + \psi_{0_c} + \hat{H}\hat{\alpha}\right|$ is simply the absolute value of the first radicand (11) plus the estimated state variable $\hat{x}_1$. Unfortunately, this radicand can become zero if $-\hat{x}_1 \equiv \psi_{0_c} + \hat{H}\hat{\alpha}$. Yet, by knowing the range of the measured variable x_1 and under the assumption that the estimation error $\tilde{x}_1$ goes to zero fairly fast, a bound for $\hat{W}$ can be obtained in the same fashion as for (11).

3.3. Simulations

All simulations were conducted open-loop, i.e. $u = 0$, with a fixed-step solver of step size 0.01. Despite the problem being formulated in continuous-time, performance evaluation was conducted in quasi-discrete-time. This is due to the fact that the system has been discretized in a consistent way utilizing the fixed-step solver and is further indicated by using a sum in the objective function (3). The oscillations in x_1 and x_2 are due to the operating point in the surge area. The simulation parameters are listed in Table 1 and the Kalman filter tunings are shown below.

3.3.1. Added noise to the measurement of x_1

We added noise to the measurement, which was band-limited white noise with a sample time of 0.01, a noise power of 10^{-5} dB and a seed of [23341].

Fig. 1 demonstrates that the ranges of W and B can be narrowed down successfully. However, there is a clear bias from the optimal point, in particular for B, since W can be identified correctly.

Fig. 2 shows the estimates for x_1, x_2 and H for the obtained optimal values of W and B. As can be seen, estimation is not perfect since there exist small oscillations, especially for $\tilde{x}_2$ and $\hat{H}$, which is an indicator for not having found the real values of W and B.

3.3.2. Filtered measurement

To demonstrate, that pre-filtering of the noisy measurement is advisable, we implemented a moving average filter for the noisy measurement introduced above. The filter utilized the respective 30 previous and subsequent measurement points to calculate the average.

Comparing Fig. 3 to Fig. 1, it can be seen that the optimal values for W and B are much closer to the real values.

Fig. 4 shows the estimates, which are still in an acceptable range, even for the not optimally identified value of B.

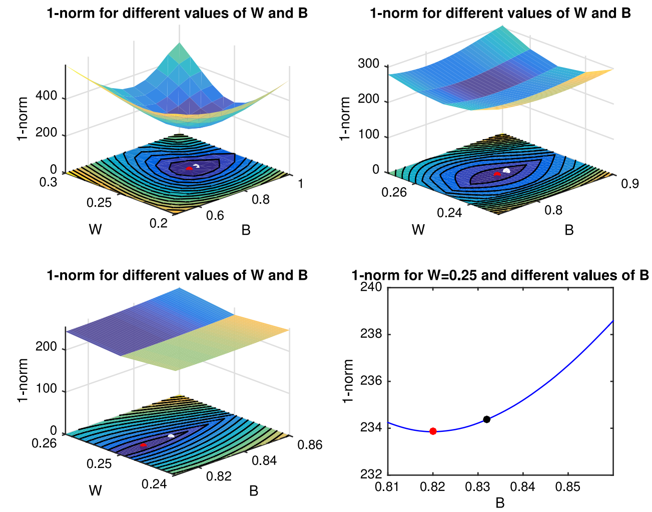

Fig. 1. Simulations for different ranges of W and B. The red dots display the obtained minima, whereas the white/black dots display the real value. (For interpretation of the references to color in the figure legends, the reader isreferred to the web version of this article.).

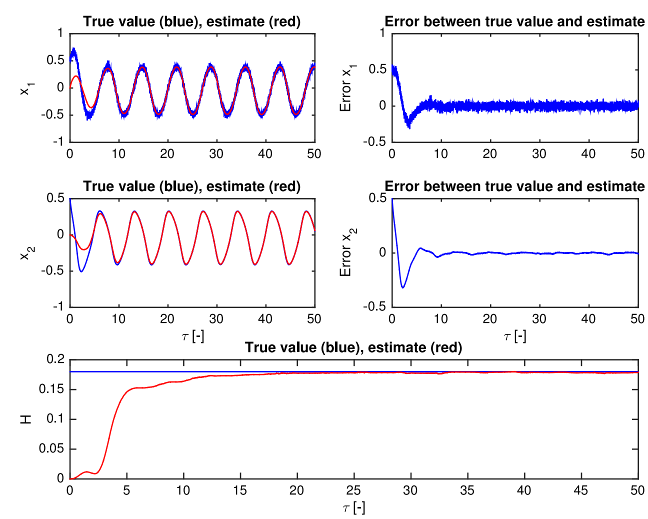

Fig. 2. Real (blue) and estimated (red) states x_1, x_2 and H for the obtained minima (closest to real values) of W and B for added measurement noise.

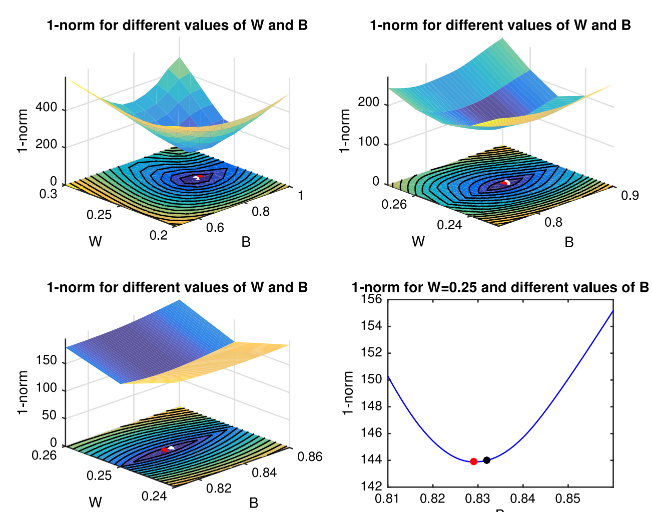

Fig. 3. Simulations for different ranges of W and B. The red dots display the obtained minima, whereas the white/black dots display the real value.

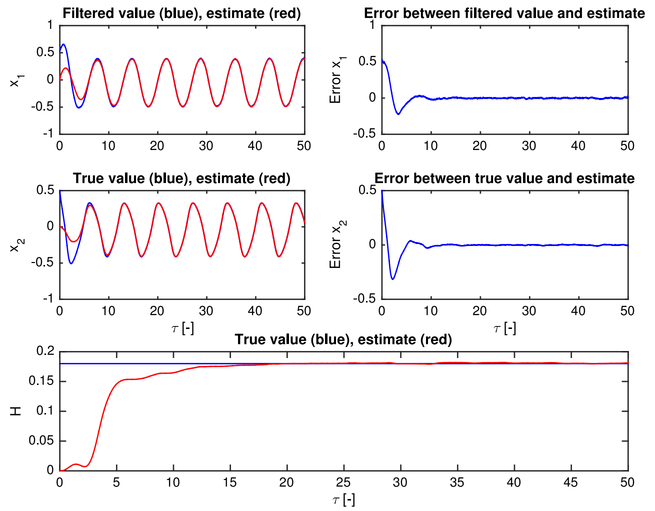

Fig. 4. Real (blue) and estimated (red) states x_1, x_2 and H for the best obtained minima (closest to real values) of W and B for the filtered measurement signal.

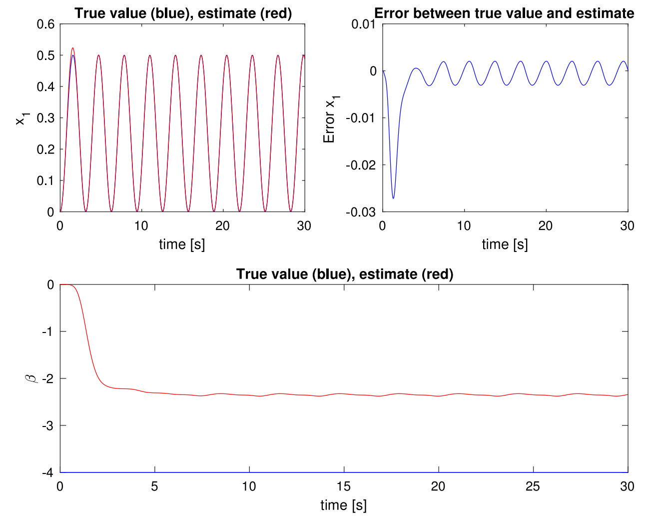

Fig. 6. True value and estimate of state x_1 together with their error as well as true value and estimate of parameter β.

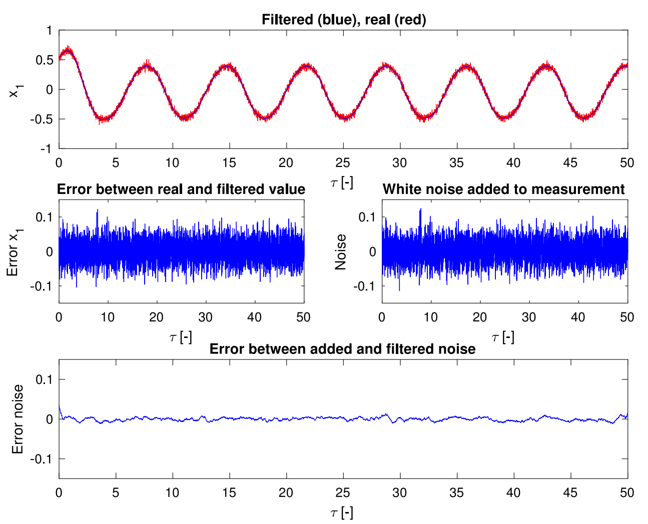

Fig. 5. Real, filtered and estimated state x_1 together with remaining noise.

In Fig. 5, we demonstrate the difference between the noisy measurement, its filtered representation and the estimate. Furthermore, the parts of the noise that could not be filtered are shown in the bottom plot.

4. Case study 2: Linear system

We base the investigation on a transfer function, $G(s) = \frac{1}{s^2+4}$, which has an oscillatory, marginally stable step response. For this transfer function, there exist multiple state space representations with respect to input–output behavior and we use the following generalized form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{\zeta}\alpha\\ \zeta\beta & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \zeta\Gamma\end{bmatrix}u$$
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{12}$$

with $\alpha = 1$, $\beta = -4$ and $\Gamma = 1$ and $\zeta \in \mathbb{R}\backslash\{0\}$. As one can see, different realizations of the system model are possible, but the original parameters have been defined for $\zeta = 1$. However, the input–output-relationships are the same even for different values of ζ. A difference to the case study presented in Section 3 is the fact that now one parameter, namely Γ, is directly affecting the gain of the input u.

We now define the model for the EKF in the following way

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\\ \dot{\hat{\beta}}\end{bmatrix} = \begin{bmatrix}0 & \hat{\alpha} & 0\\ \hat{\beta} & 0 & \hat{x}_1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \hat{\beta}\end{bmatrix} + \begin{bmatrix}0\\ \hat{\Gamma}\\ 0\end{bmatrix}u \tag{13}$$
$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{x}_1 & \hat{x}_2 & \hat{\beta}\end{bmatrix}^T$$

since β is the only parameter for which an augmentation of the system dynamics (12) is still fully observable.

Using the proposed methodology in Section 2 leads to the result in Fig. 6, where the true and estimated state variable x_1 and their corresponding error are displayed together with the true value and estimate of β. While the state estimate is acceptable, the estimate of β is substantially different from its true value.

The values of the parameters α and Γ are shown in the plot of the 1-norm in Fig. 7. A minimum is found for $\hat{\alpha} = 1.8$ and $\hat{\Gamma} = 0.6$, which are not the correct values. However, for $\zeta \approx 0.58$, the values $\alpha \approx 1.04$, $\beta \approx 4$ and $\Gamma \approx 1.03$ can be calculated. These values are close to the values associated with the model (12), however, $\hat{\beta}$ shows a clear oscillatory behavior. This calls for an extension to the proposed methodology in Section 2 in order to find parameter values that match the measurement even better.

5. Extension to the methodology

Again, as can be seen in the simulation results in Fig. 6, the estimated variable $\hat{\beta}$ oscillates around some nominal value and has a steady offset compared the actual value of β. The reason for this is found in the chosen norm (3), which appears to give non-optimal results for this case (see Fig. 7). Therefore, the methodology should investigate more measures, to obtain a conclusive result about the parameter values.

In Fig. 8, it can be seen that all estimated values $\hat{\beta}$ reach a steady-state value (or oscillate around one) after approximately $t = 10$ s. Therefore, we propose to run open-loop, forward simulations for all parameter combinations of α and Γ and their associated values of $\hat{\beta}$, for which we take the mean values between $t = 10$–30 s. The open-loop, forward simulation results

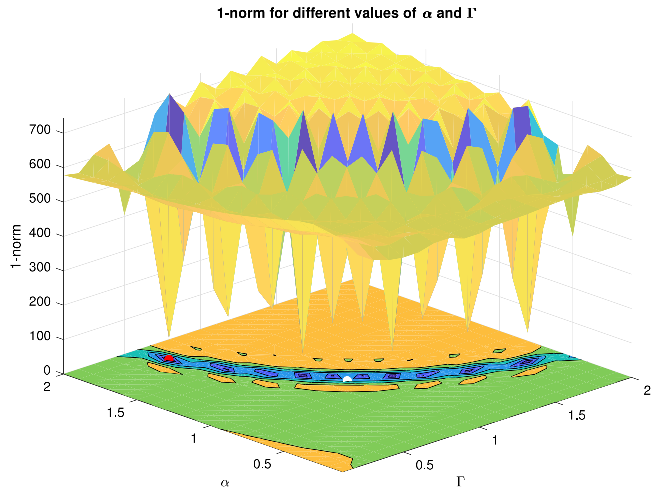

Fig. 7. 1-norm for all values of α and Γ.

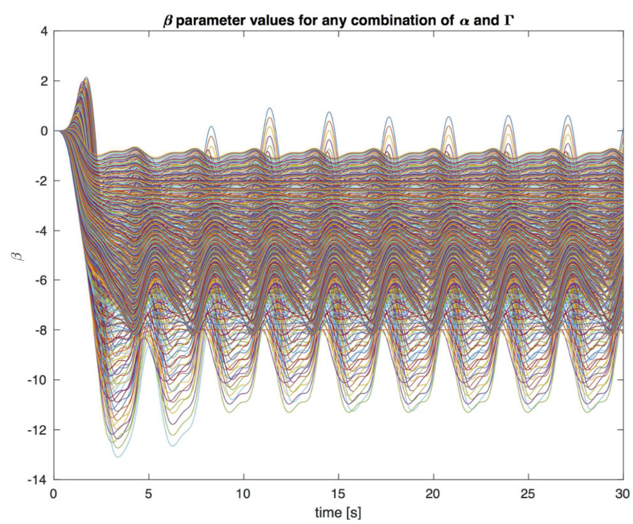

Fig. 8. All values of β over time, where one value of β corresponds to one combination of α and Γ.

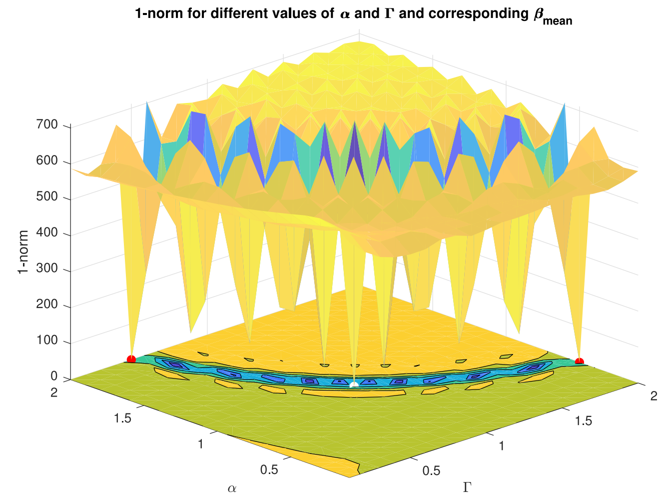

Fig. 9. 1-norm for all values of α and Γ obtained by forward simulations. Minima are marked with red dots, real values with a white dot.

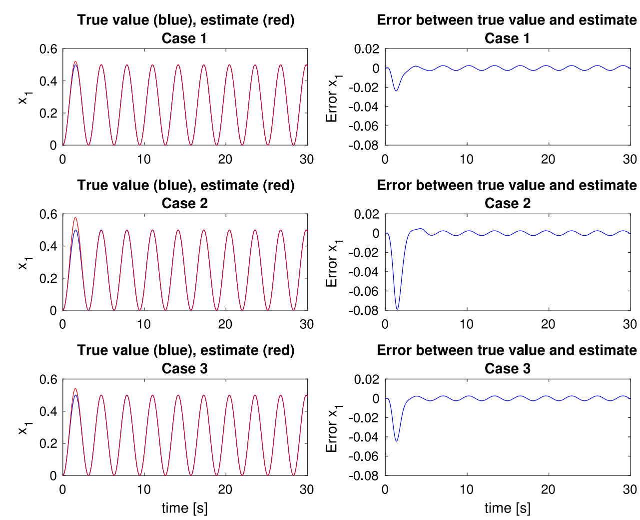

Fig. 10. True values and estimates of state x_1 together with their errors for all three cases.

are then compared to the measurement, again using the 1-norm defined in (3).

The extension to the methodology is shown in Algorithm 2.

Algorithm 2 Validation Algorithm

1: Calculate the mean value of each parameter in $\mathscr{P}_s$
   - Start calculating the mean after reaching a steady-like state
   - This steady-like state includes oscillations around a steady state

2: Use this mean value in forward simulations
   - The mean value has to be simulated together with its corresponding values of $\hat{p}_{u_i}$
   - Calculate the norm in (3), where $C_{ext}\hat{x}_{ext}$ is now replaced by the result from the forward simulation

The 1-norm for the forward simulations is presented in Fig. 9. It can be seen that now three minima can be identified, which are significantly smaller than any other values in the parameter space. These values are listed in Table 2 and correspond with the original system (12) for different values of ζ.

Table 2
Parameter values for the three obtained minima.

	$\hat{\alpha}$	$\hat{\Gamma}$	$\hat{\beta}$	ζ
Case 1	2	0.5	−2	0.5
Case 2	0.5	2	−8	2
Case 3 (true values)	1	1	−4	1

It should be mentioned that the estimated parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\Gamma}$ correspond to the respective values $\frac{1}{\zeta}\alpha$, $\zeta\beta$ and $\zeta\Gamma$ from the original system (12) and hence using these relations with $\zeta = 1$ lead to the parameters of the original system (12), namely $\alpha = 1$, $\beta = -4$ and $\Gamma = 1$.

The simulations corresponding to the values presented in Table 2 are shown in Figs. 10 and 11.

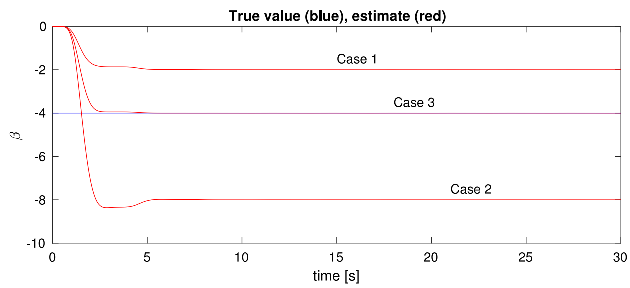

Fig. 11. True value and estimates of the parameter β for all three cases.

6. Case study 3: Stable linear system

This case is based on a stable, linear system description

$$G(s) = \frac{1}{s^2 + 2s + 1} \tag{14}$$

with double pole $s_{1,2} = -1$ and state space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\xi}\alpha \\ \xi\beta & \gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \xi\delta \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{15}$$

with $\alpha = 1$, $\beta = -1$, $\gamma = -2$, $\delta = 1$ and $\xi \in \mathbb{R}\backslash\{0\}$.

The model for the EKF is defined in the following way

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & \hat{\alpha} & 0 \\ \hat{\beta} & \hat{\gamma} & \hat{x}_1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{\delta} \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \hat{\beta} \end{bmatrix}^T, \tag{16}$$

which is fully observable.

Simulations have been performed using a biased sinusoidal input signal $u(t) = 2 + \sin(5t)$. The results applying the proposed methodology in Section 2 with its extension in Section 5 are not visualizable here, since the set of unobservable parameters has three elements. These are found to be $\alpha = 0.5$, $\gamma = -2$ and $\delta = 2$. These parameters lead to the states provided by the EKF in Fig. 12.

The results obtained are hence one state space representation of (14) with $\xi = 2$, compare (15). By fixing the value of $\delta \equiv 1$, and re-running the simulations only for the parameters α and γ, the true values $\alpha = 1$, $\beta = -1$ and $\gamma = -2$ could be obtained. This, however, is not shown here due to space limitations.

It should be mentioned that the identification of parameters was not possible for an input signal that was not biased, i.e. for a sinusoidal input oscillating around zero. However, identification of parameters was achievable using a step as input function for the system (14).

7. Discussion

In this work, a simple method for combined state and parameter estimation for general dynamic systems is introduced. The main idea is to implement an (augmented) extended Kalman filter together with an algorithm, which minimizes a criterion to estimate unmeasurable state variables as well as unknown parameter values.

The simulation results, especially for the nonlinear system in Section 3, lead to the conclusion that, in order to increase performance of the proposed method, pre-processing of the measurement signals is critical. Particularly, filtering high-frequent parts of the measurement noise increases performance. In this work, a moving average filter was implemented in order to filter the noise. However, it must be noted that the number of data points, which are included in the filter, should be chosen carefully. The reason for this lies in the fact that for oscillating systems, the filtered signal's amplitude will not reach that of the original signal for a large number of data points. Nevertheless, this is not a sole problem of moving average filters, but also for many other filters that could have potentially been implemented here, such as standard low-pass filters or the Wiener filter.

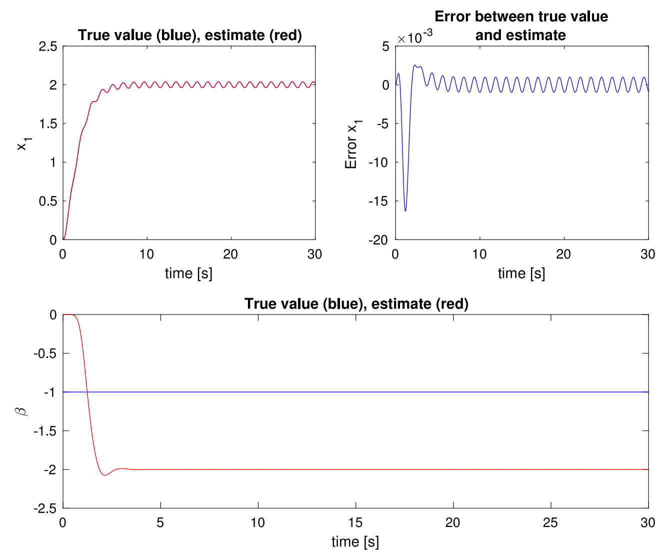

Fig. 12. True value and estimate of state x_1 together with their error as well as true value and estimate of parameter β.

Convergence time and hence the required simulation time in general are heavily depending on the tuning of the extended Kalman filter. A certain minimum simulation time is needed in order to obtain good estimates for the unobservable parameters $\hat{p}_u$. This implies that fast convergence of the EKF is crucial if the proposed method is used in an offline manner with measurements of limited duration.

A big advantage of the method is that it can still be used if the set $\mathscr{P}_s$ is empty, and hence no (parameter) states are added to the EKF. However, identification of parameters as part of the dynamic system reduces the overall computational cost since it is computationally more efficient. This means that not the full set of parameters $\mathscr{P}$ has to be identified in the Monte-Carlo fashioned approach. Therefore, the authors propose to always utilize the maximum allowable set of parameters $\overline{\mathscr{P}_s}$ in the EKF.

Persistence of excitation is often crucial to enable the identification of model parameters. The method presented in this work is no exception to that. Nevertheless, in the case studies presented in this paper, this was not an issue since the dynamic behaviors of the systems were of oscillatory nature.

Identifiability analysis has not been performed in the present, application-oriented work. Nonetheless, the authors would like to emphasize that checking identifiability of parameters is an important task and might be subject to future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

$$
\begin{aligned}
A_{13} &= -\frac{\gamma}{2\hat{B}}\left[\frac{\operatorname{sgn}\left(\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right)\hat{H}\hat{\alpha}}{\sqrt{\left|\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right|}}-\frac{\hat{\alpha}}{\sqrt{\psi_{0_c}+\hat{H}\hat{\alpha}}}\right],\\
A_{14} &= -\frac{\gamma}{2\hat{B}}\left[\frac{\operatorname{sgn}\left(\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right)\hat{H}\hat{\beta}}{\sqrt{\left|\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right|}}-\frac{\hat{H}\hat{\beta}}{\sqrt{\psi_{0_c}+\hat{H}\hat{\alpha}}}\right],\\
A_{15} &= -\frac{1}{\hat{B}^2}\left[\hat{x}_2-\gamma\left(\operatorname{sgn}(\hat{x}_1+\hat{\psi}_0)\sqrt{\left|\hat{x}_1+\hat{\psi}_0\right|}-\sqrt{\hat{\psi}_0}\right)\right],\\
A_{22} &= \hat{B}\left[-\frac{3\hat{H}\hat{x}_2^2}{2\hat{W}^3}-\frac{3\hat{H}\hat{x}_2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-1\right)-\frac{3\hat{H}\phi_0}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-2\right)\right],\\
A_{23} &= \hat{B}\left[-\frac{\hat{x}_2^3}{2\hat{W}^3}-\frac{3\hat{x}_2^2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-1\right)-\frac{3\phi_0\hat{x}_2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-2\right)\right],\\
A_{24} &= \frac{3\hat{B}\hat{H}\hat{x}_2}{\hat{W}^3}\left[\frac{\hat{x}_2^2+\phi_0\hat{x}_2+\phi_0^2}{2\hat{W}}+\left(\frac{\phi_0}{\hat{W}}-1\right)\hat{x}_2+\left(\frac{\phi_0}{\hat{W}}-2\right)\phi_0\right],\\
A_{25} &= -\frac{\hat{H}\hat{x}_2^3}{2\hat{W}^3}-\frac{3\hat{H}\hat{x}_2^2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-1\right)-\frac{3\hat{H}\phi_0\hat{x}_2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-2\right)-\hat{x}_1-u,
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
\hat{\alpha} &= -\frac{1}{2}+\frac{3}{2}\frac{\phi_0}{\hat{W}}-\frac{1}{2}\left(\frac{\phi_0}{\hat{W}}-1\right)^3,\\
\hat{\beta} &= \frac{3}{2}\frac{\phi_0}{\hat{W}^2}\left(\left(\frac{\phi_0}{\hat{W}}-1\right)^2-1\right)
\end{aligned}
\tag{18}
$$

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