MULTIPLE MODEL ESTIMATION WITH INTER-RESIDUAL DISTANCE FEEDBACK

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Abstract. This paper presents a modification of the Multiple Model Adaptive Estimation concept. A trade-off problem between tracking the process and distinguishing the models is pointed out and an adaption of the elemental filters is proposed. The adaption scheme modifies the filters such that the predicted measurements not become too close in some sense. This has considerable influence on the distinguishability of the filters and thereby the properties of the Multiple Model Adaptive Estimation algorithm. Stability of the method is considered, and a simulated example demonstrates the method.
Keywords. Adaptive systems; estimation; Kalman filters; parameter estimation; state estimation; system identification

INTRODUCTION

Multiple Model Adaptive Estimation (Anderson and Moore, 1979; Athans and Chang,1976; Maybeck,1982), have been developed for simultaneous estimation of states and parameters in dynamic systems by Magill (1965) and further refined by Lainiotis (1971). A common assumption in MMAE is that the parameters only take on a finite number of different values. This is often an approximation of the continuous parameter case, but when prior knowledge indicates that the parameters only obtain a finite number of different values, MMAE is a method to utilize this information.

The MMAE concept constitutes a bank of state estimators running in parallel, producing one residual for each filter, as illustrated in Fig.1. Each filter has a model that is different from the others and used to compute weighting coefficients which indicates the validity of each filter. These weighting coefficients are used to compute an overall state estimate and parameter estimate. The MMAE concept has also been closely tied to adaptive control (Multiple Model Adaptive Control) by use of LQG controllers together with the overall state estimate (Athans and co-workers,1977; Magill,1965). Successful operation of MMAE is highly dependent upon the distinguishability of the models and tuning of the filters (Maybeck,1982; Maybeck and Pogoda,1989). When Kalman filters are used in the bank, there is feedback from the predicted measurements which may force the residuals too close together and interrupt the discrimination property of the filter bank.

In this paper a method, Inter-Residual Distance Feedback (IRDF), for on-line modification of the filters is proposed. The objective is to maintain the discrimination property of the filter bank. This is achieved by detuning the filters through modulation of certain filter parameters. The modulation is governed by a scalar quantity computed from a distance measure between the residuals.

THE MMAE METHOD

Let $\boldsymbol{\theta}$ denote a q-dimensional vector of uncertain parameters in a dynamic stochastic state space model for a dynamic system. Assume that $\boldsymbol{\theta}$ can take on only one of N different values, $\boldsymbol{\theta}_i$, $i = 1, ..., N$. In this paper an operational mode S_i is associated with $\boldsymbol{\theta}_i$. Then the true system denoted S^*, is contained in the set $\mathcal{S} = \{S_1, ..., S_N\}$. The i-th operational mode S_i is modeled as

$$\begin{aligned} \frac{d}{dt}\boldsymbol{x}_i(t) &= \boldsymbol{f}[\boldsymbol{x}_i(t), \boldsymbol{u}(t), \boldsymbol{\theta}_i] + \boldsymbol{v}_i(t) \\ \boldsymbol{y}(t) &= \boldsymbol{g}[\boldsymbol{x}_i(t), \boldsymbol{\theta}_i] + \boldsymbol{w}(t) \end{aligned} \quad (1)$$

where $\boldsymbol{y}(t)$ is measurement vector, $\boldsymbol{x}_i(t)$ is state vector and $\boldsymbol{u}(t)$ is deterministic control input. The process noise $\boldsymbol{v}_i(t)$ and measurement noise $\boldsymbol{w}(t)$ are both assumed to be independent zero mean Gaussian with covariance matrices $\boldsymbol{V}_i$ and $\boldsymbol{W}$ respectively. Vector dimensions are: $\dim \boldsymbol{x}_i(t) = \dim \boldsymbol{v}_i(t) = n$, $\dim \boldsymbol{u}(t) = r$, $\dim \boldsymbol{w}(t) = \dim \boldsymbol{y}(t) = m$. The functions $\boldsymbol{f}[\cdot]$ and $\boldsymbol{g}[\cdot]$ may be general non-linear and vector-valued. The model in Eq. (1) is denoted by M_i and within the limitations of modeling, M_i describes the system S^* when operating in mode i. All the models constitute a set $\mathcal{M} = \{M_1, M_2, ..., M_N\}$.

At discrete time instants t_k, the MMAE algorithm calculates the probability $P_i(t_k)$ of each model M_i conditioned on the discrete time measurement history $Y_k = \{\boldsymbol{y}(t_1), ..., \boldsymbol{y}(t_k)\}$ obtained by sampling of $\boldsymbol{y}(t)$

$$P_i(t_k) \stackrel{def}{=} prob\{S^* = M_i | Y_k\} \quad (2)$$

$P_i(t_k)$ updates recursively as

$$P_i(t_k) = \frac{p[\boldsymbol{y}(t_k)|M_i, Y_{k-1}]P_i(t_{k-1})}{\sum_{j=1}^{N} p[\boldsymbol{y}(t_k)|M_j, Y_{k-1}]P_j(t_{k-1})} \quad (3)$$

where $p[\boldsymbol{y}(t_k)|M_i, Y_{k-1}]$ is the density of $\boldsymbol{y}(t_k)$ conditioned on M_i and Y_k. The overall state estimate for the filter bank is given as

$$\hat{\boldsymbol{x}}(t_k^+) \stackrel{def}{=} \sum_{i=1}^{N} \hat{\boldsymbol{x}}_i(t_k^+) P_i(t_k) \quad (4)$$

where $\hat{\boldsymbol{x}}_i(t_k^+)$ is the state vector estimate in the i-th elemental

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filter after the measurement update (Maybeck,1979). An estimate $\hat{\boldsymbol{\theta}}(t_k)$, of $\boldsymbol{\theta}$ is the conditional mean

$$\hat{\boldsymbol{\theta}}(t_k) \stackrel{def}{=} \sum_{i=1}^{N} \boldsymbol{\theta}_i P_i(t_k) \tag{5}$$

The Eqs. (3), (4) and (5) are valid for general models M_i and conditional densities $p[\boldsymbol{y}(t_k)|M_i, Y(t_{k-1})]$. However due to filter complexity, applications of MMAE have mainly dealt with linear models and Kalman filters. With the assumption of Gaussian noise and linear models, the conditional densities are

$$\begin{aligned} p[\boldsymbol{y}(t_k)|M_i, Y(t_{k-1})] &= (2\pi)^{-\frac{m}{2}}(\det[\mathcal{E}_i(t_k)])^{-\frac{1}{2}}\exp(-\nu_i(t_k)) \\ \nu_i(t_k) &= \frac{1}{2}\boldsymbol{\varepsilon}_i^T(t_k)\mathcal{E}_i^{-1}(t_K)\boldsymbol{\varepsilon}_i(t_k) \end{aligned} \tag{6}$$

where $\boldsymbol{\varepsilon}_i(t_k) = \boldsymbol{y}(t_k) - \hat{\boldsymbol{y}}_i(t_k)$ is the residual vector of the i-th elemental filter and $\mathcal{E}_i(t_k)$ is the estimated covariance matrix of $\boldsymbol{\varepsilon}_i(t_k)$ at time instant t_k. For brevity the Kalman filter including its model M_i is denoted $\mathcal{F}(M_i, \boldsymbol{K}_i)$. Both $\boldsymbol{\varepsilon}_i(t_k)$ and $\mathcal{E}_i(t_k)$ are supplied by $\mathcal{F}(M_i, \boldsymbol{K}_i)$. The Kalman filter equations for discrete time measurement update may be found in (Gelb,1984; Jazwinski,1970; Maybeck,1979; Maybeck,1982) for linear and nonlinear models.

When $S^* = M_j$ one should expect that

$$\nu_j(t_k) \ll \nu_i(t_k)\,,\ \forall i \neq j \tag{7}$$

which is denoted as regular behavior of the residuals. Now $P_j(t_k)$ increases towards unity while probabilities of the mismatched filters will decrease towards zero if the condition of Eq. (7) persists over several measurements. A more formal convergence condition is given in (Anderson and Moore,1979). If $S^* \notin \mathcal{M}$ and/or the filters are tuned improperly it is possible that

$$\nu_1(t_k) \approx \nu_2(t_k) \approx \ldots \approx \nu_N(t_k) \tag{8}$$

Then P_i is now governed by $\det[\mathcal{E}_i(t_k)], i = 1, \ldots, N$ and $P_j(t_k)$ increases if $\det[\mathcal{E}_j(t_k)] < \det[\mathcal{E}_i(t_k)]$, $i \neq j$, while $P_i(t_k)$, $i \neq j$ decrease. For Kalman filters and Extended Kalman filters $\det[\mathcal{E}_i(t_k)]$ is not dependent on which model is correct and erroneous decisions upon the valid model may result (Athans and Chang,1976). Hence the situation of Eq.(8) is undesirable. The behavior of MMAE as outlined above is related to the identifiability concept of the algorithm and tuning of the filters.

When using MMAE with changing parameters, a widely used ad-hoc modification is to fix a lower bound on $P_i(t_k)$. Without such a lower bound it is seen from Eq. (3) that a change not will be detected. A reasonable lower bound on P_i is 0.001 which gives $P_i(t_k) \in [0.001, 1-(N-1)0.001]$.

A TRADE-OFF PROBLEM IN MMAE

This section highlights the model discrimination versus tracking properties of MMAE. By tracking is meant the ability of a filter to predict the output $\boldsymbol{y}(t_k)$ given Y_{k-1}. The trade-off problem may be stated as follows: *On one hand, we want good tracking capabilities for each filter $\mathcal{F}(M_i, \boldsymbol{K}_i)$, when $S^* = S_i$. On the other, we want the residuals to be distant, to achieve fast and reliable model discrimination.* The side in the trade-off which is favored, depends a lot on how strong each filter updates its state estimates from the measurements $\boldsymbol{y}(t_k)$. To illustrate this, let the system operate in only two modes, $S^* \in \{S_1, S_2\}$, with the corresponding models M_1 and M_2. Also restrict the models to be linear with continuous time measurements.

$$\begin{aligned} \frac{d}{dt}\boldsymbol{x}_i(t) &= \boldsymbol{A}_i\boldsymbol{x}_i(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{v}_i(t) \\ \boldsymbol{y}_i(t) &= \boldsymbol{C}_i\boldsymbol{x}_i(t) + \boldsymbol{w}(t) \end{aligned} \tag{9}$$

Here $\boldsymbol{A}_i = \boldsymbol{A}(\boldsymbol{\theta}_i), \boldsymbol{B}_i = \boldsymbol{B}(\boldsymbol{\theta}_i)$ and $\boldsymbol{C}_i = \boldsymbol{C}(\boldsymbol{\theta}_i)$. Then $\mathcal{F}(M_1, \boldsymbol{K}_1)$ and $\mathcal{F}(M_2, \boldsymbol{K}_2)$ can be viewed as feedback systems with proportional gain as shown in Fig.2. Introduce $\boldsymbol{y}(t|S_i)$ as the measurement from S^* when known to be be in mode S_i and similarly $\boldsymbol{y}(\mathrm{j}\omega|S_i)$ in the frequency domain. Then from Fig. 2

$$\begin{aligned} \boldsymbol{\varepsilon}_i(\mathrm{j}\omega|S_j) &= [\boldsymbol{I} + \boldsymbol{P}_i(\mathrm{j}\omega)\boldsymbol{K}_i]^{-1}\boldsymbol{y}(\mathrm{j}\omega|S_j) \\ &\quad - [\boldsymbol{I} + \boldsymbol{P}_i(\mathrm{j}\omega)\boldsymbol{K}_i]^{-1}\boldsymbol{P}_i(\mathrm{j}\omega)\boldsymbol{B}_i\boldsymbol{u}(\mathrm{j}\omega) \\ \boldsymbol{P}_i(\mathrm{j}\omega) &= \boldsymbol{C}_i(\mathrm{j}\omega\boldsymbol{I} - \boldsymbol{A}_i)^{-1}\,,\ i,j = 1,2 \end{aligned} \tag{10}$$

Assume that both M_1 and M_2 are stochastic observable and stochastic controllable (Maybeck,1979 Chap.5). Then for increasing process noise covariance matrices $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$, both filters have the same property: $\hat{\boldsymbol{y}}_i(\mathrm{j}\omega) \rightarrow \boldsymbol{y}(\mathrm{j}\omega)$ and $\boldsymbol{\varepsilon}_i(\mathrm{j}\omega) \rightarrow \boldsymbol{0}$ regardless of the mode. This behavior may give the situation of Eq.(8) and the filters mask the differences between M_1 and M_2. This indicates that the distance, in some sense, between the residuals is crucial. One possible choice is to define

$$\begin{aligned} \boldsymbol{\varepsilon}_{ij}(t) &= \boldsymbol{\varepsilon}_i(t) - \boldsymbol{\varepsilon}_j(t),\ i \neq j \\ \boldsymbol{\varepsilon}_i(t) &= \boldsymbol{y}(t) - \hat{\boldsymbol{y}}_i(t) \end{aligned} \tag{11}$$

and use some vector norm $\|\boldsymbol{\varepsilon}_{ij}\|$ as the distance between any two residuals. Denote $\|\boldsymbol{\varepsilon}_{ij}\|$ as the *Inter-Residual Distance* and the vector $\boldsymbol{\varepsilon}_{ij}(t_k)$ as the *Inter-Residual Difference.* When filter gains become large, we have that

$$\boldsymbol{\varepsilon}_{ij}(t) = \hat{\boldsymbol{y}}_j(t) - \hat{\boldsymbol{y}}_i(t) \rightarrow \boldsymbol{0}$$

which highlights the fact that the filters in the bank should not be tuned totally independently.

FILTER GAIN MODULATION

A method for modulating the filter gains according to a measure of $\boldsymbol{\varepsilon}_{ij}$, $i \neq j$ is now proposed. A simple quadratic form

$$J_{ij}(t) = \boldsymbol{\varepsilon}_{ij}^T(t)\boldsymbol{\Gamma}_{ij}\boldsymbol{\varepsilon}_{ij}(t)\,,\ i \neq j \tag{12}$$

is chosen as the distance measure of $\boldsymbol{\varepsilon}_{ij}$, where $\boldsymbol{\Gamma}_{ij}$ is a positive definite diagonal scaling matrix. The number of filters is here restricted to $N = 2$ but extensions to more filters is outlined at the end of this chapter. The main principle of the method is to keep the inter-residual distance measure $J_{12}(t)$ above a specified limit J_{12}^0 by adjusting the filter gains. A general way to achieve this is by varying the process noise covariances $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$. In filter calculations, $\boldsymbol{V}_i$ is now replaced by modulated process noise covariance matrices $\boldsymbol{V}_i'(t)$ defined as

$$\boldsymbol{V}_i'(t) = \eta(t)\boldsymbol{V}_i\,,\ i = 1,2 \tag{13}$$

where $\eta(t) \in [\eta_{min}, 1.0]$ and the lower bound η_{min} can be chosen to give a lower bound on $\boldsymbol{V}_i'(t)$. The restriction $\eta_{min} > 0$, makes $\boldsymbol{V}_i'(t) \geq \boldsymbol{0}$ and the upper bound $\eta \leq 1.0$ is chosen to secure that $\boldsymbol{V}_i'(t) \leq \boldsymbol{V}_i(t)$.

The time derivative of the modulating variable $\eta(t)$, should be an odd function of $J_{12}(t) - J_{12}^0$, being zero for $J_{12}(t) = J_{12}^0$ and for $\eta \notin \langle \eta_{min}, 1.0\rangle$. One choice is

$$\frac{d}{dt}\eta(t) = \begin{cases} \zeta\,[J_{12}(t) - J_{12}^0] & ,Cond\,1 \\ 0 & ,Cond\,2 \end{cases} \tag{14}$$

where the conditions in Eq. (14) are:

$$Cond\,1 \;:\; \eta \in \langle \eta_{min}, 1.0\rangle$$

$$Cond\,2 \;:\; \begin{cases} (\eta = \eta_{min} \quad AND \quad \zeta\,[J_{12}(t) - J_{12}^0] < 0)\ OR \\ (\eta = 1.0 \quad AND \quad \zeta\,[J_{12}(t) - J_{12}^0] > 0) \end{cases}$$

These conditions provides anti-integration windup (AWU), see Fig. 3 and 4. The constant $\zeta > 0$, must be specified together with the lower inter-residual distance limit J_{12}^0 or a lower inter-residual difference limit ϵ_{12}^0, such that $J_{12}^0 = \epsilon_{12}^{0\,T}\Gamma_{12}\epsilon_{12}^0$. Note that Eq. (14) is an integrator and ζ should be selected to provide proper attenuation of noise on $\eta(t)$. The concept is shown in Fig. 3 for $N = 2$. From the standard Kalman filter equations it follows that increasing values of V_1 and V_2 increase the filter gains. This in turn, reduces the value of J_{12}. Small values of V_1 and V_2 make J_{12} more dependent on the differences between M_1 and M_2, and presumably greater in mean square sense. Eqs. (12), (13) and (14) adjust $\eta(t)$ to an equilibrium in mean value such that $J_{12}(t) = J_{12}^0$. This is shown inside the dashed lines in Fig. 3 and described by Eqs. (15) through (19). These equations constitute a system of nonlinear differential equations ((15) through (19)) of order $n^2 + 3n + 1$ knowing that the state estimate covariance matrix $X_i(t) = X_i^T(t)$.

$$\begin{aligned} \frac{d}{dt}\hat{x}_1(t) &= [A_1 - X_1(t)C_1^T W^{-1} C_1]\hat{x}_1(t) \\ &\quad + X_1(t)C_1^T W^{-1} y(t) + B_1 u(t) \end{aligned} \qquad (15)$$

$$\begin{aligned} \frac{d}{dt}\hat{x}_2(t) &= [A_2 - X_2(t)C_2^T W^{-1} C_2]\hat{x}_2(t) \\ &\quad + X_2(t)C_2^T W^{-1} y(t) + B_2 u(t) \end{aligned} \qquad (16)$$

$$\begin{aligned} \frac{d}{dt}X_1(t) &= A_1 X_1(t) + X_1(t) A_1^T \\ &\quad - X_1(t)C_1^T W^{-1} C_1 X_1(t) + \eta(t) V_1 \end{aligned} \qquad (17)$$

$$\begin{aligned} \frac{d}{dt}X_2(t) &= A_2 X_2(t) + X_2(t) A_2^T \\ &\quad - X_2(t)C_2^T W^{-1} C_2 X_2(t) + \eta(t) V_2 \end{aligned} \qquad (18)$$

$$\frac{d}{dt}\eta(t) = \begin{cases} \zeta\,\left[(C_2\hat{x}_2(t) - C_1\hat{x}_1(t))^T \Gamma_{12} \right. \\ \left. (C_2\hat{x}_2(t) - C_1\hat{x}_1(t)) - J_{12}^0\right] & ,Cond\,1 \\ 0 & ,Cond\,2 \end{cases} \qquad (19)$$

Since $V_i'(t)$ is a function of time, filter gains cannot be precomputed even for linear models.

A simplification of the method described above is obtained if, instead of modulating V_i, the new information $K_i\epsilon_i(t)$, is modulated as

$$K_i'(t)\epsilon_i(t) = \eta(t) K_i \epsilon_i(t)\,,\ i = 1, 2\,,\ \eta(t) \in [\eta_{min}, 1.0] \qquad (20)$$

The individual filter gains K_i are now precomputable, and only the modulation is computed on-line. This simplified method is shown inside the dashed lines of Fig. 4 and described by Eqs. (19), (21) and (22).

$$\begin{aligned} \frac{d}{dt}\hat{x}_1(t) &= (A_1 - \eta(t)K_1C_1)\hat{x}_1(t) \\ &\quad + \eta(t)K_1 y(t) + B_1 u(t) \end{aligned} \qquad (21)$$

$$\begin{aligned} \frac{d}{dt}\hat{x}_2(t) &= (A_2 - \eta(t)K_2C_2)\hat{x}_2(t) \\ &\quad + \eta(t)K_2 y(t) + B_2 u(t) \end{aligned} \qquad (22)$$

Eqs. (21) and (22) together with (19) constitute a $2n+1$ dimensional nonlinear state-space system with $u(t)$, $y(t)$ and J_{12}^0 as inputs. It should be noticed that in general, modulation of $K_i\epsilon_i$ as in Eq. (20) and modulation of V_i as in Eq. (13) does not give the same filter gain. This is due to the filter gain calculation in Eq. (23) together with the Riccati equations (17) and (18).

$$K_i(t) = X_i(t) C_i^T W^{-1} \qquad (23)$$

The question of stability and which constraints this imposes on $\eta(t)$ and ζ is now considered. As can be seen from Eqs. (15) through (18) and Eqs. (21) and (22), analysis is simpler when modulating $K_i\epsilon_i$ directly, because the Riccati equations are eliminated. For examination of stability we assume that the models are linear and by utilizing a Lyapunov function, we can establish a *sufficient* condition for asymptotic stability of the autonomous parts of the Eqs. (19), (21) and (22). We consider the Lyapunov function

$$V(\tilde{x}) = \tilde{x}^T Q \tilde{x} \ \in \mathbb{R} \qquad (24)$$

where $Q = Q^T > 0$ and

$$\tilde{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \eta \end{bmatrix} \qquad (25)$$

Global asymptotic stability is guaranteed if

$$\frac{d}{dt}V(\tilde{x}) = \left[\frac{d}{dt}\left(\tilde{x}^T\right) Q\tilde{x} + \tilde{x}^T Q \frac{d}{dt}\left(\tilde{x}\right)\right] < 0\,,\ \forall\,\tilde{x} \qquad (26)$$

We now make the rather restrictive and simplifying choice $Q = I$ and substitute Eqs. (19), (21) and (22) into Eq. (26) which gives

$$\begin{aligned} \frac{d}{dt}V(\tilde{x}) &= \hat{x}_1^T\left\{A_1^T + A_1 - \eta([K_1C_1]^T + K_1C_1)\right\}\hat{x}_1 \\ &\quad + \hat{x}_2^T\left\{A_2^T + A_2 - \eta([K_2C_2]^T + K_2C_2)\right\}\hat{x}_2 \\ &\quad + 2\zeta\eta\left\{\hat{x}_1^T C_1^T \Gamma_{12} C_1 \hat{x}_1 + \hat{x}_2^T C_2^T \Gamma_{12} C_2 \hat{x}_2 \right. \\ &\quad \left. - \hat{x}_1^T C_1^T \Gamma_{12} C_2 \hat{x}_2 - \hat{x}_2^T C_2^T \Gamma_{12} C_1 \hat{x}_1\right\} \end{aligned} \qquad (27)$$

Reorganize Eq. (27) into

$$\frac{d}{dt}V(\tilde{x}) = -\left[\hat{x}_1^T, \hat{x}_2^T\right] P(\eta,\zeta) \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \qquad (28)$$

where the symmetric matrix $P(\eta,\zeta)$ is partitioned into the four submatrices

$$\begin{aligned} P_{11}(\eta,\zeta) &= -A_1^T - A_1 + \eta([K_1C_1]^T + K_1C_1) \\ &\quad - 2C_1^T \Gamma_{12} C_1 \zeta\eta \\ P_{12}(\eta,\zeta) &= P_{21}^T(\eta,\zeta) \ = \ 2C_1^T \Gamma_{12} C_2 \zeta\eta \\ P_{22}(\eta,\zeta) &= -A_2^T - A_2 + \eta([K_2C_2]^T + K_2C_2) \\ &\quad - 2C_2^T \Gamma_{12} C_2 \zeta\eta \end{aligned}$$

Now, if a region $\mathcal{R}$ in the η, ζ plane can be found such that

$$\mathcal{R} = \{\eta,\zeta | P(\eta,\zeta) > 0,\ \eta \in [\eta_{min}, 1.0]\,, \zeta > 0\} \qquad (29)$$

then Eq. (26) is satisfied and the system formed by the Eqs. (19), (21) and (22) is asymptotic stable for all $\eta, \zeta \in \mathcal{R}$. The region $\mathcal{R}$ may however be a conservative restriction on η and ζ. An approximation of $\mathcal{R}$ can be found numerically by partitioning the η, ζ plane into a grid and testing $P(\eta,\zeta)$ for positive definiteness at all grid points. This is demonstrated for a simple system in the simulated example. Note that it is only necessary to analyze $Cond\,1$ in Eq. (19) by the Lyapunov function, since under $Cond\,2$ stability is only concerned with the linear Eqs. (21) and (22). Stability analysis is more complicated when modulating V_i instead of modulating $K_i\epsilon_i$, although there are some results which may be used. For stable linear models, both filters $\mathcal{F}(M_1, K_1)$ and $\mathcal{F}(M_2, K_2)$ are separately stable for all $\eta \in [\eta_{min}, 1.0]$ and for all $\zeta > 0$ because $\eta V \geq 0$. See (Jazwinski,1970 pp.234–244). This does however *not* guarantee *asymptotic* stability, LUND 4

because η may oscillate within the interval $[\eta_{min}, 1.0]$.

For Extended Kalman filters and higher order filters there is no computational benefit in modulating the new information vectors rather than the process noise covariance matrices, because filter gains are computed on-line. Thus direct modulation of new information should only be considered for steady state filters. But even for steady state filters direct modulation of new information may introduce errors. Nonlinear models make Eqs. (15) through (19) even more intractable for analysis, and extensive simulations should be carried out in order to evaluate a bank of nonlinear filters with IRDF.

In order to use the concept of filter gain modulation for three or more models, one possible approach is to scan $(N^2-N)/2$ distance measures and select $J_{min}(t) = \min_{i,j} J_{ij}(t)$, $i \neq j$. Then substitute $J_{12}(t)$ with $J_{min}(t)$ and J_{12}^0 with an overall lower distance measure J^0 in Eq. (14). Another method is to consider only the inter-residual distance between the two most probable models. However this method suffers from the fact that one of the probabilities approach unity while the others become zero. This problem can be solved by reducing the time horizon of the probability calculation, (Magalhães and Binder,1987).

SIMULATION TESTS

We shall demonstrate the IRDF concept by applying it on a second order linear SISO system S^* given in a continuous discrete time formulation

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ 0 & a \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.9 \end{bmatrix} u(t_k)$$
$$y(t_k) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t_k) \\ x_2(t_k) \end{bmatrix} \qquad (30)$$

Here $y(t_k)$ is discrete time measurement and $u(t_k)$ is control input changed only at discrete time instants t_k. The operational modes are determined by the parameter a, which has the values $a = 0.5$ in mode S_1 and $a = 1.0$ in mode S_2. The corresponding models M_1 and M_2 are given by

$$\frac{d}{dt}\begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix}_i = \begin{bmatrix} -0.5 & 1 \\ 0 & -a \end{bmatrix}_i \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix}_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t_k) + \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}_i$$
$$\hat{\boldsymbol{y}}_i(t_k) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{x}_1(t_k^-) \\ \hat{x}_2(t_k^-) \end{bmatrix}_i + w(t_k) \qquad (31)$$

where $t \in [t_k, t_{k+1})$, $i = 1,2$ and $\hat{\boldsymbol{x}}_i(t_k^-)$ is the estimate of $\boldsymbol{x}_i(t)$ at sample instant t_k before adding new information. The sample interval is 0.1 time units. Note that the control input matrix in Eq. (30) differs 10% from the one in Eq. (31). Hence, S^* is not a member in $\mathcal{M}$. In this example we shall apply direct modulation of filter gains, $\boldsymbol{K}_i = [k_1\ k_2]_i^T$, as shown in Fig. 4. An approximation of the stability region $\mathcal{R}$ given by Eq. (29) is shown in Fig. 5. Here the scaling matrix is a scalar and chosen as $\boldsymbol{\Gamma}_{12} = 1$. The stable region corresponds to $\boldsymbol{P}(\eta,\zeta) > \boldsymbol{0}$ and the other region to $\boldsymbol{P}(\eta,\zeta) \leq \boldsymbol{0}$. By choosing $\eta_{min} = 0$, asymptotic stability is guaranteed for $\eta \in [0, 1.0]$ and $\zeta \in \langle 0, 0.7]$. In simulations ζ is chosen to 0.5 if no other value is stated. For small values of η; ζ may be larger than 0.5, still not violating the stability constraint. This stable region is obtained when $\boldsymbol{K}_1 = [0.92, 0.55]^T$, $\boldsymbol{K}_2 = [0.92, 0.36]^T$ and when the process and measurement noise have covariances $\boldsymbol{V}_1 = \boldsymbol{V}_2 = 10\boldsymbol{I}$ and $\boldsymbol{W} = 0.1$ respectively. In all simulations, the system and the filters were excited by a square wave $u(t_k)$ with *amplitude* = 0.5, *mean* = 2.0 and *period* = 40.0, all being dimensionless quantities. Excitation $u(t_k)$ and system output $y(t_k)$ are shown in Fig. 6

The Kalman filters were implemented in continuous-discrete time formulation with measurement updating at every 0.1 time unit. Between two successive sample instants t_k and t_{k+1}, both the system and filter state equations are integrated numerically using an explicit variable step length Runge-Kutta method of order 5(4). The operational mode changes from S_1 to S_2 at 80 time units and back to S_1 at 170 time units. Initial values of the system were $\boldsymbol{x}(0) = \hat{\boldsymbol{x}}_1(0) = \hat{\boldsymbol{x}}_2(0) = 0$ and $\eta(0) = 1.0$. The residuals obtained without IRDF are compared to those obtained with IRDF ($\zeta = 0.5$) in Figs. 7 and 8. For clarity, no measurement noise was added to the system output here. The lower inter-residual difference limit was specified to $\varepsilon_{12}^0 = 0.3$. Figure 9 shows ε_{12} with and without IRDF. Observe that ε_{12} varies around -0.3 instead of 0.3 which is irrelevant due to Eq. (12). The variation in ε_{12} is induced by the excitation, $u(t_k)$. Filter gains k_1 and k_2 are shown for both cases in Figs. 10 and 11, and η is shown in Fig. 12. The Figs. 7, 8 and 9 illustrate the trade-off problem.

A lower limit of $P_i(t_k) = 0.001$ was applied in order to avoid that the probability of the invalid model lock on to zero. The probability of M_1 is shown in Fig. 13 with and without IRDF. Initial probabilities were $P_1(0) = P_2(0) = 0.5$. Without the IRDF $P_1(t_k)$ is seen to approach zero despite M_1 is expected to be closest to S^* before 70 time units. Also the change of mode at 170 time units is not detected without IRDF. This is due to the small difference between $\varepsilon_1(t_k)$ and $\varepsilon_2(t_k)$, which was denoted as irregular residual behavior. However, with the IRDF, the residuals behave in a regular way and the probabilities become right.

In order to examine the influence of ζ, simulations were carried out for ζ outside the stability region, $\mathcal{R}$. For $\zeta = 4.0$, Fig. 12 shows $\eta(t_k)$ and Fig. 14 shows $\varepsilon_{12}(t_k)$. The other parameters are unchanged. The variations in ε_{12} about ${\varepsilon_{12}}^0$ is now smaller and η adjusts faster than for $\zeta = 0.5$. However the stability is maintained, which indicates that the stability region of Fig. 5 is rather conservative.

Simulations with zero mean Gaussian noise of covariance 0.1 added to the system output were also carried out. For values of ζ inside the stability region in Fig. 5, IRDF behaved similarly as without noise. Modulation of $\boldsymbol{V}_i$ has also been simulated and similar results were observed for this system.

CONCLUSIONS

A method for on-line modulation of the filters used for Multiple Model Adaptive Estimation has been proposed. Stability of the method was investigated for a bank of two linear filters, and simulations were carried out using a second order SISO system to demonstrate the properties of the proposed method. The Inter-Residual Distance Feedback was a successful method to enhance the model discrimination properties of MMAE. Further investigations should include other distance measures between the models especially in a probabilistic sense. Also some means of determining η_{min} from an overall tracking capability of the filterbank should be considered. The concept of MMAE with IRDF extends naturally to discrimination between nonlinear models with different structures and different orders, only the dimension of measurement vectors should be equal.

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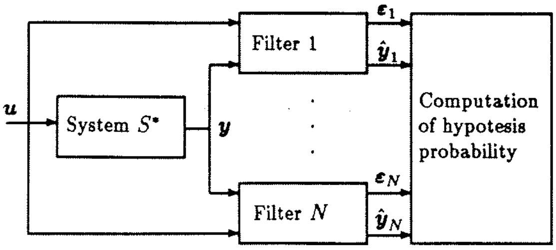

Fig. 1. Multiple Model Adaptive Estimator.

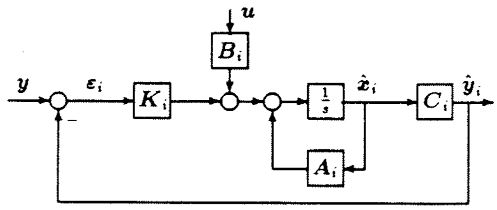

Fig. 2. $\mathcal{F}(M_i, K_i)$ viewed as a proportional feedback system.

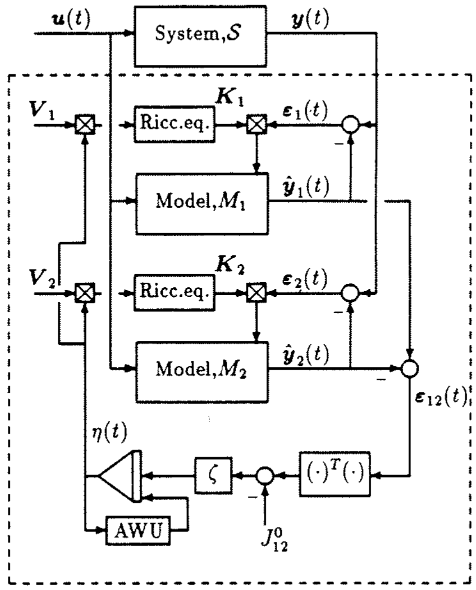

Fig. 3. Modulation of process noise covariance. AWU is anti-integration windup, see Eq.(14).

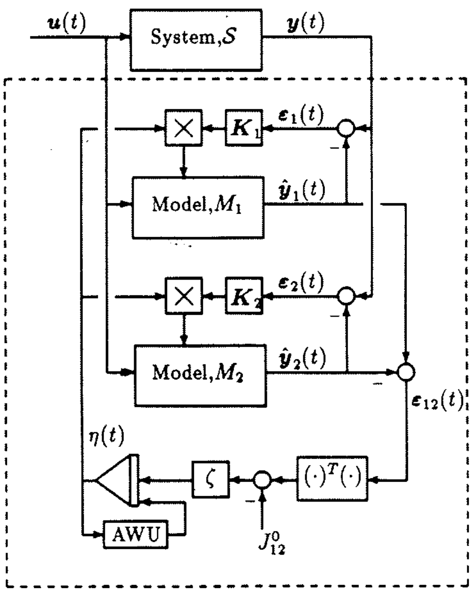

Fig. 4. Modulation of new information in the filters. AWU is anti-integration windup, see Eq. (14).

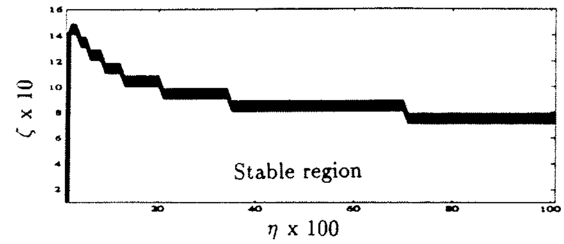

Fig. 5. Approximated stability region in the η, ζ plane. See Eq. (29)

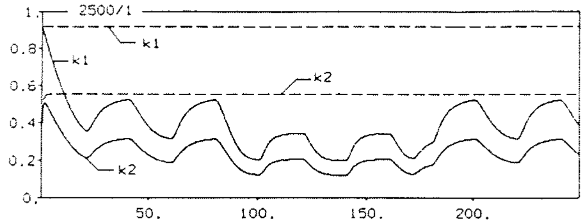

Fig. 10. Filter gains k_1 and k_2 of $\mathcal{F}(M_1, \boldsymbol{K}_1)$ with (solid line) and without (dashed line) IRDF.

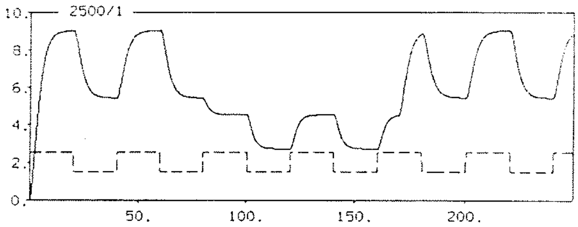

Fig. 6. System output $y(t_k)$ (solid line) and input $u(t_k)$ (dashed line).

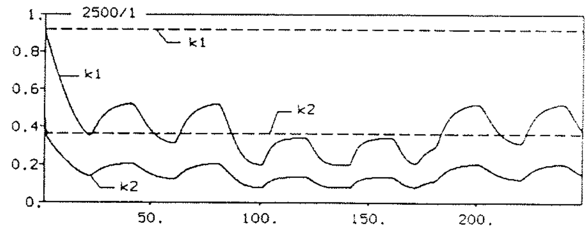

Fig. 11. Filter gains k_1 and k_2 of $\mathcal{F}(M_2, \boldsymbol{K}_2)$ with (solid line) and without (dashed line) IRDF.

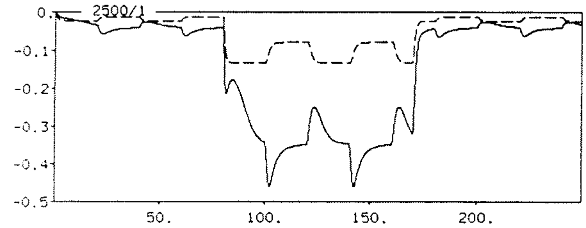

Fig. 7. Residual $\boldsymbol{\varepsilon}_1$ of $\mathcal{F}(M_1, \boldsymbol{K}_1)$ with (solid line) and without (dashed line) IRDF.

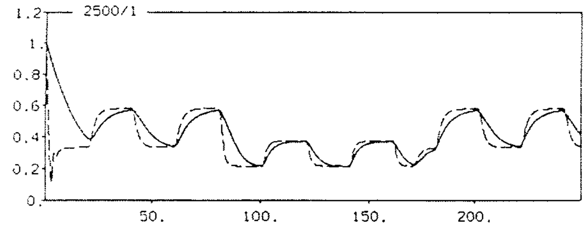

Fig. 12. Modulating variable $\eta(t)$, for $\zeta = 0.5$ (solid line) and for $\zeta = 4.0$ (dashed line).

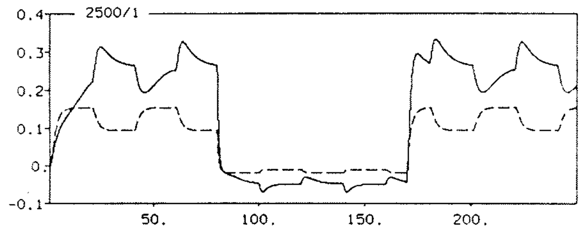

Fig. 8. Residual $\boldsymbol{\varepsilon}_2$ of $\mathcal{F}(M_2, \boldsymbol{K}_2)$ with (solid line) and without (dashed line) IRDF.

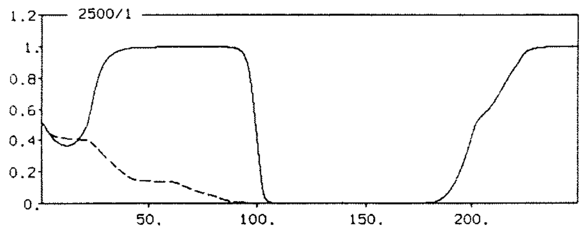

Fig. 13. Probability P_1 of M_1, with (solid line) and without (dashed line) IRDF. ($P_2 = 1 - P_1$)

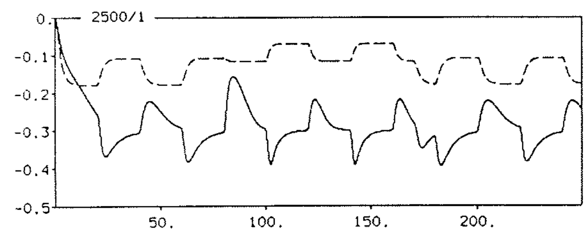

Fig. 9. Inter-residual difference ε_{12} with (solid line) and without (dashed line) IRDF

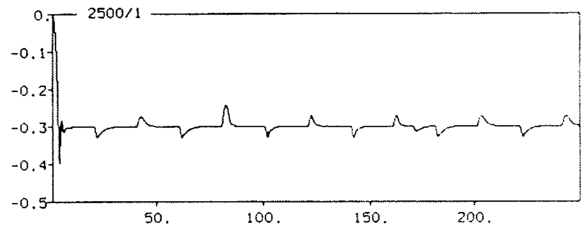

Fig. 14. Residual difference ε_{12}, with IRDF and $\zeta = 4.0$