# Overvaluation-not volatility-is the main danger in stock markets 

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#### Abstract

Real-world stock markets are volatile and express such traits as overvaluation, psychological moods, cycles and crashes. This paper develops and explores a model which has these properties. The model is aggregated, continuous and non-linear. It is developed in stages. In the initial stage it is applied to the price dynamics of one type of stock only. Later on it is applied to a weighted price index of different stocks, to try to capture the dynamics of a stock exchange as a whole. The purpose of the model is to gain insight both into short-term dynamics and stability properties, and the dynamics of long-range cycles and crashes. Based on the model, the transaction tax reform proposal to stabilise stock markets is discussed. Another and new stabilising idea is presented-substituting a stock with a type of bond.


## Introduction

There are different motivations for research into and modeling of stock market dynamics:

- To try to make a profit by using one's model as a tool for speculation.
- To use the model to gain insight into stock market dynamics.
- To use the model to gain insight into stock market dynamics and systemic weaknesses, and based on this, suggest reforms.

The first motivation seems by a large margin to be the most popular. This paper, however, stems from the third motivation. I am well aware that many market participants and (also academic) observers would react to the third motivation with the reply that such activity is uneccessary, or (having an open mind) futile since there are no interest in such reforms whatsoever in circles having the power to institute them. To this I can only reply that if there are serious problems they ought to be remedied, and one should say so even if the current probability of being ignored is high. And this type of research and discourse should be more urgent with today's free-flow and interconnected global economy, where the effects of a large event in for instance the US stock market propagates round the world in a fraction of a day, feeds back to the place of origin, and through such a process may initiate serious financial crises.

A much-discussed suggestion for stock market (and currency trade) stabilisation is to introduce a transaction tax (fee). But some empirical studies show that volatility ${ }^{1}$ is not reduced when transaction taxs are implemented - a frequently-quoted paper is Jones and Seguin (1997). Some skeptics argue that transaction taxs are harmful, since they reduce liquidity in the market (Davidson, 1998). While being agnostic ${ }^{2}$ on whether such transaction taxes will have the desired stabilising effects, this paper suggests a possible explanation for volatility not neccessarily being reduced with a(n) (increased) transaction

[^0]tax. By this I take the position as 'devil's advocate', since I believe that reforms to stabilise stock markets (and also currency markets) are very much needed.

This paper however, also argues that volatility as such is not the main problem. Instead two other phenomena which are related - but not equivalent - to volatility, are focused: instability, and gross long-term overvaluation. It will furthermore be argued that instability is only a problem in connection with gross long-term overvaluation. A system of 'voting bonds' instead of stocks is suggested to remedy this.

The paper is organised in four main sections: Short-term and long-term dynamics, a reform proposal, and finally an appendix with details about how the model has been developed and how parameter values have been decided.
2. This agnosticism is based on the belief that socio-economic processes are so complex that the only way to decide issues like this is to try out proposals - just as one in the physical sciences has to do laboratory experiments to decide controversies that cannot be solved through theoretical research and discourse only.

## Short-term dynamics

## The short-term model

Consider the block diagram in figure 1 :


Figure 1 about here
The symbols in the diagram are defined as follows (denomination is shown in brackets [], empty brackets mean that the corresponding entity is dimensionless):
$p_{r} \quad=$ 'real' or 'sustainable' value of the stock [ ], expressed by the price/dividend ratio it can yield in the long run. At $p_{r}$ the stock is neither over- nor undervalued. For convenience I will use the term 'price' or 'value' in the following, even if I am talking about the price/dividend ratio. $p_{r}$ is assumed constant in the following.
$p \quad=$ current market price (that is, price/dividend ratio) of stock [ ].
$\frac{\dot{p}}{p} \quad=$ price change rate $\left[\right.$ day $\left.^{-1}\right]$. The dot implies differentiation with respect to time.
$s \quad=$ differentiation operator $\left[\right.$ day $\left.^{-1}\right]$. See footnote 3.
$n \quad=$ net current aggregate demand for stock [number of units]. This demand may be negative, that is, when there is a net surplus of stocks offered.
$c_{1} \quad=$ constant factor $[1 /$ (number of units $\cdot$ day) $]$ transforming net demand into price increase rate. The total number of stocks issued is incorporated in this
factor. There is, as indicated in the figure, saturation in price decrease rate, since current surplus offered cannot exceed the total number of stocks issued. In the model, this is translated into a maximum rate of price decrease. This saturation will only be reached in connection with panics, treated in a later section.
$c_{2}=$ constant factor [number of units] transforming price deviation into corresponding net demand. The total number of stocks issued is incorporated also in this factor.

Surplus aggregate demand is now assumed to consist of three components,

$$
\begin{equation*}
n=n_{r}+n_{b}+n_{e} \tag{1}
\end{equation*}
$$

We will from now on use the term 'demand' in the sense of 'surplus aggregate demand'. We have
$n_{r} \quad=$ Component due to 'fundamentalist' agents' belief about the sustainable value of the stock.
$n_{b} \quad=$ Component due to agents watching price increase/decrease rate and doing 'technical trading' based on this. The sustainable value of the stock has no direct influence on this component. Subscript $b$ signifies 'bandwagon'.
$n_{e} \quad=$ Component due to fundamentalist agents having different estimates of the sustainable value of the stock, due to bandwagon agents taking differing action based on the same price increase rate, and due to the influence of external events (the 'news process'). $n_{e}$ also incorporates modeling errors and simplifications. It is assumed to be a zero mean stochastic process. Subscript $e$ signifies 'error'.

The differential equation

$$
\begin{equation*}
T_{b} \frac{d n_{b}}{d t}=-n_{b}+K_{b}\left(\frac{\dot{p}}{p}\right) \tag{2}
\end{equation*}
$$

which corresponds to the transfer function ${ }^{3} h_{b}(s)$ from $\dot{p} / p$ to $n_{b}$, shown in the block diagram in figure 1,

$$
\begin{equation*}
h_{b}(s)=\frac{K_{b}}{1+T_{b} s} \tag{3}
\end{equation*}
$$

is the chosen model for the speculative component of demand: There is a positive feedback through $h_{b}(s)$ from price increase rate to the demand component $n_{b}$. If for instance $\dot{p} / p$ is large and positive at a certain moment, many agents will jump on the bandwagon and buy now in the hope that prices will continue to rise. Of course some technical trading strategies are more elaborate than this, for instance action in 'counter-phase', buying when prices are falling in the expectation that they will rise later on. It is assumed however, that herd mentality is the dominant type of speculative behaviour. Invoking Occam's razor, the simplest model that accounts for this is given by equations (2), (3). The parameter $T_{b}$ expresses the small time lag from acquiring price information to buying (or selling), that speculative action cannot get around. This lag is due to delays in acquiring information, considerations, and then getting the trading done. The gain $K_{b}$ expresses how strong speculative action is, based on the available price change rate information.

Note the term 'action', as opposed to 'agents'. Individual agents may of course operate in a purely speculative/bandwagon mode, others may again be pure 'real investors'. But most have composite motives (real-economic more or less off the mark, and speculative). When the market as a whole is considered this discussion becomes unimportant, since the market as a whole must neccessarily have a 'composite motive'.

The surplus demand component $n_{e}$ accounts for the aggregate effect of agents making erroneous and different assumptions about the stock value, but in such a way that the mean surplus demand error is assumed to be zero. We also incorporate the effects of
3. Here $s$ is a differentiation operator, so that $y(t)=\frac{1}{1+\tau s} x(t) \Leftrightarrow(1+\tau s) y(t)=x(t) \quad$, shall be interpreted as $y(t)+\tau \frac{d y}{d t}=x(t)$; a linear differential equation with input $x$ and output $y$.
differences in individual speculative behaviour into this noise process, since in reality each speculative agent will act according to unique dynamics, which will also be nonlinear with parameters and structure that will change with time. All this individual behaviour is averaged into the linear, time invariant transfer function (3). What is lost through this simplification is then assumed to be to a sufficient degree represented through the error process $n_{e}$. Thus, $n_{e}$ has at least two components: erroneous estimates of the stock's sustainable value, and modeling errors due to aggregation. A third component is the effect of different exogenous economic news that influence the valuation of the stock.

If we now consider a situation where the price of the stock is at its sustainable value, the market should have no real-economic incentive to trade. But trading will take place all the same, since exogenous changes influence the market - this is the 'news' component of the process $n_{e}$. And individual more or less rational, more or less well-informed agents have their own differing assessments, and trade based on this, even when there are no exogenous 'news'-related impulses. In the system-theoretic language of this paper, we may say that the error process $n_{e}$ is a 'disturbance' that excites the system, so that the market is never in equilibrium, but fluctuates around it.

For small fluctuations $\Delta p$ around a constant $p_{r}$, so that $p=p_{r}+\Delta p$, we have a linear system which is excited by the error process $n_{e}$. The transfer function from $n_{e}$ to $\Delta p$ is

$$
\begin{equation*}
h_{p, n_{e}}(s)=\left(\frac{p_{r} c_{1}}{T_{b}}\right) \frac{1+T_{b} s}{s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}}, \tag{4}
\end{equation*}
$$

where the undamped resonance frequency is $\omega_{0}=\sqrt{\frac{c_{1} c_{2}}{T_{b}}}$,
and the relative damping factor is $\quad \zeta=\frac{1+c_{1} c_{2} T_{b}-c_{1} K_{b}}{2 \sqrt{c_{1} c_{2} T_{b}}}$

The two eigenvalues of the system are indicated as small dots in figure 2 :


Figure 2 about here
Consider a case where $K_{b}$ is increased while $T_{b}$ is held fixed, that is, speculative action is stronger, while the information/decision time lag remains the same. From (5) we see that $\omega_{0}$ is independent of $K_{b}$, while (6) implies that $\zeta$ decreases with increasing $K_{b}$. In terms of figure 3, this means that the eigenvalues of the system move along the circle towards the imaginary axis.


Figure 3 about here
The system approaches the border of instability, which means increased volatility: For a given variance in the error process $n_{e}$, the variance in price will increase with $K_{b}$. This follows from the two expressions

$$
\begin{equation*}
\phi_{p p}(\omega)=\left.\left|h_{p, n_{e}}(s)\right|_{s=j \omega}\right|^{2} \phi_{e e}(\omega) \tag{7}
\end{equation*}
$$

$$
\begin{equation*}
\operatorname{var}(\Delta p)=\frac{1}{2 \pi} \int_{p p}^{\infty}(\omega) d \omega \tag{8}
\end{equation*}
$$

where $\phi_{e e}(\omega)$ and $\phi_{p p}(\omega)$ are the power spectral density functions for the input noise process $n_{e}$ and the output zero-mean process $\Delta p$, respectively. If $h_{p, n_{e}}$ is close to unstable, it has a high resonance peak near frequency $\omega_{0}$, which is then mirrored in $\phi_{p p}(\omega)$, following (7). This translates into a large area under $\phi_{p p}(\omega)$ and a correspondingly high $\operatorname{var}(\Delta p)$, according to (8).

There is an interesting insight that emerges from the model at this stage. Again consider figures 2 and 3 . Numerical values for $K_{b}$, and all other parameters have been chosen through a procedure described in the appendix. The eigenvalues are located on the imaginary axis $(\zeta=0)$ and the system is on the edge of instability, for $K_{b 1}=14378$. On the other hand the system is overdamped (i.e. 'non-volatile') for $\zeta>1$, which we have when $K_{b}<12077=0.84 K_{b 1}$. Since real stock markets seem to be (see appendix) underdamped $(0<\zeta<1)$, this suggests that there is some adaptive mechanism at work to keep $K_{b}$ close to the gain $K_{b 1}$ that makes the system unstable.

That mechanism may be the following: One one hand, bandwagon-type agents are attracted by short-term movements, to exploit them through trading. By their participation, they increase $K_{b}$ and volatility, by this attracting even more trading. But when the market gets too turbulent, some of them abstain, thus decreasing $K_{b}$, and temporarily stabilising the system. This type of behaviour is confirmed by prominent Norwegian speculative traders in an interview (Aftenposten report, 2000).

The resulting fluctuations between system stability and instability also explain the irregular bursts of volatility that characterise time series of stock prices. This issue is central in Lux and Marchesi (2000), who attribute such bursts to fundamentalists switching to bandwagon behaviour. For the purposes of this paper, we will see that it is not neccessary to account for the burst phenomenon. It suffices to model short-term dynamics
as time invariant, but such that the system is close to instability.

## The ideal stock market

The textbook argument for a stock market is that it is an optimal way to channel surplus money into investment: The sum of all participants' trading activity channels society's surplus into such enterprises as are considered by the collective mind of the market to have the best future prospects. If we relate this conception of a stock market to the model in figure 1 , this corresponds to the case $K_{b}=0$. All action is then taken on the basis of each agent's best valuation of a firm's prospects, regardless of what other agents do. Demand is

$$
\begin{equation*}
n=n_{r}+n_{e}, \tag{9}
\end{equation*}
$$

there is no 'bandwagon' component $n_{b}$. The transfer function (4) from $n_{e}$ to $\Delta p$ now reduces to the first-order expression

$$
\begin{equation*}
h_{p, n_{e}}(s)=\frac{c_{1}}{s+c_{1} c_{2}} \tag{10}
\end{equation*}
$$

If we use this in (7) and (8), we find that this market will have much lower volatility than for the case $K_{b}>0$. More important, the system is now dramatically more stable. In the section on long-term dynamics we will make the point that a smaller value of the inner loop gain $K_{b} c_{1}$ has the advantage of making stock market crashes much less probable. A possible stabilisation solution is therefore some institutional market reform that reduces $K_{b}$ without increasing $c_{1}$. We will now discuss whether it is probable that the transaction tax proposal will have this desired effect.

## A reform idea: A transaction tax

The conclusion of this subsection will be that a stock market with transaction taxes will not neccessarily be less volatile and more stable (but it will probably have a lower
trading volume). The main argument is that the bid/ask spread, cet. par., should increase with a transaction tax, because both sellers and buyers have to compensate for this tax in the price they offer. An increased spread translates in our model into a proportionately larger coefficient $c_{1}$, thus countering the reduction in bandwagon loop gain because of a lower $K_{b}$.

I will use the Oslo Stock Exchange (OSE) as a basis for the argument ${ }^{4}$. On the screen the brokers at any time see an 'order list' (OL) for any specific stock. On this OL, stocks bid or asked are listed with quantity and corresponding price, sorted by price. If any prices bid and asked coincide, the OSE system executes a trade. The range that the price jumps up or down when a trade occurs, must be roughly proportional to the average bid/ask spread. If there is a net surplus $n$ of stocks being bid for, the probability of a price jump upwards is larger than for a jump downwards, and vice versa. This is the first argument for the equation

$$
\begin{equation*}
\frac{\dot{p}}{p}=c_{1} n \quad, \text { with } c_{1} \text { proportional to the bid/ask spread. } \tag{11}
\end{equation*}
$$

If no trades occur in a given time interval, the OL is still updated continually as a result of what new bid/asks the totality of brokers choose to input, based on the current OL and other information. In this situation, if there is - say $-n$ more stocks being bid for than offered, there is still a 'pressure' for price increase at work, since brokers will then adjust bid and ask prices upwards based on this observed imbalance. Again (11) is confirmed.

While a(n) (increase in the) transaction tax will reduce $K_{b}$ (which cet. par. would make the system more stable - the rationale for the transaction tax reform), one will also get the unwanted side effect of an increase in $c_{1}$, as explained above. If this increase is stronger than the reduction in $K_{b}$, the gain $K_{b} c_{1}$ in the positive (bandwagon) feedback loop in figure 1 will increase, and the system will become less stable with the increased

[^1]transaction tax. This is reflected in a lower relative damping factor, see equation (6). The term $c_{1} c_{2} T_{b}$ in (6) is negligible in relation to $1-K_{b} c_{1}$, so that increased $c_{1}$ has roughly the same destabilising effect as increased $K_{b}$. Such near-instability, due to the bandwagon feedback loop, is one condition for panics and crashes in the model. This is discussed in the next section.

We should not, however, conclude by the above that transaction taxes will be futile. This can only be definitely decided by trying them out. The point has merely been to present a reason for why transaction taxes may not neccessarily do the job.

Finally, note that volatility in the sense of price variance has not been an issue here - the issue is systemic (in)stability. Price variance may be be high if activity and volume is high, even if the system is quite stable.

To sum up this section on short-term dynamics, the goal has been to ensure system stability by reducing the gain $K_{b} c_{1}$ in the positive feedback loop. The main motive has been not to decrease volatility (price variance), but to increase stability and through this block dangerous panics in a stock market, 'dangerous' in the sense that such events can have grave impacts on the rest of the economy. Since reducing $K_{b} c_{1}$ seems to be difficult, we will from now demand that a satisfactory crash-preventing proposal must work in spite of the market being in a 'normal' near-unstable condition.

## Long-term dynamics

From now on let us consider the model not to represent a specific stock, but a 'composite stock', composed of stocks from all firms listed on a stock exchange. The composite stock $\mathrm{p} / \mathrm{d}$ ratio is roughly proportional to a deflated stock exchange index (for short-term movements inflation may be ignored). The $\mathrm{p} / \mathrm{d}$ ratio for the 'composite stock' is defined as the total value of all stocks traded on the exchange, divided by the total sum of dividends. All stocks are assumed to have roughly similar dynamics. These assumptions mean that the composite stock $\mathrm{p} / \mathrm{d}$ ratio (from now on for convenience called the 'index' or 'price') will also fluctuate around $p_{r}$, with dynamics that are similar to those for one specific stock. The difference is that a price shock for one stock only, does not impact very strongly on the index (as opposed to for instance a Central Bank interest rate change, see examples in appendix).


Figure 4 about here
The aggregation step from one stock to an index is comparable to the earlier step of aggregating all agents into one composite agent. We uphold all variable and parameter
names and numerical values introduced for the one-stock model, with the note that they now pertain to the index. By now we are ready to consider an augmented model as shown in figure 4. Again we assume that model imperfections and approximations, and fluctuations in demand for the 'composite stock' implied by the index, is accounted for by a zero mean noise process $n_{e}$ as introduced earlier. We recognise the short-term model in the lower half of the figure. Ignore until further notice the nonlinear function outlined in bold in the upper part of figure 4 ; assume this to be unity for the time being. Consider the block in the upper right-hand corner. The instantaneous price increase rate is an input to the block, which is a low pass filter with time lag $T_{f}$ of a one year magnitude. The rationale for this filter is that the increase rate of 'optimism' (or 'confidence', 'bullishness', 'animal spirits') is assumed proportional to the long-term trend in index increase. Hourly, daily, even weekly and (to some degree) monthly fluctuations are disregarded; there is a sluggishness in market mood. Price appreciation has to be persistent over a long time before the market really picks up. On the other hand, when the price culminates and starts falling, the market will need a corresponding amount of time for such a change of affairs to sink in. The increase rate in optimism is set equal to the market's perception of the longterm index increase rate. Optimism is given a numerical value, and a range which is both positive and negative. Thus 'pessimism' corresponds to negative optimism.

By now it should be clear why it has been neccessary to transit from an invidual category of stock to an index: Market mood is a function of the behaviour of the aggregate of all stocks, not one category only.

In the absence of any perception of a long-term tendency for price to change (that is, a flat price level over a long perod), the current level of optimism will slowly erode to zero, through the "forgetting factor" $c_{3}$. The argument for this is that the market will gradually forget its initial mood and tend towards a neutral attitude (zero optimism or pessimism) if the mood is not maintained by a sustained increase or decrease in stock price.

Let us see what happens if we isolate the upper subsystem encircled by a dotted line from the model, input a rectangular price increase rate pulse, and observe the response in optimism. We assume a one-year (defined as 250 trading days) constant price increase rate pulse. This pulse, and the corresponding response in optimism, is illustrated in figure 5.


Figure 5 about here
The input pulse is not shown to scale. The choice of parameter values is discussed in the appendix. Note how optimism culminates after around 500 trading days (2 years). There is thus a very large difference between the choice of fast dynamics of the bandwagon loop (minutes, hours), as opposed to the newly added long-term mood loop (years).

To complete the explanation of the long-term mood loop, consider the nonlinear function in figure 4, assumed to be unity until now.


Figure 6 about here
This function, shown in figure 6 , introduces a weakening of the coupling from price
change rate to mood change rate when the price is low.
Without this modification, simulations break down by the system "diving" at an accelerating rate into zero price during the downswing phase. This happens since when $p$ is small and falling, the price change rate $\frac{p}{p}$ grows strongly negative, thus accelerating the negative mood. There are at least two possible modifications to counter this: One is to simply introduce a limit to mood on the negative side, another is the one chosen - to reduce the coupling from the price change rate to mood change rate when prices are low. This should be reasonable also in a real word sense: Agents are probably less mood sensitive to price changes when the price is low - they are then holding out and waiting for better times.

We have in this subsection introduced two first-order linear blocks and one nonlinar relation, which together form the long-term mood loop. Parameter values have been decided through a simulation-based trial-and-error process described in the appendix. With the resulting choice of parameters (and no external noise exciting the system; $n_{e}=0$ ), the price cycles look like in figure 7.


Figure 7 about here
The first cycle is somewhat different. This depends on the choice of initial values. But after a while the system settles down to the same regular (limit) cycle, regardless of the choice of initial values.

Note that the model is not dependent upon a crash-and-subsequent-recovery mechanism for cycles to occur, or upon being excited by any exogenous variable, for instance $n_{e}$. This system can never be in equilibrium, it will always self-oscillate. The dynamics stem from endogenous mechanisms whch may be verbally described as follows: The upswing is due to the spreading and self-reinforcing belief that 'if I get in now, I can always cash in my investment with a profit at some later time'. This upswing, however, sooner or later has to culminate at some level, when enough agents feel that current prices have grown far too high in relation to the sustainable value of the stock, and act based on this. In the model this is accounted for by a gradually more negative demand component $n_{r}$. This leads to a stagnating price growth trend which is reflected in slower growth of optimism, which again feeds back through demand to further slowing of the price growth rate, and the price will eventually culminate. The optimistic mood will start deteriorating when price has stopped increasing, and this gradually leads into a downswing. But the downswing also needs time to build up momentum. Years later, things will pick up again after the pessimistic mood culminates because of a sustained and now positive demand component $n_{r}$. The upswing starts, and one cycle is completed. The period time of a cycle is in the main decided by the inertia of market perception: It needs time to absorb a persistent tendency in price change, and it needs time to forget.

## Panics and crashes

The reader may at this stage object that the downswing predicted by our model is very slow and well-behaved. Where are the panics, which may erase a substantial part of an index in part of a trading day? Different panic mechanisms are conceivable. Whether the panic is triggered by an easily identifiable event (for instance a strong and unexpected Central Bank interest hike), or simply a random large price dip, a common feature should be the occurence of a strong negative mood impulse. We will focus on systemic fragility
in connection with such an event.
A conceivable panic mechanism is modeled by the additions and changes indicated by shading in figure 8 .


Figure 8 about here
A "panic subsystem" emits a negative pulse that abruptly reduces optimism and thus the demand component $n_{o}$. This happens when two conditions are fulfilled: The price is far above $p_{r}$ and there is a negative spike in $\dot{p} / p$ that is so large that it is noticed by the market. Two inputs are multiplied with each other. They are the factor

$$
\begin{equation*}
c_{5} \frac{p}{p_{r}} \tag{12}
\end{equation*}
$$

where $c_{5}$ is a constant, and a negative spike which is
$\dot{p} / p-z$ when $\dot{p} / p<z$,
else the spike is zero. The value $z<0$ defines a 'dead-zone' ${ }^{5}$, which means that any price change rate less negative than $z$, is not noticed as exceptional.

[^2]Equation (12) is an expression that will be larger the further above sustainable value the current price is. This is assumed to give a measure of 'wariness' in the market. Agents will on the average be more sensitive to large downward blips in price when the current price is much higher than sustainable value. This property is modeled by multiplying (12) and (13) and inputting the result to the optimism subsystem. If price is not far away from sustainable value, even noticeable downward blips are not reacted to as danger signals.

There is a further modification to the system: The until now fixed saturation level of price decrease rate is changed to a 'dynamic saturation', which works like this: If there has been a recent strong downward price blip, saturation will occur at a less negative value for some time afterwards. The argument is that if there is large-scale selling at one instance (which is the reason for a sharp price decrease), those who have just bought will stick with the newly acquired stock for some period instead of selling it immediately again at a lower price. Simulations without this modification result in crashes where the price falls steeply to near zero in a day or two, which is obviously unrealistic. The details of this mechanism is explained in the appendix.

Simulation runs of the model with the the panic mechanism are shown in figure 9.


Figure 9 about here (may be divided between two pages)

The first run, now displayed in three different time scales, is shown in figure 10


Figure 10 about here
The system is excited by the process $n_{e}$, which is chosen as a normally distributed lowpass filtered white noise process. The choice of statistical properties for this process is not critical for our purposes. Choice of process parameters is discussed in the appendix.

The same smooth arc repeated (for comparison) in all graphs in figure 9, is the peak part of a system period with $n_{e}=0$, see figure 7 . The differences between each run with $n_{e} \neq 0$ are only due to different initial states in the random noise generator, i.e. the statistical properties of the noise process $n_{e}$ are the same for all runs. The spikes shown below each crash are the corresponding "panic pulses" $z$ (shown not to scale, and with the opposite sign). We note that all runs coincide with the smooth arc before the first crash (except for the fuzz due to $n_{e} \neq 0$ ). Any crash contributes to an earlier downswing than what we get with $n_{e}=0$. We also note that an early crash will "defuse" the system somewhat because the system will not reach the same maximum price level afterwards, as
indicated by run no. 8 .
Admittedly, the panic mechanism posited here is one candidate out of several conceivable. Johansen and Sornette have in a series of papers, among them Johansen and Sornette (2000), analysed empirical data and made a case for crashes being the end result of a frenzied phase with a characteristic log-periodic shaped price curve in the weeks leading up to a crash. The model in this paper cannot generate that sort of dynamics. An attempt to realise such a model by modification of the model presented here, would probably have to include one or more medium-term feedback loops between the short- and the long-term loop, with dynamics adapting because of agents changing their behaviour due to their running observations of what sort of behaviour is most successful. But this is outside the scope of this paper. What it has in common with Johansen et. al., however, is the position that crashes are outliers in a statistical sense. In our model that is accounted for by an additional feedback connection being activated in a crash situation, which means that the model is abruptly changed, and remains different and strongly unstable for a short time until it lapses back into the 'ordinary' nearly unstable model again. Johansen and Sornette's log-periodic path also breaks down at the crash point, but they do not suggest any mechanism for the crash.

Regardless of the above, such differences in the approach to crashes do not impact significantly on the suggested long-range limit cycle dynamics and gross overvaluation as indicated in figure 7, which constitute the main focus of this paper.

## The interaction between short- and long-term dynamics

Consider figure 11, which is identical to figure 8, except for removal of the shading, and the emphasising of two positive feedback loops:


Figure 11 about here
The panic mechanism unfolds like this: Assume a strong random downwards price blip in a situation where $p » p_{r}$. A large 'panic pulse' results, and optimism is abruptly reduced, leading to a corresponding abrupt fall in $n_{o}$. This again leads to a sudden decrease in $\dot{p} / p$, which is amplified immediately through the bandwagon feedback loop. The gain $K_{b} c_{1}$ is crucial. Simulations show that a slight reduction in the loop gain $K_{b} c_{1}$ is sufficient to block the evolving of panics that occur when $K_{b} c_{1}$ is at a value that corresponds to near-unstable short-term-dynamics (i.e. the real-world situation). But we have already seen in the previous section that finding a scheme that reduces the bandwagon loop gain $K_{b} c_{1}$, is difficult. So the next subsection will be dedicated to another proposal which, in spite of not reducing the high bandwagon loop gain and volatility, still could do the trick of inoculating the system against crashes.

## A reform idea: 'Voting bonds'

This idea ${ }^{6}$ (Andresen, 2000) is admittedly even more 'unrealistic' than the proposal of a transaction tax, not in the sense that it will not work, but in the sense that it implies a fairly large institutional change to stock markets.

The method is to ensure that gross overvaluation, that is, $p$ » $p_{r}$ never occurs, so that big random downward price blips will not have any dramatic impact when they occur in a normally near-unstable stock market. This may be achieved by changing the character of stocks from perpetuities (financial claims that never mature) to 'stocks that are repaid at nominal value', that is, to what we could perhaps call 'voting bonds' (from now on, 'VBs'). Before explaining the (hopeful) advantages of this proposal, first a description of what a VB is:

A new or existing firm in need of capital issues VBs instead of stock. VBs give dividends just as stocks do, and the dividend rate is decided each year just as with stocks, by those holding the firm's equity (that is, its VBs). But after a predetermined amount of years, the VB matures, and will be paid back at its nominal value, just as a bond.

If there is a situation with VBs that were issued at some earlier time, and the firm wants to float additional ones (this will be the typical case), the voting power of already existing bonds is reduced correspondingly. After such an additional float, every VB, old and new, has a voting power proportional to its nominal value. (As a possible modification to compensate for inflation, new VBs may be assigned a slightly lower voting power, so that inflation since the last float is compensated for.) Whenever a firm floats new VBs, the new and reduced voting power of all earlier and still non-matured VBs are calculated and

[^3]published. All this is easy in a computer- and net-mediated market environment. Note that there is a nice built-in balancing mechanism here: Whether to float additional VBs is decided by the current owners, that is, those who hold a majority of existing VBs. Thus they have an incentive to hold back before floating new (and additional) VBs.

So far on technicalities. What is the point of this proposal?
Its main purposes are twofold:

- To remove the phenomenon of gross long-term overvaluation, with ensuing long term upswings and downswings, and occasional panics and crashes, that have dangerous effects on the real economy.
- To enable the stock (now: VB) market do what the textbooks say it is supposed to do, channel fresh cash into activities that the population of market participants thinks have a future.

To the first point: In a VB market the participants know that if demand for a given firm's VBs drives the price upwards, the firm may very well decide to float additional VBs. In fact, the firm very probably has to do that sooner or later, at the least to redeem earlier floated VBs when they mature, so this will happen. And any VB har a limited lifespan, and will be redeemed at nominal value. This knowledge in the market is a strong incentive aginst buying existing VBs far above nominal value. Thus gross long-term overvaluation should not occur. This again should remove the primary reason for the biggest panics and crashes. This is what could be called a 'robustness' argument for the VB proposal - the main argument for it.

Now to the second point: Today only a negligible part of stock market buying is fresh cash drawn in for new venture. Most of it is a game of musical chairs in already existing stocks: '....The social object of skilled investment should be to defeat the dark forces of time and ignorance which envelop our future. The actual, private object of most skilled investment today is to beat the gun, as the Americans so well express it, to outwit the
crowd, and to pass the bad, or depreciating, half-crown to the other fellow.' (Keynes ([1936] 1973, p. 155).

VBs however, are paid back when they mature. Firms have to renew their equity continually (as already stated). This should stimulate the market to do what it is supposed to do: In an 'optimal' way channel and shift money to those ventures that are the most promising, based on the collective wisdom/knowledge of the aggregate of market participants. This is an 'efficiency' argument for the VB proposal.

There is, however, an important argument against the proposal: How can a VB market take care of the needs of an entrepreneur who runs a big risk investing in a new venture, sticks with the initially issued equity without drawing dividends, hoping to be able to sell it some years later at a price that covers the large risk premium plus a reasonable appreciation? The answer is that it can't, so some modification is needed. A possible amendment to ensure an incentive for entrepreneurship in a VB-dominated environment, is to give buyers a choice between how the stock acquired/created is to be treated:

- As a VB with high liquidity but low chance of long-term appreciation because of finite time to maturity.
- As a stock with unlimited life time, but with a transaction tax levied when it is sold. The tax rate should be initially high and fall exponentially with the time the stock is held by the owner. If the owner sticks with the stock for - say - ten years, the tax rate may be approximately zero.

The character of the stock has to be decided by the buyer before purchase/initial creation, logged in a stock registry, and remain unalterable until it is sold. This modification to the VB proposal complicates the issue somewhat, but is still technically easy to implement.

Finally, to another counter-argument against the VB proposal: "Who owns the firm if all VBs are redeemed and no one wants to by new ones?". Firstly, this is a situation that will arise only when bankruptcy is imminent - then the firm will cease to exist anyway.

Secondly, the reform could mandate that a certain small minimum share of any firm's stock must be perpetuities. If this limit sitiuation has been reached (which should be very seldom) a prospective buyer will be informed that the equity currently under consideration must be bought as a perpetuity, not a VB.

Speculators will of course try to circumvent such a proposed new regime. I cannot at the time of writing see any serious flaws with the proposal, but suggestions about how circumvention might be carried out, are welcome. Hopefully this can contribute to a comprehensive discussion of reforms to stabilise stock markets - a crucial issue in today's world.

## Appendix: Building the model, choice of parameter values

The general problem with modeling the dynamics of an index, based on real-world data, is that we lack information about what sort of processes excite the system. We can measure the system output (the index), but know very little about the input. The method chosen here is therefore to search out events where one specific and known input was so dominant for some reasonable time window, that this together with the index behaviour for the same period can be used to extract a coarse system model.




Figure 12 about here
Such an event is a Central Bank interest decrease. Referring to figure 1, this corresponds to inputting a small positive stepwise increase to $p_{r}$ (or equivalently, except for a constant
multiplicative factor, to $n_{e}$ ).
Figures 12 and 13 show the responses for some U.S. indices in connection with two different Fed interest rate reductions, on 3 January and 18 April 2001, respectively. (The April graphs show only the pertinent part of the trading day.) The horizontal bars corresponds to one tenth of a trading day of 6.5 hours.


Figure 13 about here
We note that a common feature is a steep index increase, overshooting upwards and slightly downwards before the reaction to the interest rate cut tapers out so much that it, except for the system remaining at a higher price, is drowned by the effects of other and not knowable input processes.

The simplest model to account for this type of underdamped step response is a second-order linear system, which is the one initially chosen and shown in figure 1 . The model given by equations (4) - (6) has been simulated for a range of settling times and
relative damping values. A step response that was considered acceptably close (by visual inspection) to the data in figures 12 and 13 is shown in figure 14:


Figure 14 about here
The system that generated this response has undamped resonance frequency

$$
\begin{equation*}
\omega_{0}=\frac{2 \pi}{T_{0}}=62.8, \text { where the period time is } T_{0}=0.1 \tag{14}
\end{equation*}
$$

and the relative damping factor is $\zeta=0.4$
We now choose a reasonably short time lag for bandwagon behaviour,

$$
\begin{equation*}
T_{b}=0.00128 \text { [trading days] }(=\text { half a minute }) \tag{16}
\end{equation*}
$$

We may freely decide the coefficient $c_{1}$, and choose $c_{1}=7 \cdot 10^{-5}$. By this, the second.order model is uniquely determined, since $c_{2}$ and $K_{b}$ may now be calculated from equations (5) and (6).

The sampling time for storing simulation results was chosen as $T=1 / 13$, but the discretisation interval for numerical simulations had to be chosen smaller than $T_{b}$, and was therefore set to $T / 100=0.6 T_{b}=0.3$ [minutes]. The numerical simulation algorithm chosen is the fixed-step Euler method. The 'sustainable price' $p_{r}$ is chosen as a constant; $p_{r}=10$, throughout.

At this stage, let us digress to again consider the special idealised case with $K_{b}=0$, i.e. no bandwagon behaviour, see equations (9) and (10). The time lag for this 'pure
fundamentalist' model is then

$$
\begin{equation*}
T_{r}=\left(c_{1} c_{2}\right)^{-1}=\left(T_{b} \omega_{0}^{2}\right)^{-1}=0.198[\text { trading days }] \approx 1.5 \text { hours } \tag{17}
\end{equation*}
$$

(Other reasonable parameter values leading to other values of $T_{r}$ have been tried out, and it turns out that different choices are not critical for the analysis and conclusions.) With $K_{b}=0$, all agents have the same perfect information about $p_{r}$. Any change in $p_{r}$ is responded to by each agent in the same manner. However, action is assumed to be dispersed in time. While such agents receive perfect information, they do not act instantaneously, and slower than bandwagon agents. With $K_{b}=0$, a small upwards jump in $p_{r}$, for instance because of a reduction in the Central Bank overnight rate, results in a price adjustment path that is a first order stable exponential step response as shown in figure 15 :


Figure 15 about here

We have no overshoot, no oscillations, no unpredictable excursions, just a smooth and asymptotically perfect price adjustment. This is of course a completely unrealistic representation of what occurs in the real world. At the same time, it should be noted that this is the way a stock market ought to work, reflecting real-economic changes impacting the price, and nothing else.

Now to the long-range model, but without crashes, as introduced in figure 4. First, choices were made for the recognition time lag in connection with long-term price change; $T_{f}=200$, and for the forgetting time in the optimism subsystem; $1 / c_{3}=714$. These
choices are fairly arbitrary, but they should be chosen within the range of a year or three (one 'trading year' is defined to be 250 trading days). Several simulations show that values over this range give the same qualitative behaviour. The gain $c_{4}=234046$ was determined by demanding that the period of one long-range cycle should be in the order of 8 years $=2000$ trading days. An increased $c_{4}$ leads to shorter periods, and vice versa. Finally, the nonlinear 'decoupling function for low prices' shown in figure 6,

$$
\begin{equation*}
f(x)=\frac{1}{1+\left(p_{r} / p\right)^{c_{6}}} \tag{18}
\end{equation*}
$$

was determined by experimenting, ending with a choice $c_{6}=2$ that gave a smooth exponential rise in the upswing part of the cycle, and reasonable peak and bottom prices. It turned out that system dynamics were bounded and cyclical for a wide range of values, but the cycles were rather big in amplitude and 'squarish' for larger values of $c_{6}$.

The final model with crashes was determined by first considering the character of the noise process $n_{e}$. The crash mechanism was still disconnected at this stage. Simulations were done to decide a reasonable variance and autocorrelation for the noise process, which was chosen normally distributed. For several parameter sets, the graph of the resulting simulated price over one trading day was visually compared to some one-day graphs of the three U.S. indices. Acceptable similarity was achieved when $n_{e}$ was chosen as low-passfiltered white noise with a filter time constant $=3$ days.

The most difficult part of the parameter-determining process was when implementing the crash mechanism introduced in figure 8. The first step was tinkering with two parameters simultaneously, the dead-zone limit $z$ (see equation (13)), and the 'wariness at high price level' coefficient $c_{5}$ (equation (12)). The choice of $z$ after many simulation runs was $z=-0.155$, which corresponds to a price decrease per half-hour of $1.3 \%$. If $z$ is too small in magnitude crashes occur too often - and vice versa. But this also depends on $c_{5}$. If $c_{5}$ is too small crashes never occur, not even at the peak of a cycle, and
$c_{5}$ too large leads to crashes too far below the peak price level. The choice after many simulations was $c_{5}=0.9$.

The final touch to the crash mechanism was how to avoid price diving to zero during a crash. This has already been mentioned; here are the details: Consider figure 16, which shows a detailed view of a lower subsystem in figure 8:


Figure 16 about here
The net aggregate supply of stock ( $n$ is negative when supply is larger than demand) can now never go lower than a value $\underline{n}$ ' that is a variable saturation in net supply. If there is a situation with no recent panics, we set $\underline{n}^{\prime}=\underline{n}=\frac{1}{c_{1}}\left(\frac{\dot{p}}{p}\right)_{\text {min }} . \underline{n}$ is a constant - chosen as $\underline{n}=-0.25$, i.e. a floor of $-25 \%$ price fall during a trading day is assumed. If there has been recent panics, however, $\underline{n}^{\prime}$ will be $>\underline{n}$. This modification blocks the otherwise occuring unrealistic event of several contiguous panics bringing price towards zero. After a couple of days, however, agents will again be ready to sell stock they bought in the last panic, and further panics are therefore possible. This is ensured by $\underline{n}^{\prime}$ sinking gradually towards $\underline{n}$ through decay of the output from the transfer function in the lower part of figure 16 , with a time constant set to 2 days.

## References

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[^0]:    1. 'Volatility' is in this paper equated with the variance of stock $\mathrm{p} / \mathrm{d}$-ratio.
[^1]:    4. Personal communication with a staff representative, the Oslo Stock Exchange.
[^2]:    5. Dead-zone eeffects are common in physical systems, and frequently treated in the control engineering literature.
[^3]:    6. The proposal was originally launched for discussion on the Internet economics mailing list 'Post Keynesian Thought' (PKT) in December 1998. It is archived on the PKT website, at [http://csf.colorado.edu/forums/pkt/dec98/0159.html](http://csf.colorado.edu/forums/pkt/dec98/0159.html). The ensuing exchange is easily found with this URL as a starting point
