Curso: "Continuous-time monetary macroeconomic stock-flow models, applied to financial sector -growth, debt crises and electronic money".

Profesor: Trond Andresen
Fecha: 2, 4, 6, 9 y 11 de septiembre de 2013

Horario: 7:00 – 9:00 am

Lugar: FLACSO

Literature: download this collection containing several articles.

Below I refer to the big page numbers in the upper right hand corner in the collection.
Page numbers are given like this in the text below: [12 – 14].
Note that there are overlaps in the referenced excerpts. Also that not all material mentioned in the summary below is in the supplied collection. This will be remedied by also distributing my powerpoints after each presentation. These powerpoints will as a general rule be easier to read than the supplied collection.
Session 5 is quite open as you see. I expect contributions from the audience there ... **;-)**
Finally, there are topics in this collection that I do not mention in the summary below, for instance [99 – 100].

Session 1:
The basic building block, Phillips' 1954 *first order time lag* (a first order linear differential equation), the time lag both as a *stock flow unit* (SFU), but also as a behavioral relation (can be used for both) [1 – 4], [22 – 24]. Introducing the block diagram representation [26], used in control engineering, but very convenient for economics and easier to grasp than equivalent sets of differential equations. Why macroeconomic systems should be modeled in continuous, not discrete time (the last is by a large margin the most common) [52 - 53].

Session 2:
Connecting SFU units together for an equivalent aggregate macro network [5- 8, and/or 27 – 30], in this way in fact coming up with an alternative answer to the persistent mainstream demand for "micro foundations for macro". Discussing the importance of and representation of money velocity (which simply is the inverse of the time lag in the first order SFU) [6, 94 - 95], as a necessary extra factor in addition to circulating money stock (the latter is not sufficient to explain inflation and deflation as the monetarists believe). Presenting a set of rules for an algebra of time lags in networks, and how this allows dramatic simplifications and aggregations without losing the essence. [10 – 19, but will be explained much simpler than given there].

Session 3:
Incorporating a financial sector and financial accumulation to the network [42 - 47].
High powered money (base money) versus bank-created credit money. I prove that credit money is net created within the Basel banking framework [55 - 63]. The *three races*: between money growth, debt growth and real growth. The debt-related development of fragility and crisis because the system gradually gets dominated by what I call "non-discretionary" flows [77 - 79]. Discussing different crisis mechanisms, in the spirit of Steve Keen, Hyman Minsky and Michael Hudson.

Session 4:
Discussing the macro stock flow model with finance and a government incorporated, in the light of MMT = Modern Monetary Theory (= neo-chartalism) which I support.
Introducing purely electronic money, and discussing its advantages [89, 91 - 98]. A special application is parallel (= complementary) currencies, which could be used in euro zone crisis countries [85 - 88], by near-bankrupt U.S. states, and (possibly) by dollarized countries like Ecuador. One question for Ecuador: is should the CBE issue electronic but "genuine" dollars, or should it issue electronic "tax credits" that may have an echange rate to the USD somewhat below par.
Also presenting the concept of a hierarchical “ecology” of complementary currencies in addition to the official national one: initiated as urban, regional, community currencies – all electronic but allowed to run autonomously and under locally-decided rules and frameworks.

Session 5:
Presenting and discussing policy options building on three pillars: 1) the stock-flow continuous model 2) MMT 3) money being electronic. Discussing policy options in the light of the challenges given by

- dollarization
- a foreign-trade, open economy setting,
- the strength of political and psychological opposition and possible sabotage.

This last session should use more time for discussions.

The Macroeconomy as a Network of Money-Flow Transfer Functions

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Keywords: *Macroeconomics, network, block diagram, time lag, simulation*

An introduction on A.W. Phillips' "hydraulic" macroeconomic models is given. His (and others economists') notion that a macroeconomy may reasonably be considered to have dynamics corresponding to a first order time lag transfer function, is justified in this paper by aggregation of individual micro agents. In connection with this economic application, we derive and discuss a theorem and some rules for general networks of time lagged blocks. Finally, Monte Carlo simulations of networks of micro agents are undertaken, supporting the validity of the first order time lag aggregate model.

1. Introduction

A macroeconomy evolves through time, and may be viewed as a collection of sub-entities interacting with each other. It is thus a dynamic system. Dynamics are mathematically and conceptually much more complicated than (comparative) statics. When doing dynamics, algebraic equations (corresponding to the intersecting schedules widely used in economics) are substituted with differential or (in the discrete time case, difference) equations. These equations are difficult to work with in the sense that one can hardly—as one can in a static framework—find graphic or algebraic (when possible) solutions to them without computer-implemented solution software. Furthermore, it is very difficult to gain any qualitative insights about the behaviour of a dynamic system by inspecting its differential equations. The method of representing the system graphically through block diagrams lends itself much easier to such insights. This way of representing a system may be considered an interface between the user and the differential equation based model.

In two seminal papers (Phillips 1954, 1957), A.W. Phillips (who today is known in economics almost exclusively for the *Phillips curve*) did the above[1]. He modeled the macroeconomy as a dynamic system consisting of interconnected sub-entities. In the first paper he found the algebraic solutions to the models, while in connection with the latter paper he had access to an analog computer for numerical simulation. At the time, this was pioneering work. It was a second stage after his initial and simpler physical hydraulic simulation model with vessels interconnected by tubes. Phillips had the advantage of a background as an electrical engineer, and acquainted himself with the fairly new discipline of control engineering and theory that had evolved strongly doing the second world war. He saw that it could be applied to economics.

After an initially enthusiastic reception, however, this research was subsequently disparaged by many in the economics profession as “hydraulic Keynesianism". This was possible not the least since at that time very few economists had access to, or were acquainted with, the methods and the few and expensive tools available for simulating dynamic models.

Figure 1 shows a facsimile from Phillips’ 1954 paper, the simplest model with fixed prices. The thick dotted line is added here. Above this line is the control system. It compares actual output P to desired output P_d, and the error ε is fed into a PID controller. Note that these variables are money *flows* [currency unit / time unit] since the model is in continuous time, as opposed to what is usual in dynamic economics, where the time axis is partitioned into periods, and variables therefore are money *amounts*. The PID controller decides the intervention strategy of the government. The control action is government spending, indicated by the symbol E_c (I have put a circle around the symbol to indicate that it is inserted here and not part of Phillips’ original figure). It is not the purpose of this paper to discuss the control strategy suggested by Phillips, but only his model of the demand-to-output relationship, equation (1) below. The dotted rectangle (inserted by me) indicates this part of his model, a “vessel” fed by the incoming aggregate demand flow E. The block is called L_p in Phillips’ notation, and is a first order time lag,

1. I am indebted to Professor P.N. (Raja) Junankar, who pointed out these articles to me, after observing similarities with my own work.

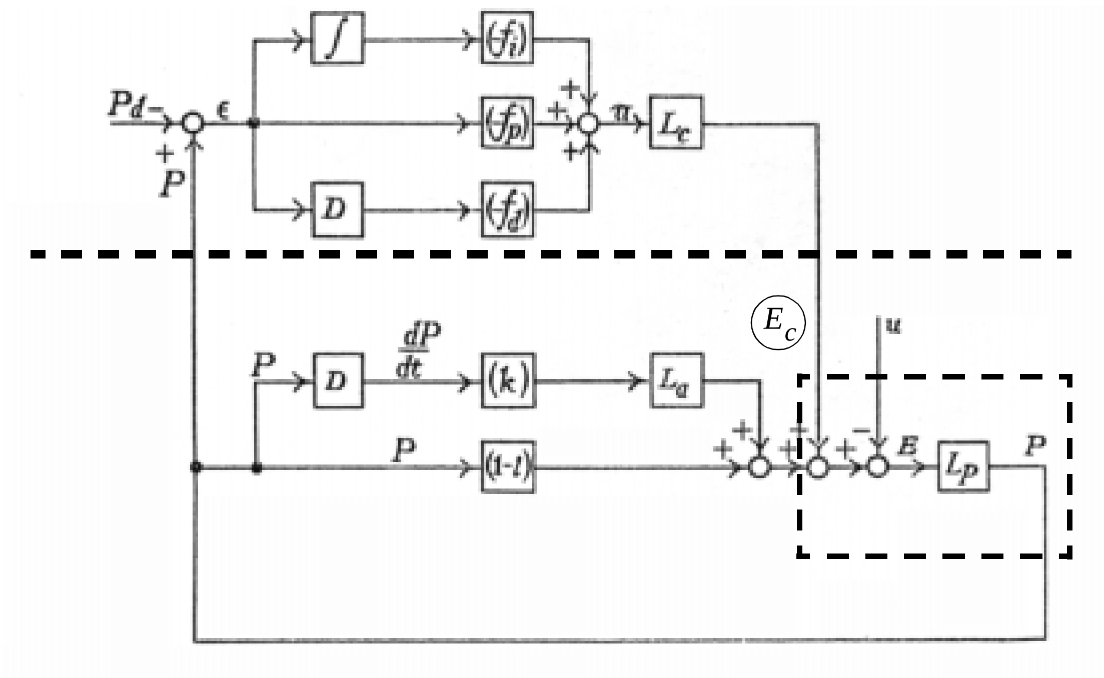

Figure 1: Facsimile of original Phillips block diagram

$$\frac{P}{E}(s) = h(s) = \frac{1}{1 + T_p s} \qquad (1)$$

The rationale for the model (1) has traditionally been explained like this: The economy needs time to adjust to a change in demand. The first order time lag is the *simplest* model for such dynamics. Thus Phillips and others (for instance Godley and Cripps, 1983) use an “Occam’s razor” type of justification for their choice of model..

The time lag model corresponds well (at least as a linearized approximation) to a vessel: A sudden increase in the incoming flow will initially increase the level of fluid (in our case: money), which leads to increased outflow in the next round. This is portrayed in figure 2:

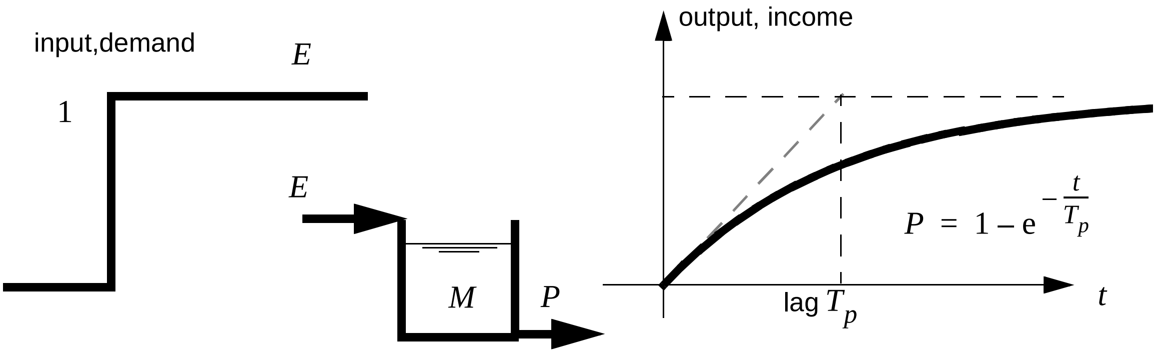

Figure 2: First-order time-lagged response

The economy is assumed to react to a jump of one unit in the demand flow with a time-dispersed exponential output response asymptotically approaching the incoming flow level.When the output flow P (theoretically) has reached that asymptotic level, we have equilibrium. T_p is the time lag describing the speed of adjustment..

This paper will strengthen the validity of the time lag model by deriving it from the fact that the economy is an aggregate of a large number of individual agents (firms, households or both together). Our approach may introductorily be explained as follows: Assume a jump in demand. The increased flow of money percolates through the interconnected network of tens of thousands of firms that constitute the demand-to-output part of the economy, and gradually (but not immediately) the effect will show up as increased income for the factors of production (i.e. output). The lag for the aggregate is a consequence of *two different factors*: The time lag of each firm on the micro level, and the degree to which the average unit of money flows to many other firms before it leaves the aggregate as income to the factors of production.

Money stock *M* must be the integrated difference between demand- and output flows. We have the stock/flow balance equation

$$\dot{M}(t) = -P(t) + E(t) \tag{2}$$

At the same time we want a step response as in figure 2, corresponding to the transfer function (1). If we choose

$$P(t) = M(t)/T_p, \tag{3}$$

this is satisfied. Equation (3) is intuitively appealing in the sense that the outgoing flow is proportional to money stock, which can be regarded (by the physical “vessel” analogy) as a “pressure” driving this flow. And the larger the time lag T_p, the less flow *P* for a given *M*, i.e. a large time lag means that money has to accumulate significantly before it leads to increased spending.

In Phillips’ model in figure 1, we note that time lags also show up in two other places in his diagram, labeled L_c and L_a: The first is a lag in the government’s (control action) spending, which may be interpreted as either sluggishness in ascertaining the current economic situation, sluggishness in implementing the intervention policy, or a combination of both. The second lag accounts for a sluggishness in the investment spending reaction of investors to the rate of change in output (*D* is a differentiation operator in the block diagram), through an “accelerator” coefficient.

So far on the Phillips model. The interested reader is referred to the original papers.

2. From micro to macro agents

We now choose the first-order time lag model as a candidate description of the behaviour of an individual "micro" agent; a household, a firm, a bank, a government. This generic economic agent concept is indicated in figure 3:

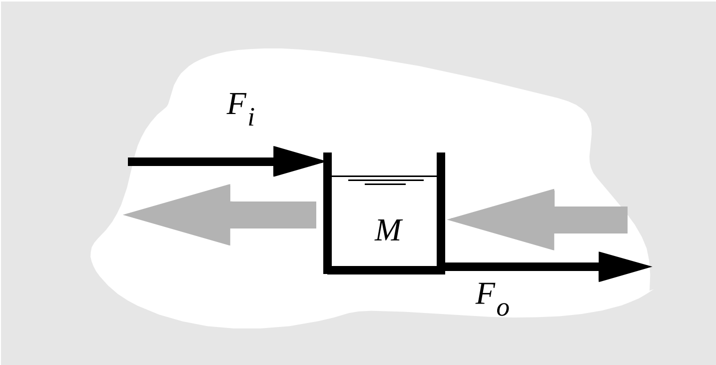

Figure 3: A generic microeconomic agent

The agent is again compared to a vessel with varying volume of fluid. Income money flow F_i and spending flow F_o are shown as black arrows. Real flows (labour, goods, services) are suggested by the thick shaded arrows in the figure. The grey shaded area surrounding the agent is simply the set of all other agents, i.e. the macroeconomic system.

Money stock M for the agent is the volume in the vessel at a given instant. M may be interpreted as the agent's necessary liquid buffer to handle discrepancies between in- and outgoing money flows. This buffer is needed since both income and spending—seen from the individual agent—will fluctuate in a more or less unpredictable manner. This uncertainty is the only rationale for an agent to hold money, as opposed to non-liquid return-yielding financial assets.

Money stock may also be interpreted as due to a neccessary "decision + action time delay" τ for the agent before received money is passed on again. We may think of this time delay in terms of a specific "packet" of money arriving at the inlet, appearing at the outlet τ time units later. For the special case with $F_o = F_i = F$ constant, M will also be constant. We have

$$M = F\tau \text{, or } \tau = M/F \tag{4}$$

From (4) follows that a *local velocity of money* is:

$$v = 1/\tau \tag{5}$$

The delay associated with flows in general (as in process plants, pipelines, etc.), will in the case of money be the time a given amount spends between arrival and departure *at a given agent.* Flows *between* agents may be reckoned as immediate. Thus money always resides at some agent.

The agent is, just as in the aggregate case, assumed to react to a monetary step function income flow with a time-dispersed exponential spending response asymptotically approaching the incoming flow level, as already depicted in figure 2.

As in the aggregate case, we have

$$F_o(t) = \frac{1}{\tau}M(t) \tag{6}$$

The larger the time lag τ, the less flow F_o for a given M, i.e. a large time lag (lower money velocity $1/\tau$) means that money has to accumulate significantly at the agent before the agent increases spending. The parameter τ is our first behavioural assumption for our generic agent. One may let τ ble influenced by other system variables, for instance let it increase sharply due to mood changes in a recession/depression (τ is a measure of liquidity preference, see corollary 1.4 further below) or decrease with increasing interest rates. Such modifications will make a model consisting of several such agents, nonlinear. But for the time being we will stick to the assumption of a constant and identical τ for all agents.

We have considered the dynamics of an agent with initial zero money stock and a constant inflow of money starting at $t = 0$. If we alternatively consider a situation with a certain initial money stock M_0 but no income, i.e. $F_i(t) = 0$, then our agent, following (1) and (6), spends her money following a decaying exponential curve, which seems quite reasonable in a situation with zero income. See figure 4:

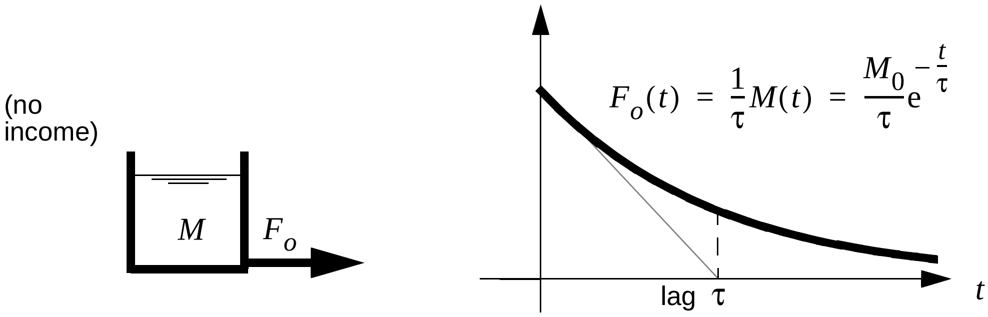

Figure 4: Time path for a micro agent with money but no income

2.1 An aggregation theorem

Let us now return to the aggregate, and let it consist of a large number of individual agents as described above. An aggregate "agent" (a sector) may for instance represent all firms, as in Phillips' macroeconomic model. The individual agents that constitute a given sector will of course have different "sizes" in the sense that money stock and flow magnitudes will vary widely between them. But we assume that (6) holds for all agents in a given aggregate, i.e. that the spending flow from an agent is proportional to the agent's money stock, by a common constant velocity factor $1/\tau$ (this assumption will be relaxed later on). Thus all agents in a given sector is represented by a transfer function of the type[1]

$$h(s) = \frac{1}{1+\tau s} \tag{7}$$

1. Here s is a *differentiation operator*, so that $y(t) = \frac{1}{1+\tau s}x(t) \Leftrightarrow (1+\tau s)y(t) = x(t)$ shall be interpreted as $\frac{dy}{dt} = \frac{1}{\tau}(-y(t)+x(t))$, i.e. a linear differential equation with input x and output y.

We furthermore assume that any (in an average sense) individual agent's outgoing money flow is divided into a share ρ (out of the sector) and $(1-\rho)$ (to other agents within the sector), where $0 < \rho \le 1$. We call ρ an *outside spending coefficient*. See figure 5. The shaded arrows indicate a network of interactions, where any individual agent in principle interacts with any other agent. Our interest is focused on two aspects, input-output characteristics of the aggregate, and the dynamics of aggregate money stock.

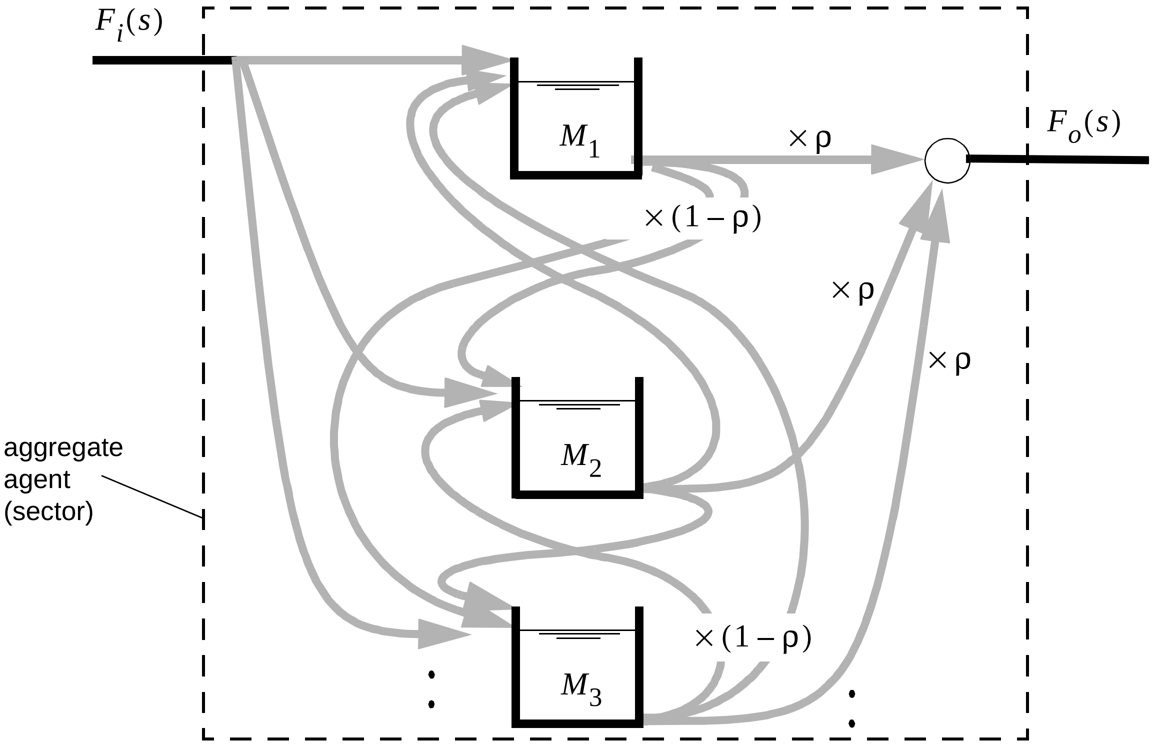

Figure 5: A flow network of "vessel" agents

Under the above assumptions the transfer function for the sector turns out to be surprisingly simple. It is given by the following "network aggregation theorem":

Theorem 1: Given a network of an infinite number of identical blocks which are first order transfer functions of the type (7), and which are interconnected by arbitrary coefficients, such that all transfer functions have identical outside spending coefficients ρ*, and such that the remaining output coefficients for each transfer function sum to* $1-\rho$*. Then the transfer function for the network, between any block input and any block output, is*

$$h_a(s) = \frac{1}{1+T_a s}\ , \text{ where } \quad T_a = \frac{\tau}{\rho} \tag{8}$$

Before proceeding with the proof, some comments to indicate that this result is intuitively satisfying. Let us first consider a type of sector where the population of agents have a low volume of monetary transactions between them, even if the number of agents may be large: A case in point is the aggregate of all households. In this case ρ is close to unity. Referring to figure 5, this means that the agents are simply laid out "in parallell", with negligible flows between them. Money arriving at a specific agent will emerge from the the agent and also the household sector, without having to "percolate" via other household agents first. People use most of their income for purchasing goods and services from firms, not paying it to other households. Thus one should expect the aggregate of households to have the same fast response as an individual agent. This also fits with (8), since $T_a = \tau$ in the limit when $\rho = 1$.

The other extreme is when the "aggregate agent" is such that agents mostly do their transactions with other agents *within* the aggregate. This case fits well with what financial sectors have developed into for the last decade. An outside agent who injected money into such an aggregate, would—if she had the means to trace that packet of money—observe that it would take a very long time before the last residue of the injected amount emerged from the aggregate. This case corresponds to ρ being close to zero. It is consistent with (8), where a small ρ means a large lag T_a, giving just the type of low-amplitude, drawn-out response that seems reasonable.

We will now prove Theorem 1.

Proof: In deriving the transfer function for the aggregate agent, we may assume that the outside incoming monetary flow arrives at one agent only, because of the symmetry between the agents, and because of the superposition principle that applies to a linear system: If the incoming flow was instead distributed between several agents, the resulting response would be the sum of responses to each component of the incoming flow, transmitted through identical transfer functions, which would then sum up to the same result we get when the incoming flow is assumed to arrive at a single agent only.

Consider the structure in figure 6.

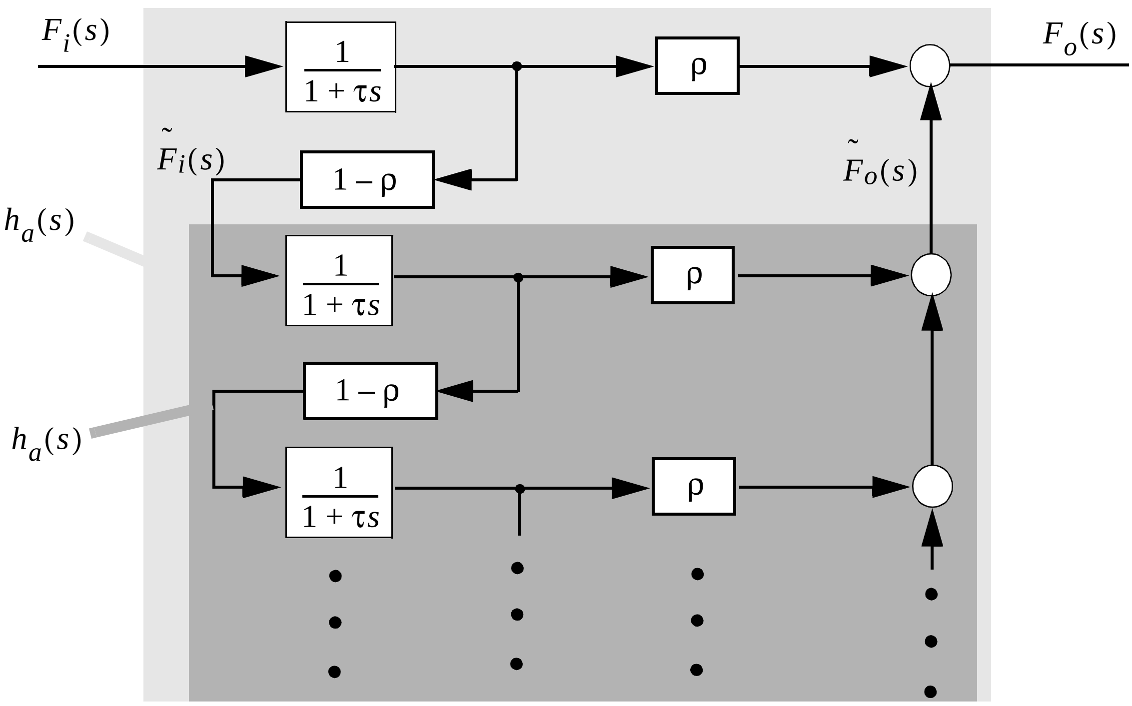

Figure 6: An input-output equivalent network that is without feedback loops

This block diagram accounts for the way an incoming monetary flow branches through the aggregate of agents. As already argued we may assume that the flow enters at one single agent, in figure 6 chosen as the uppermost. This results in a spending flow which is partioned into a share ρ leaving the aggregate, and a share $1-\rho$ to another identical agent within the aggregate. The latter flow again results in a flow that is partioned into a share ρ

leaving the aggregate, and a share $1-\rho$ to another agent within the aggregate, and so on.

Note that feedback loops are indirectly accounted for by the structure in figure 6, since the effect of any feedback loop may be equivalently represented by an infinite succession of series and parallell connections through identical transfer functions.The transfer function for the aggregate,

$$h_a(s) = F_o(s)/F_i(s) \tag{9}$$

is indicated in figure 6 by the light shaded area.

If we now remove the upper single agent from the aggregate, and assume that the remaining number of agents is so large that this does not significantly affect the dynamics of the aggregate, then $h_a(s)$ may also be found as indicated by the dark shaded area,

$$h_a(s) = \tilde{F}o(s)/\tilde{F}i(s) \tag{10}$$

Employing rules for manipulating block diagrams where blocks are in parallell and in series, we get

$$h_a(s) = \frac{\rho}{1+\tau s} + \frac{1}{1+\tau s}(1-\rho)h_a(s) \tag{11}$$

Solving for $h_a\ (s)$, we get (8). ■

(The proof could alternatively be undertaken by summing up an infinite series of transfer functions).

From theorem 1 follows a "look-inside corollary":

Corollary 1.1: Given a network as defined in theorem 1, and an output flow from the network, $F_o(t)$. Then the sum of internal flows (with)in the network is

$$F_w(t) = \frac{1-\rho}{\rho}F_o(t) = \frac{T_a-\tau}{\tau}F_o(t) \tag{12}$$

If we now allow the time lag for the network to vary, we have an "instantaneous time lag (money velocity) corollary":

Corollary 1.2: Given exogenously determined input and output flows $F_i(t)$ and $F_o(t)$. $\dot{M}(t) = -F_o(t) + F_i(t)$ *is then also exogenously determined. The instantaneous time lag of the network is*

$$T_a(t) = \frac{M(t)}{F_o(t)} \tag{13}$$

The instantaneous money velocity is $v_a = 1/T_a(t)$ (14)

From (13) and (12), and assumptions given below, we have a "flow depletion corollary":

Corollary 1.3: Given exogenously determined input and output flows $F_i(t) < F_o(t)$. *Then* $\dot{M}(t) < 0$. *Assume non-decreasing output flows, i.e.* $\dot{F}_o(t) \geq 0$ *and some lower bound* τ *on the agent time lag. Then the flows within the network will decrease relative to* $F_o(t)$,

$$\frac{d}{dt}\left(\frac{F_w(t)}{F_o(t)}\right) < 0 \tag{15}$$

Depletion of internal flows within a sector, as a consequence of the sector being forced to yield an exogenously decided output flow, is what happens in an economy with increasing debt burdens. A model of this is derived and discussed in (Andresen, 1999).

From $T_a = \tau/\rho$, and (12) and (13), we have a "liquidity preference corollary":

Corollary 1.4: Assume that liquidity preference is on the increase, which in our model is expressed by an increasing τ. *It means that, all other things being equal , both aggregate output flow* $F_o(t)$ *and the aggregate of internal flows,* $F_w(t)$, *will decrease, with*

$$F_o(t) = \rho\frac{M(t)}{\tau(t)} \quad , \textit{ and } \quad F_w(t) = (1-\rho)\frac{M(t)}{\tau(t)} \tag{16}$$

Such a mechanism is at work during serious economic crises, and contributes to a possible deflationary collapse followed by a depression (ibid.).

2.2 An algebra of time lags in a network

The reader may at this stage protest that the assumptions made until now are quite restrictive: All agents have the same first order dynamics with constant and identical time lags, all agents have the same proportion ρ of spending outside its sector. We will from now on rescind the assumptions about first order dynamics, identical time constants and identical inside/outside spending proportions. The price we have to pay is that we cannot say anything definite about the specific response of the aggregate, only about its time lag. We can (obviously) still say that the response must be positive for all *t*, regardless of differing and time-varying dynamics for all the agents in the aggregate.

We will from now on examine agent dynamics through their impulse responses. In our continuous-time economic application, an impulse or delta function $\delta(t)$ corresponds to the agent receiving one unit of money at $t = 0$. For the first order impulse response where the transfer function is given by (1), the time lag τ is also the position along the time axis of the centroid of the area under the impulse response. For an arbitrary higher order impulse response, we now define the time lag τ in the same way. If the area under the impulse response is not unity, we divide by the area,

$$\tau = \frac{\int_0^\infty th(t)dt}{\int_0^\infty h(t)dt} \quad , \text{ or } \quad \tau = \int_0^\infty th(t)dt \quad \text{if the area is unity} \tag{17}$$

If nothing else is said, we assume unit area from now on. This holds for all our economic agents, since money is not created or destroyed by any such agent. We call such an impulse response *PUA* (positive, unit area). In the *s* domain (Laplace transformed), a unit area impulse response corresponds to unity static gain of the transfer function, we have

$$h(s)\big|_{s=0} = 1 \tag{18}$$

Consider a PUA transfer function

$$h(s) = \frac{n(s)}{d(s)} = \frac{1 + b_1 s + \ldots + b_{n-1}s^{n-1}}{1 + a_1 s + \ldots + a_{n-1}s^{n-1} + a_n s^n} \tag{19}$$

The time lag τ may be found through the relation

$$\tau = \int_0^\infty t h(t) dt = \mathcal{L}[th(t)]\big|_{s=0} = -\frac{d}{ds}h(s)\bigg|_{s=0} \tag{20}$$

We write $\tau = -h'(0)$ for convenience. (20) applied to (19), gives

$$\tau = \frac{n(0)d'(0) - d(0)n'(0)}{d^2(0)} = \frac{d'(0) - n'(0)}{1} = a_1 - b_1 \tag{21}$$

For the special case of the first order transfer function, this confirms that τ is identical to the time constant, as already stated. For higher-order transfer functions we note that τ is independent of the coefficients b_i and a_i , $i > 1$.

For N PUA transfer functions of the type (19) in a series connection, the resulting transfer function will also be PUA. (20) gives

$$\begin{aligned}\tau &= -h'(0) = -(h_1 h_2 \ldots h_N)'(0) \\ &= -(h_1'(0) + h_2'(0) + \ldots + h_N'(0)) = \tau_1 + \tau_2 + \ldots + \tau_N\end{aligned} \tag{22}$$

This is reasonable, considering that τ may be seen as the lag of a flow which is transmitted through a chain of N "agents" (referring to our economic application). The impulse response of a series connection of sub-systems as in (22), corresponds to convolution of PUA impulse responses in the time domain. A PUA response is mathematically similar to a probability distribution function, which also is PUA. From probability theory we know that if we convolve N p.d.f.s, the result is the p.d.f. for the sum of the respective random variables (assuming they are independent)(see for instance Casella and Berger, 1990, p. 210). And the Central Limit Theorem (ibid., pp. 216-218) tells us that the resulting p.d.f. will tend to a normal distribution when N is large. If we return to the time domain and apply this to a system consisting of a large number of N serially connected PUA subsystems, this means that the impulse response of the system will have a shape approaching that of the normal distribution, with time lag corresponding to its mean value. To explore this, consider a transfer function

$$h_N(s) = \frac{1}{\left(1 + \frac{\tau}{N}s\right)^N} \tag{23}$$

It corresponds to a chain of N serially connected identical first order time lags. Since each lag in the chain is τ / N, rule (22) tells us that the aggregate lag will be invariant = τ.

Incidentally, we note that

$$\lim_{N \to \infty} h_N(s) = e^{-\tau s} \tag{24}$$

so that the impulse response in the limit is trivially an impulse delayed by τ. We want, however, to examine the responses for large but finite N. The impulse response corresponding to (23) is

$$h_N(t) = \frac{1}{(N-1)!}\left(\frac{N}{\tau}\right)^N t^{N-1} e^{-\frac{Nt}{\tau}} \tag{25}$$

Figure 7 shows a selection of responses, with $N = 1, \ldots, 30$ and $\tau = 1$. (The corresponding normal distribution shape is indicated with a dotted line.)

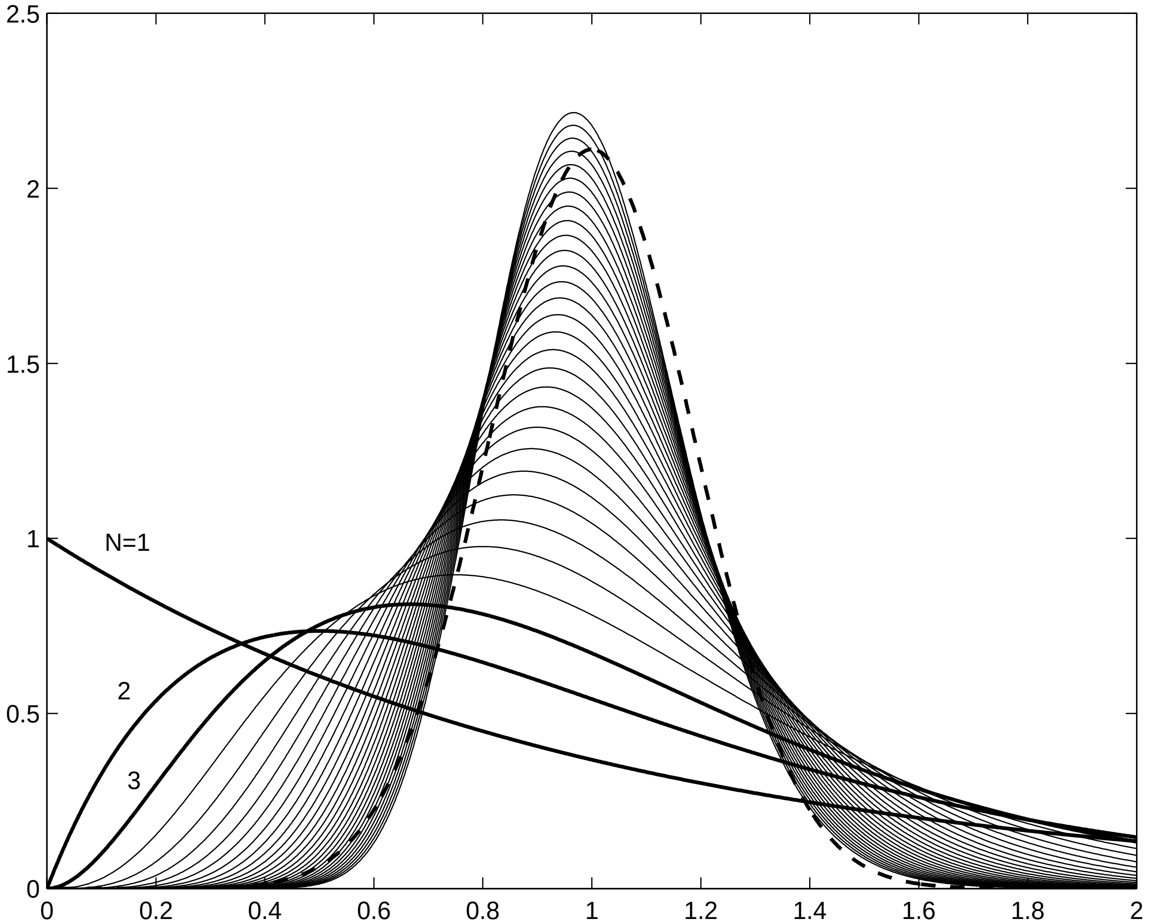

Figure 7: The impulse response of a chain of N transfer functions (23)

Note that these responses are all from transfer functions with no zeroes. We will introduce a zero and use $N = 2$ for Monte Carlo simulations of an economic network further below. It will be demonstrated that a zero and two poles are sufficient to generate a wide range of realistic responses.

Now to the case of PUA transfer functions connected in parallell:

$$h = \rho_1 h_1 + \rho_2 h_2 + \ldots + \rho_N h_N \tag{26}$$

Here we require $\rho_1 + \rho_2 + \ldots + \rho_N = 1$ so that the unit area condition is not violated. The time lag is then a weighted average,

$$\begin{aligned} \tau &= -h'(0) = -(\rho_1 h_1 + \rho_2 h_2 + \ldots + \rho_N h_N)'(0) \\ &= \rho_1 \tau_1 + \rho_2 \tau_2 + \ldots + \rho_N \tau_N \end{aligned} \tag{27}$$

The series (22) and parallel (27) rules for calculating time lags might be employed to check the time lag of a network of more arbitrary transfer functions. Assume that transfer functions of the type (19) differ from agent to agent, but with similar coefficients $\tau = a_1$ and a common outside spending coefficient ρ. Consider figure 6, but now with these more arbitrary transfer functions in the blocks. We apply rules (22) and (27), and get an equation for the time lag T_a for the aggregate by employing a similar argument as that which led to (11):

$$T_a = \tau + (\rho \cdot 0 + (1 - \rho) T_a) \tag{28}$$

This gives $T_a = \tau / \rho$, as expected.

Finally, consider a linear monovariable system on state space form with a positive—not necessarily unit area—impulse response:

$$\begin{aligned} \dot{\boldsymbol{x}} &= A\boldsymbol{x} + \boldsymbol{b}u \\ y &= \boldsymbol{c}^T \boldsymbol{x} \end{aligned} \tag{29}$$

The transfer function is

$$h(s) = \boldsymbol{c}^T (sI - A)^{-1} \boldsymbol{b} \tag{30}$$

The area under the response is

$$h(s)\big|_{s=0} = h(0) = -\boldsymbol{c}^T A^{-1} \boldsymbol{b} \tag{31}$$

We also have

$$-\frac{d}{ds} h(s)\bigg|_{s=0} = -h'(0) = -\boldsymbol{c}^T \{[-(sI - A)^{-1} I (sI - A)^{-1}]_{s=0}\} \boldsymbol{b} = \boldsymbol{c}^T A^{-2} \boldsymbol{b} \tag{32}$$

(31) and (32) in (17) then give the time lag for the system,

$$\tau = \frac{\int_0^\infty t h(t) dt}{\int_0^\infty h(t) dt} = \frac{-h'(0)}{h(0)} = \frac{\boldsymbol{c}^T A^{-2} \boldsymbol{b}}{\boldsymbol{c}^T A^{-1} \boldsymbol{b}} \tag{33}$$

3. Monte Carlo simulations

We propose that a second order transfer function with one zero and real poles is sufficient for furnishing the neccessary variability in individual agent dynamics. We now express the agent PUA transfer function in the form

$$h(s) = \frac{\alpha_1 + \alpha_1 T_z s}{\alpha_1 + \alpha_2 s + s^2} \tag{34}$$

The point of the zero $-1/T_z$ is to account for some agents spending a certain share of incoming money immediately after receipt. For a large T_z, the agent's spending reaction will start with a fairly strong initial pulse followed by a correspondingly small exponential tail. Figure 8 shows a collection of spending impulse responses for 10 agents, where all have a transfer function of the type (34). They are all PUA, and they have the same time lag, here $\tau = 1$. Each response corresponds to a specific parameter set T_z, T_{p1}, T_{p2} where $-1/T_{p1}, -1/T_{p2}$ are the poles of (34). Each set is generated by uniform probability density functions, through the following procedure: First a T_z is generated in the range $0 < T_z < 0.5\tau$ (the factor 0.5 is fairly arbitrary). According to (21) and (22), the sum of the denominator time constants must then be $\tau + T_z$. The next step is generating the pair T_{p1}, T_{p2} by a uniform p.d.f., but scaled afterwards such that this condition is satisfied.

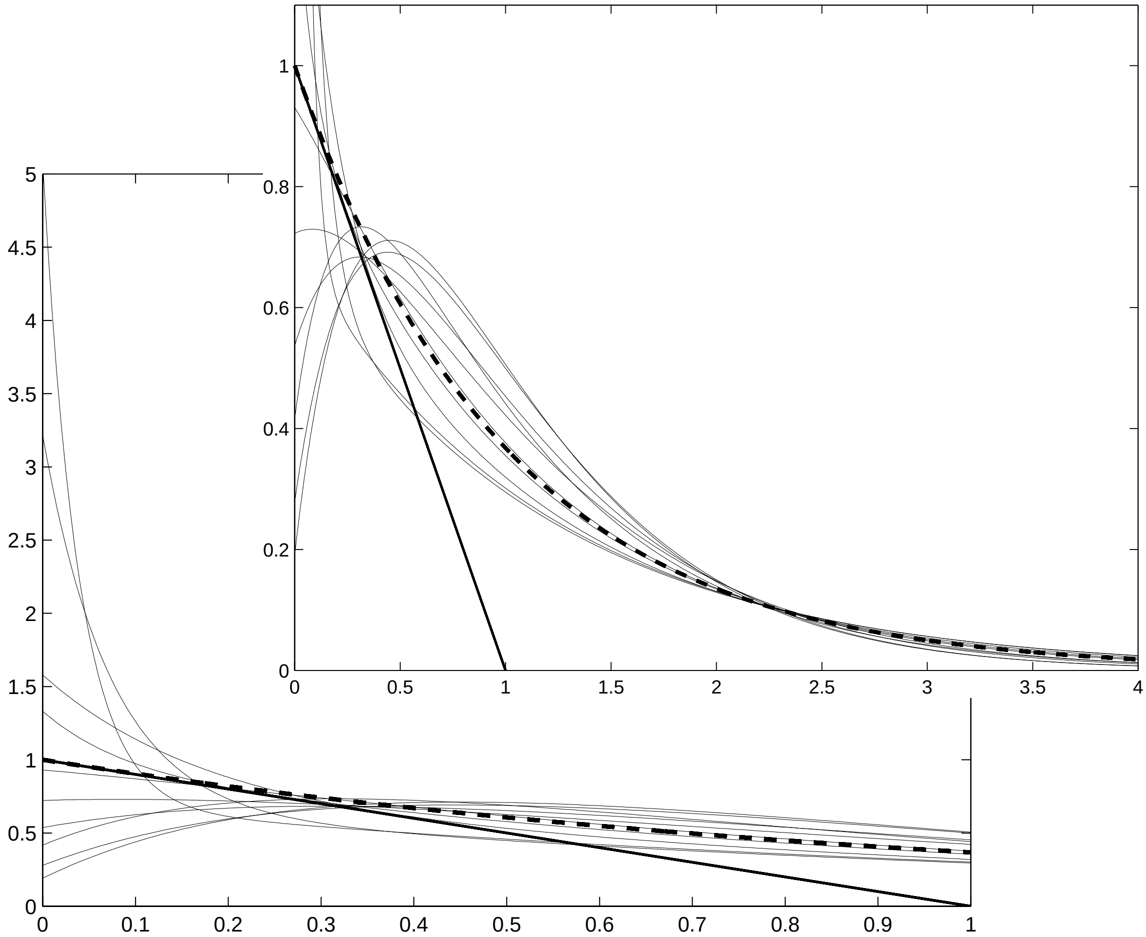

Figure 8: Ten impulse responses of (34), with a common time lag = 1

The same set of responses is shown in both windows, but with different scaling. In the lower left window we note the large initial amplitude of some responses; these are the cases with a fairly large T_z. The first-order time lag response is shown with a thick dotted line for comparison, and its tangent (and thus τ) is also indicated. We observe that the first order time lag is not a good approximation to most of the responses that are generated.

Our conjecture, however, is that if we interconnect a large number of agents with such differing responses as shown above, we will observe that *the first order time lag as an approximation improves with the number of agents*. To explore this proposition, a state space model is defined. It consists of subsystems of the type (34), interconnected such that the total system is still PUA. Let

$$\begin{aligned} \boldsymbol{x} &= A\boldsymbol{x} + \boldsymbol{b}u \\ y &= \boldsymbol{c}^T\boldsymbol{x} \end{aligned} \tag{35}$$

The matrix A is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \dots \\ -\alpha_{11} & -\alpha_{12} & r_{21}\alpha_{21} & r_{21}\alpha_{21}T_{z,2} & r_{31}\alpha_{31} & r_{31}\alpha_{31}T_{z,3} & \dots \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots \\ r_{12}\alpha_{11} & r_{12}\alpha_{11}T_{z,1} & -\alpha_{21} & -\alpha_{22} & r_{32}\alpha_{31} & r_{32}\alpha_{31}T_{z,3} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \tag{36}$$

With n subsystems we have a system dimension $2n$. Each subsystem has transfer function (34), and is realized as a controllable canonical form (see for instance Belangér, 1995, pp. 100-104). The subsystems (agents) are indexed $i = 1, \dots, n$. The definitions for α_{ij}, $T_{z,i}$ correspond to (34). Note that indices in (36) are not matrix element indices in the conventional sense, since they pertain to the subsystems. Each coefficient r_{ik} accounts for the flow from subsystem *i* to subsystem *k*. For each subsystem *i* there is an outside spending coefficient ρ_i. Then r_{ik} must satisfy

$$\sum_{k=1, k \neq i}^{n} r_{ik} = 1 - \rho_i \tag{37}$$

The column vector $\boldsymbol{b}$ has $2n$ elements and is $\boldsymbol{b} = \begin{bmatrix} 0 & b_1 & 0 & b_2 & \dots & 0 & b_n \end{bmatrix}^T$. It must satisfy

$$\sum_{i=1}^{n} b_i = 1 \tag{38}$$

Conditions (37) and (38) are neccessary to achieve unit area impulse response for the total system, i.e. such that no money is created or destroyed within it.

The row vector $\boldsymbol{c}^T$ has $2n$ elements and is

$$\boldsymbol{c}^T = \begin{bmatrix} \rho_1\alpha_{11} & \rho_1\alpha_{11}T_{z,1} & \rho_2\alpha_{21} & \rho_2\alpha_{21}T_{z,2} & \dots & \rho_n\alpha_{n1}T_{z,n} \end{bmatrix}, \text{ with all } \rho_i < 1 \tag{39}$$

Now to the procedure for assigning values to the above parameters: For each Monte Carlo run, a complete new set is generated. All probability distributions employed are uniform. The procedure is executed for each subsystem *i*: First, a lag τ_i is drawn. Based on this, the parameters α_{ij}, $T_{z,\,i}$ are generated as already described. Then an outside spending coefficient $\rho_i < 1$ is drawn. Next, coefficients r_{ik} are also drawn, but scaled afterwards such that condition (37) is satisfied. By this, we also have the next two elements in $\boldsymbol{c}^T$. After repeating this for all subsystems, all elements in $\boldsymbol{b}$ are drawn and then scaled such that (38) is satisfied.

One should expect widely differing responses, since all parameters are allowed to vary quite independently, and the distributions employed are chosen to have a fairly wide range. This makes the test of our conjecture more severe. The ranges chosen are:

$$0.1 < \tau_i < 1.9\,,\quad 0.0 < T_{z,\,i} < 0.5\tau_i\ ,\ 0.1 < \rho_i < 0.7 \tag{40}$$

The mean time lag is 1.0 and the mean outside spending coefficient is 0.4. Based on this, the approximative first order time lag response for the total system is predicted to be

$$h(t) = 0.4\mathrm{e}^{-0.4\,t} \tag{41}$$

We start simulations with a system of only 10 agents.

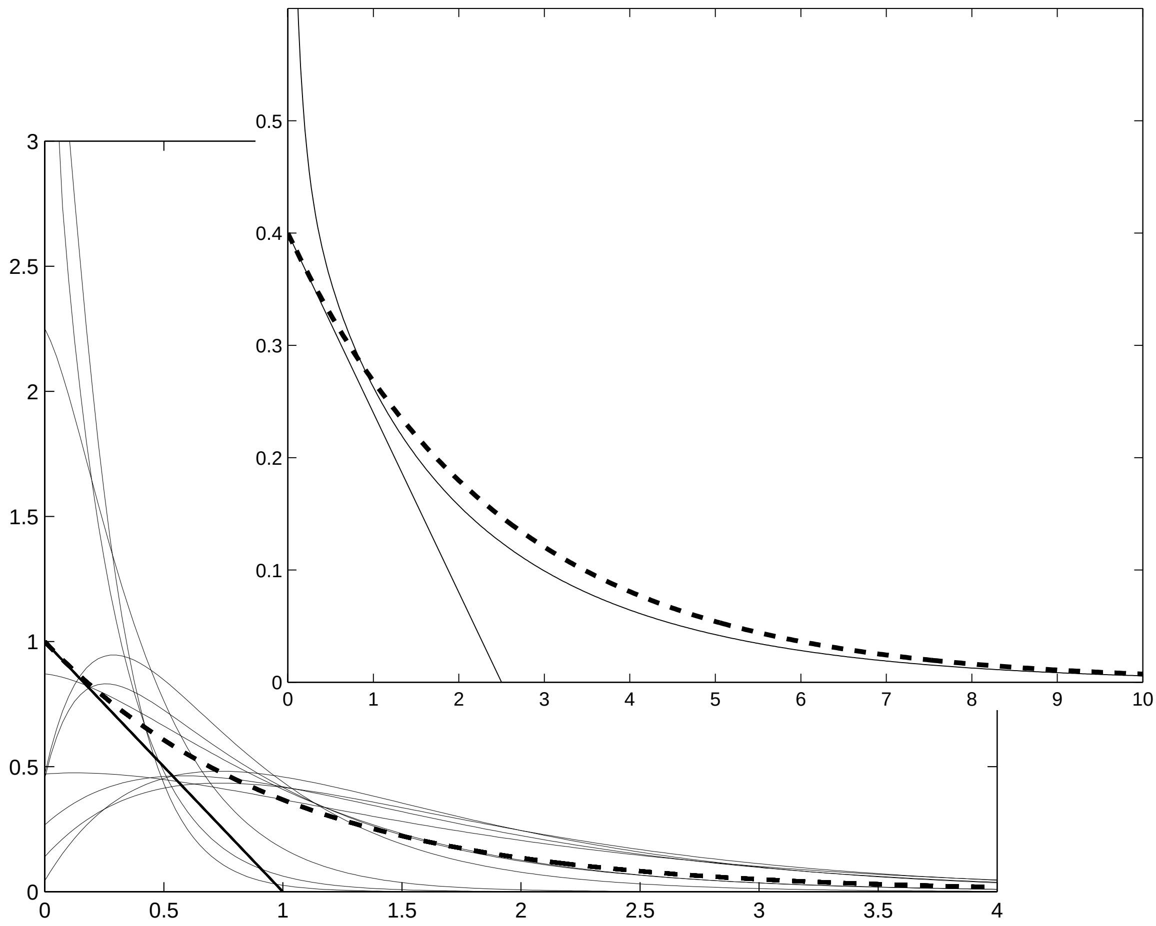

Figure 9: Ten individual responses (LL), and the corresponding aggregate response (UR)

The lower left window in figure 9 shows the responses for each individual agent. We note that the responses, as opposed to those in figure 8, are much more dispersed now since also the time lags differ between them (in the range 0.1 to 1.9). The mean first order response and its tangent is also indicated. The upper right window shows the impulse response of the "sector" consisting of these ten agents, together with the first order time lag response (41). We observe that this proposed approximation is not too bad, as predicted.

A population of as little as ten agents in a sector is quite unrealistic. We therefore do the same with a 150-agent system, which means a 300 x 300 system matrix. This is a fairly heavy computing task, so experiments are not carried out for higher numbers of agents. In figure 10 are given 10 responses for a 150-agent system, in the lower left window. For comparison, 10 responses for a 10-agent system are also given, in the upper right window.

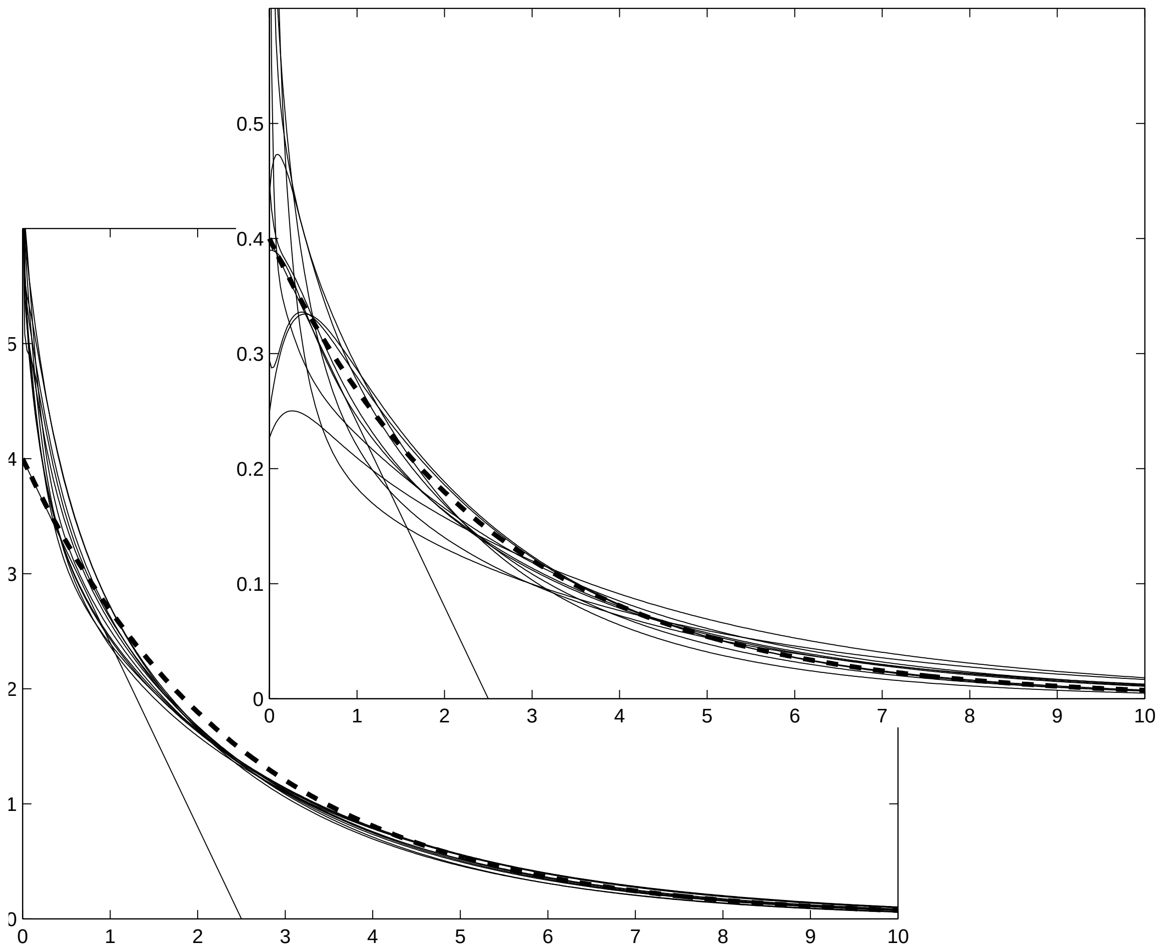

Figure 10: Aggregate responses for a 150-agent system (LL), and a 10-agent system (UR)

We observe that the first order approximation is better for the sector with the higher agent population. But the approximation cannot fully represent the strong spread in individual agent dynamics, with lags from 0.1 to1.9. This explains the steep part of the true system response at its start, which is due to outputs from the agents with the fastest dynamics).

Now to the impact of the outside spending factor, until now chosen in the range $0.1 < \rho_i < 0.7$. If instead all ρ_i were close to unity, this would mean that agents do not interact, but spend most of their money directly out of their own sector, like in the household case. In this case the sector response is simply a weighted mean of individual responses. On the other hand, if all ρ_i are $\ll 1$, this means that a unit of money in an average sense has to pass many agents before it is spent out of the sector.

We have simulated such a case, with $0.05 < \rho_i < 0.15$, i.e. a mean ρ of 0.1. Figure 11 shows 10 Monte Carlo responses for a 150 agent system.

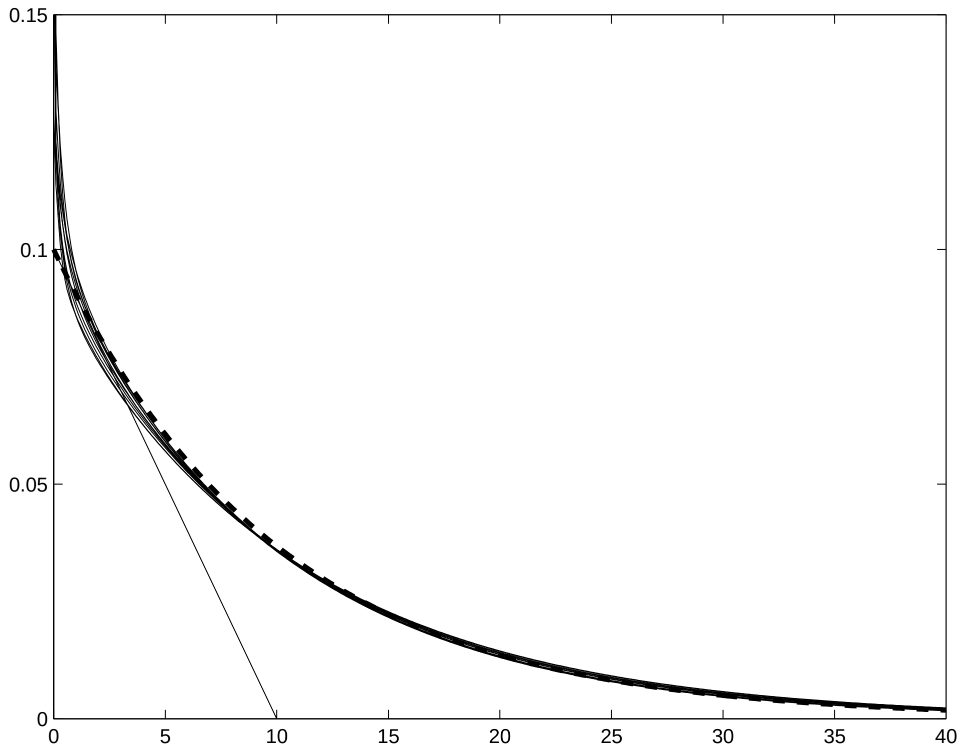

Figure 11: Aggregate responses for a 150-agent system with a low ρ

In this case the time lag is 10 for the aggregated system, in accordance with (8). Compared to figure 10, lower left window, we see that

- the responses are closer to the proposed first-order approximation.
- the initial spikes which are due to zeroes in transfer functions for individual agents in the network, still leave a mark on the aggregate response in the form of a corresponding initial peak, but a smaller and narrower one.

We will now try to explain this by exploiting an intermediate result from the proof given for Theorem 1. We again make the simplifying assumption that all agent transfer functions and all outside spending coefficients ρ are identical. But now the agent transfer functions are of the type (34), which we here write as

$$h(s) = \frac{1 + T_z s}{1 + a_1 s + a_2 s^2} = \frac{d(s)}{n(s)} \tag{42}$$

The intermediate result (11) may be generalized to

$$h_a = \frac{\rho d}{n} + \frac{d}{n}(1 - \rho)h_a , \tag{43}$$

where dependence on s is omitted for brevity. Solving for h_a and using (42), we get

$$h_a(s) = \frac{1 + \rho T_z\left(\frac{s}{\rho}\right)}{1 + (a_1 - T_z[1-\rho])\left(\frac{s}{\rho}\right) + \rho a_2\left(\frac{s}{\rho}\right)^2} \tag{44}$$

We compare this to the first-order approximation, which, following (8) and (21), is

$$h_a(s) = \frac{1}{1 + (a_1 - T_z)\left(\frac{s}{\rho}\right)} \tag{45}$$

Eq. (44) confirms that the zero will make itself felt also for the aggregate system, as already observed through the Monte Carlo runs shown in figure 11. But (44) also tells us that its relative influence on system dynamics is less when ρ is small, which is supported by a comparison of figures 10 and 11. Furthermore, when ρ is reduced, the influence on system dynamics of the second-order term in the denominator in (44) decreases in relation to the first-order term. This also supports the observation that the system response is closer to that of the first-order approximation (45), when ρ is small.

4. Conclusion

The first order time lag approximation for a sector is vindicated. It gets better for large number of agents (which is what we have in macroeconomic applications), and still better for sectors with strong interaction (for instance firms, as opposed to households).

The model may be applied to any type of sector, and any (reasonable) size sector, and these aggregates may be interconnected.

Since agents' spending preferences are constant in each simulation, behavioural assumptions have been quite restrictive. But the model can easily incorporate time-varying coefficients, or more realistically: coefficients dependent upon system states. In the last case the model will become non-linear. This is a topic for further research.

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A block diagram approach to macroeconomic dynamics, and why IS/LM is fatally flawed

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A dynamic model of an individual, and then an aggregate (sector), economic unit is developed. This model and other building blocks are employed to create macroeconomic models represented through block diagrams. A simulation tool based on block diagram representation is applied to a simple textbook economy with firms and households. Finally, a dynamic extension of the IS/LM static model is presented in block diagram form, and it is demonstrated through the dynamic extension that IS/LM's way of treating money stock is flawed to a degree that implies that IS/LM must be discarded.

JEL classification: B50, C02, C60, C65

1. Introduction

Dynamics are mathematically and conceptually much more complicated than (comparative) statics: Algebraic equations are substituted with differential equations. These equations are difficult to work with in the sense that one cannot – as in a static framework – find graphical solutions to them without computer-implemented solution algorithms.

Furthermore, it is difficult to gain insights about the properties of a system by inspecting its differential equations. It will hopefully be demonstrated that the method of representing the system graphically through block diagrams lends itself easier to such insights. Representing a system this way may be considered an interface between the user and the differential equation based model. This paper – among other things – tries to convey the usefulness of the (graphic) block diagram approach.

The structure of the paper is as follows: We start in section 2 with choosing the simplest possible dynamic model: an economic unit with the approximate dynamics of a vessel with money flowing through it. This is in the "hydraulic Keynesian" tradition of A.W Phillips (1954,1957). Vessel dynamics is compared to an alternative of "pipeline dynamics", and argued to be superior.

Some basic control systems concepts and tools for continuous-time modeling are introduced in subsection 2.1. This subsection may be skipped or fast browsed by readers with this type of background.

We then argue in subsection 2.2 that the "vessel dynamics" model is not only useful in the sense that it is the simplest one that gives meaningful behaviour (an "Occam's razor" choice which was Phillips' reason), but that it may be additionally justified when one considers that a sector is the aggregate of a large number of individual units. A theorem about this is presented and proved.

In section 3 two basic "textbook" macro models are presented and discussed using the earlier introduced concepts and tools.

Section 4 argues for a fairly dramatic claim, a claim that may be the more controversial since the argument given is quite simple. The claim is that the IS/LM model is fundamentally inconsistent and therefore should be discarded.

2. A money stock/flow model for a generic economic unit.

An economic unit in our terms may be a household, a firm, a bank, a government. The generic economic unit concept is shown in figure 1:

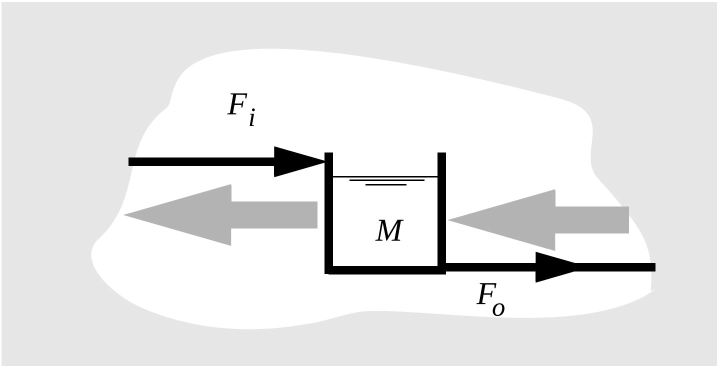

Figure 1

The unit may be compared to a "vessel" or "reservoir" with varying volume of "fluid". Money flows F_i (in) and F_o (out) are shown as black arrows. While money in the real world moves between units in discrete "packets", we will consider money flows to be continuous. This is reasonable for the time scale (weeks, months, years) of the dynamics that is to be considered. Real flows (labour, goods, services) are suggested by the thick shaded arrows in the figure. The grey shaded area surrounding the unit is simply the aggregate of all other units, i.e. the macroeconomic system.

Money stock M for the unit is the volume in the vessel at a given instant. Its size depends on the unit's precautionary, speculative and transaction motives.

Money stock may also be interpreted as due to a neccessary *decision+action time delay* τ for the unit before received cash is passed on again.

For the special case with $F_o = F_i = F$ constant, M will also be constant. We may then think of the time delay in terms of a specific "particle" of money arriving at the inlet, appearing at the outlet τ time units later. We have

$$M = F\tau \text{, or } \tau = M/F \tag{1}$$

From (1) follows that a *local velocity of money* is:

$$v = 1/\tau \tag{2}$$

The delay associated with flows in general (as in process plants, pipelines, etc.), will in the case of money be the time a given amount spends between arrival and departure *at a given unit*. Flows *between* units may be reckoned as immediate. Thus money always resides at some unit.

We now introduce the unit step function $\mu_1(t)$ and the corresponding step response *k(t)*. The step function simply means that at time $t = 0$, an incoming flow of money with amplitude = 1[currency unit/time unit] begins, and the flow F_o resulting from this specific input, is the step response. For the unit we could conjecture that the money flow is delayed exactly τ time units, resulting in the trivial step response shown to the right in figure 2 (assuming that the unit starts out with zero money stock):

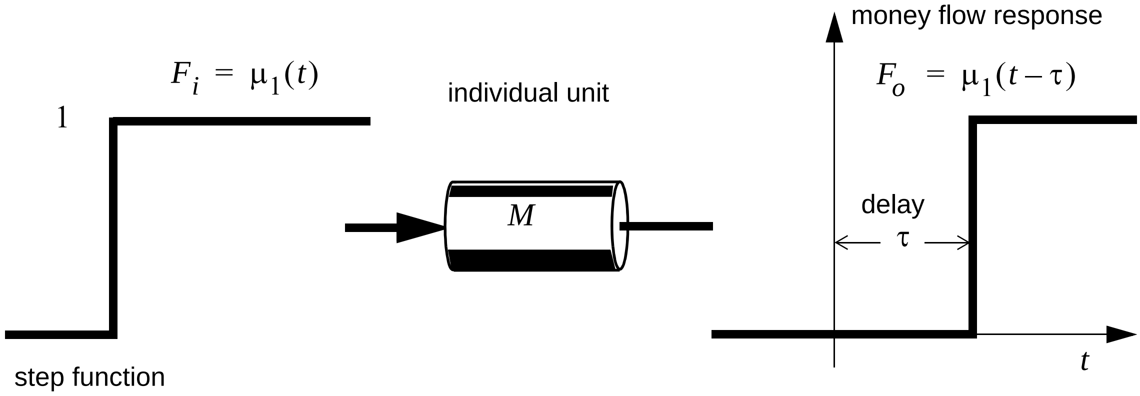

Figure 2

Such a response would have occured if the unit had been comparable to a "pipeline", as suggested in the figure. The vessel analogy, however, is obviously more realistic, and its response is shown in figure 3. Fluid has to rise in the vessel to build up the neccessary "pressure" before an outflow starts. More specifically, we assume the following dynamics: The unit reacts to a monetary step function type flow with a time-dispersed exponential spending response asymptotically approaching the incoming flow level.

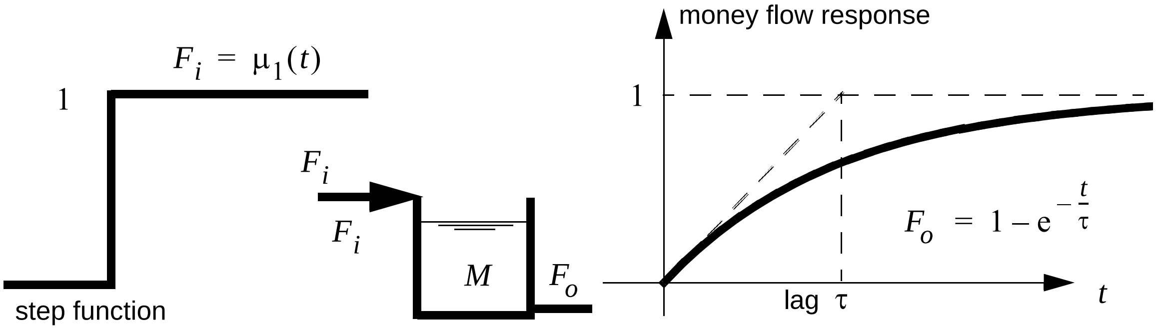

Figure 3

The term τ is now not a time delay, but what in systems theory lingo is called a time *lag*. Geometrically, it corresponds to the position along the time axis of the intersection between the tangent at $t = 0$, and the horizontal asymptote. The unit react to a sudden incoming money flow by gradually increasing its spending, and the parameter describing the speed of adjustment is τ. When spending flow F_o (theoretically) has reached the asymptotic level, we have equilibrium. The money stock of our unit (the "volume of the vessel" in figure 1) must be the integrated difference between income- and spending flows. We then have the differential equation

$$\dot{M}(t) = -F_o(t) + F_i(t) \tag{3}$$

At the same time we demand that the step response F_o *(t)* shall be as in figure 3:

$$F_o(t) = 1 - e^{-\frac{t}{\tau}} \tag{4}$$

If we choose

$$F_o(t) = \frac{1}{\tau}M(t) \quad , \tag{5}$$

it may be shown that both (3) and (4) are satisfied. We now observe that as long as we confine ourselves to $F_o(t)$, (5) corresponds to definitions (1) and (2) for respectively time delay and money velocity. These definitions were based on the the very unrealistic assumptions of constant and equal in- and outflows, and a pipeline model. These assumptions may now be rescinded. The absence of $F_i(t)$ in (5) is reasonable, since the unit exercises control only over $F_o(t)$.

Equation (5) is intuitively satisfying in the sense that the outgoing flow is proportional to money stock, which can be regarded by physical analogy as a "pressure" driving this flow (pressure is proportional to fluid level in a vessel, which again is proportional to fluid volume when the vessel is cylindrical). The larger the time lag τ, the less flow F_o for a given *M*, i.e. a large time lag (lower velocity) means that money has to accumulate significantly at the unit before the unit increases spending. The parameter τ is the first behavioral assumption for our generic unit. One may let τ be influenced by other system variables, for instance let it increase sharply in a recession/depression (increased liquidity preference) or decrease with increasing interest rates. Such modifications will make a model consisting of such units nonlinear. But in this paper we confine ourselves to the simple assumption of constant τ.

2.1 The Laplace transformation and block diagrams

(This subsection may be skipped or browsed by readers familiar with control systems literature and concepts.)

Finding time responses for systems of the type introduced above, requires solution of linear differential equations. A tool that makes this task easier, both in the stage where the problem is to construct and understand a model, and in the subsequent solution (or numerical simulation) stage, is the *block diagram*. We will later employ this tool extensively. Block diagrams are based on the Laplace transformation, which is described in most undergraduate mathematical textbooks. The main advantage of the Laplace transformation is that differential equations are substituted with algebraic equations. We will develop the topic through the example given by eqs. (3)-(5). Substituting (5) in (3), we get:

$$\dot{M}(t) = -\frac{1}{\tau}M(t) + F_i(t) \tag{6}$$

Laplace transforming both sides leads to

$$sM(s) - M_0 = -\frac{1}{\tau}M(s) + F_i(s) \tag{7}$$

where *s* is the Laplace transformation[1] free variable. M_0 is the initial value for $t = 0$.

This equation may be solved for $M(s)$,

$$M(s) = \frac{\tau}{1+\tau s}F_i(s) + \frac{\tau}{1+\tau s}M_0 = h_{mi}(s)F_i(s) + \frac{\tau}{1+\tau s}M_0 \quad (8)$$

Here $h_{mi}(s) = \frac{\tau}{1+\tau s}$ (9)

is the *transfer function* from $F_i(s)$ to $M(s)$, enabling us to find the response $M(t)$ when $F_i(t)$ (i.e. also $F_i(s)$) is given.

Consider the case in figure 3; which has initial zero money stock, $M_0 = 0$, and a step input flow $\mu_1(t)$, which has the Laplace transform $1/s$. Then (8) gives

$$M(s) = h_{mi}(s)\frac{1}{s} = \frac{\tau}{(1+\tau s)s} \quad (10)$$

which has the inverse transform

$$M(t) = \tau\left(1 - e^{-\frac{t}{\tau}}\right) \quad (11)$$

This is the step function response for the money stock. Employing (5), we get the spending flow step response $F_o(t)$ already given in (4):

$$F_o(t) = 1 - e^{-\frac{t}{\tau}} \quad (12)$$

The transfer function from $F_i(s)$ to $F_o(s)$ also follows from (5), giving

$$h_{oi}(s) = h_{mi}(s)\frac{1}{\tau} = \frac{1}{1+\tau s} \quad (13)$$

So far on the dynamics of a unit with initial zero money stock and a constant inflow of money starting at $t = 0$. If we alternatively consider an unit with a certain initial money stock M_0 but no income, i.e. $F_i(t) = 0$, then we may also find the time path of $F_o(t)$. From (8) we now get

$$M(s) = \frac{\tau}{1+\tau s}M_0 \quad (14)$$

which inverse transformed is $M(t) = M_0 e^{-\frac{t}{\tau}}$, leading to

$$F_o(t) = \frac{1}{\tau}M(t) = \frac{M_0}{\tau}e^{-\frac{t}{\tau}} \quad (15)$$

We conclude that our unit spends its money following a decaying exponential curve, which seems reasonable in a situation with zero income. See figure 4:

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1. Both functions of time and Laplace transforms are written with the same symbol. The context, or explicitly written dependence on *t* or *s* will suffice to distinguish between them. Note also that *s* is here not the savings coefficient. We avoid confusion in this paper by using the propensity to consume instead, $c = 1 - s$.

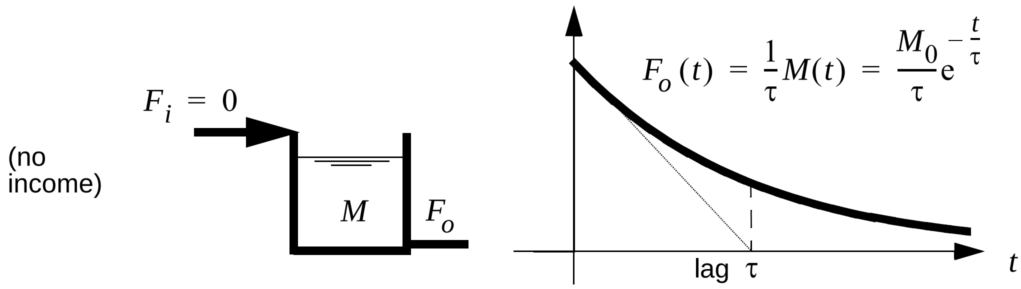

Figure 4

We may now introduce the block diagram, which embodies the same information as that represented through differential equations, but which is better for understanding a system. This is the rationale for modeling and simulation packages such as Simulink (Mathworks, 2007) being based on block diagram description.

Consider figure 5:

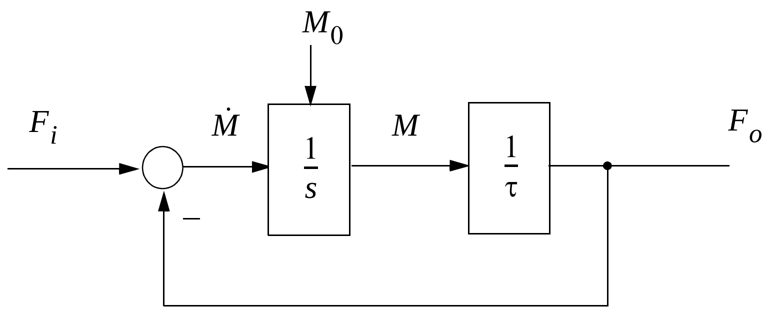

Figure 5

The rules for drawing and interpreting this diagram are as follows:

1. The variable exiting a rectangular block is the product of the variable entering the block and the expression within the block. Thus we have (5): $F_o = M/\tau$.
2. The small dot to the right signifies a *branching point*. This means that the variable F_o is both an output of the system, and also fed back to the system's input side. In this case we have a negative feedback.
3. The circle to the left is a *summation point*. The arrow leaving the circle is the sum of arrows entering the circle. An arrowhead with a minus sign associated with it, means that the variable corresponding to this arrow is to be subtracted in the summation. Thus we have (4); $\dot{M} = -F_o + F_i$.
4. A block of the type $1/s$ signifies (in accordance with rule 1) that the variable exiting this block is $1/s$ times the variable entering it. But dividing by *s* in Laplace symbolism signifies integration: $M = \int \dot{M}dt$. The block of the type $1/s$ is accordingly called an *integrator*.
5. The vertical arrow on top of the integrator specifies the initial value of the output variable from the integrator. This arrow is often rescinded for convenience, or if the corresponding initial value is zero.

Note that we have avoided signifying whether F_i, M, F_o in the block diagram depend on the Laplace variable *s*, or time *t*. The reason for this is that the block diagram may represent *both* the Laplace-transformed case and the time domain case. We just have to keep in mind that the integrator in the Laplace-transformed interpretation of the block diagram means multiplying with $1/s$, as opposed to when we want the diagram to represent relations between time-varying variables.Then the block $1/s$ is an *integration operator* - it stenographically signifies the relationship $M = \int \dot{M} dt$. For more on operator interpretation of s, see for instance Rowell (1997, 207 - 211).

The block diagram in figure 5 corresponds exactly to the model given by equations (3) and (5). By inspecting the figure, we see how the spending flow F_o is caused by “pressure” from money stock M, while F_o at the same time feeds back negatively and depletes the same money stock.

The block diagram in figure (5) is called an *elementary* block diagram, because it contains only “simple elements” like integrators (one in this case) and constants ($1/\tau$ in this case). Such a block diagram may be changed (*reduced*) into an equivalent (in an input/output sense) diagram, where a simpler structure is achieved at the cost of more complex expressions in the blocks that remain after the procedure. For our example, the reduced block diagram turns out to be as shown in figure 6. This diagram, reduced to only one block, simply results in the transfer function $h_{oi}(s)$, see (13).

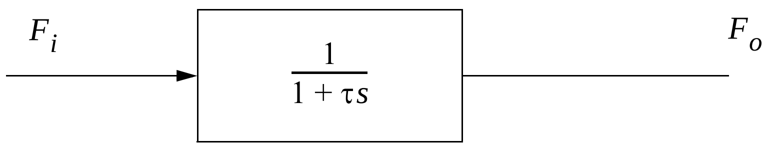

Figure 6

Manipulating block diagrams into different but equivalent diagrams will be done later in examples.

2.2 An aggregate unit

We now will consider a generic aggregate unit (a sector), which consists of a large number of individual units. Such an aggregate unit may for instance represent all households, or consumption goods firms, or all firms, or all banks, etc.

Let us confine the discussion to units within a given sector. Individual units there will of course have different “sizes” in the sense that money stock and flow magnitudes will vary widely. But we assume that (5) holds for all units in a given aggregate, i.e. that the money stock of a specific unit is proportional to the spending flow from the unit, by a common factor τ. Thus all units in a given sector may be represented by the transfer function (13).

We furthermore assume that any (in an average sense) individual unit’s outgoing money flow is divided into fractions ρ (out of the sector) and $(1 - \rho)$ (to other units within the sector), where $0 < \rho < 1$. This is illustrated in figure 7:

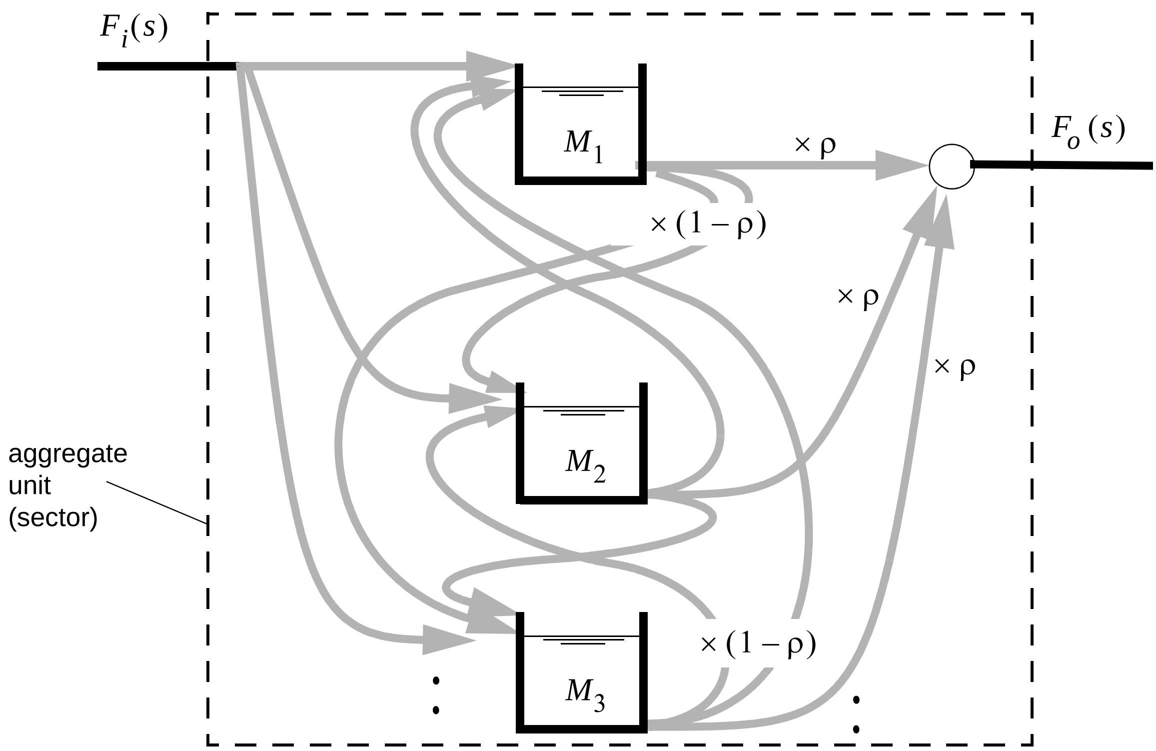

Figure 7

Note that this is a “physical” flow chart, not to be confused with the mathematical block diagram introduced earlier. The shaded arrows indicate a network of interactions, where any individual unit in principle interacts with any other unit. Our interest is still focused on two aspects, input-output characteristics of the aggregate unit, and the dynamics of aggregate money stock. The surprisingly simple result is that - under the above assumptions - the transfer function for the aggregate unit turns out to be

$$h_a(s) = \frac{1}{1+T_a s}, \text{ where } T_a = \frac{\tau}{\rho} \geq \tau \tag{16}$$

Before proceeding with the proof, some comments to indicate that this result is intuitively satisfying. Let us first consider a type of sector where the population of units have a low volume of monetary transactions between them, even if the number of units may be large: A case in point is the aggregate of all households. In this case ρ is close to unity. Referring to figure 7, this means that the units are simply laid out “in parallell”, with negligible flows between them. Money arriving at a specific unit will emerge from the the unit and also the aggregate, without having to “percolate” via other household units first. Thus one should expect the aggregate to have the same fast response as an individual unit. This also fits with (16), since $T_a = \tau$ in the limit when $\rho = 1$.

For the firm sector, we will have $\rho < 1$, since each firm will direct a significant part of its money outflow to other firms, not out of the sector.

The extreme case $\rho \ll 1$ is when the “aggregate unit” is such that units mostly do their transactions with other units *within* the aggregate. This case fits well with what financial sectors have developed into for the last decades. An outside unit who injected money into such an aggregate, would – if she had the means to “trace” that packet of money – observe that it would take a very long time before the last residue of the injected amount emerged from the aggregate. It is consistent with (16), where a small ρ means a large lag T_a, giving just the type of low-amplitude, drawn-out response that seems reasonable.

We will now prove (16).

Proof: In deriving the transfer function for the aggregate unit, we may assume that the outside incoming monetary flow arrives at one unit only, because of the symmetry between the units, and because of the superposition principle that applies to a linear system: If the incoming flow was instead distributed between several units, the resulting response would be the sum of responses to each component of the incoming flow, transmitted through identical transfer functions, which would then sum up to the result we get when the incoming flow is considered to arrive at a single unit only.

Consider the structure in figure 8. It is a block diagram with transfer functions. This block diagram accounts for the way an incoming monetary flow branches through the aggregate of units. As already argued we may assume that the flow enters at one single unit, the uppermost in figure 8. This results in a spending flow which, according to figure 7, is partioned into a share ρ leaving the aggregate, and a share $1-\rho$ to another unit within the aggregate. This share again results in a flow that is partioned into a share ρ leaving the aggregate, and a share $1-\rho$ to another unit within the aggregate, and so on. The transfer function

$$h_a(s) = F_o(s)/F_i(s) \tag{17}$$

is indicated in the figure by the light shaded area.

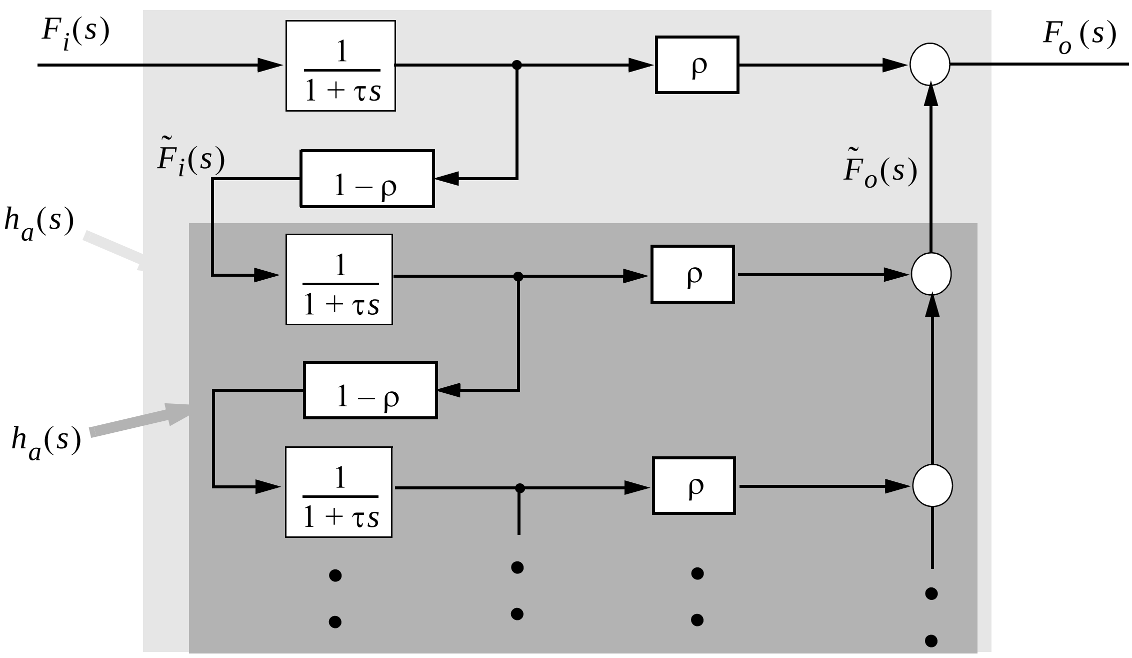

Figure 8

If we now extract the upper single unit from the aggregate, and assume that the remaining number of units is so large that this does not significantly affect the transfer function of the aggregate, then $h_a(s)$ will also be found as indicated by the dark shaded area,

$$h_a(s) = \tilde{F}_o(s)/\tilde{F}_i(s) \tag{18}$$

Employing rules for manipulating block diagrams where blocks are in parallell and in series, we get

$$h_a(s) = \frac{\rho}{1+\tau s} + \frac{1}{1+\tau s}(1-\rho)h_a(s) \tag{19}$$

Solving for $h_a(s)$, we get (16). This completes the proof.

A more comprehensive treatment of this theorem and its ramifications, is given in (Andresen, 1998).

Note that this aggregation theorem makes a stronger case for the choice of a first order time lag ("vessel") model of a macroeconomy. Phillips (1954, pp. 291 - 292) chooses this model because it is the simplest one among many that has the property of a gradual response to a sudden change in the input. A similar model and reasoning is found in Godley and Cripps (1983).

Phillips (1957) discusses whether his first order time lag model from 1954 is too simple. But the above theorem strengthens the case for the 1954 model, since it is derived from the fact that an economic aggegate is a network of interacting units. Monte Carlo simulations of networks with up to 150 interacting units with randomly selected individual time lags (around a mean), and with randomly selected coefficients for flows between them, is done in Andresen (1998). These confirm that the time lag representation is a a fair approximation for the aggregate, even when the variance around the mean for generated parameters is chosen quite large, for instance unit time lags that may vary by a factor of ten.

3. A “textbook economy” with firms and households

We will consider an economy with households, firms, no government and no financial sector. Consider the diagram in figure 9: This a “physical flow chart” representation of this economy. Further below we will introduce the mathematical block diagram of the same system. As is clear from figure 9, we assume that there are no external sinks or sources of money. This assumption will be rescinded later on, among other things to discuss the multiplier.

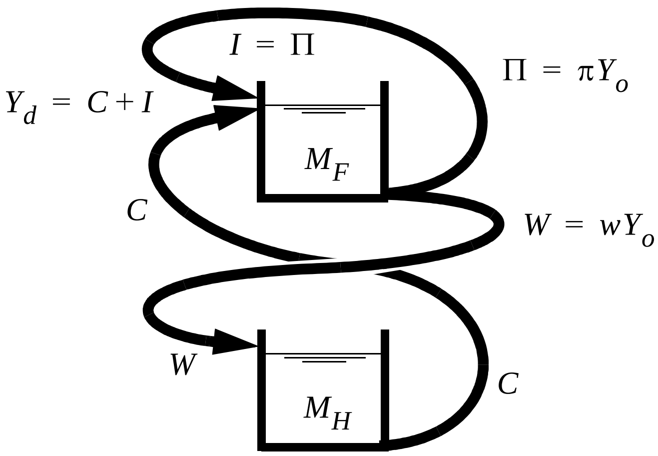

Figure 9

In the figure we have these flows:

Y_d	=	aggregate demand [currency unit / time unit]
Y_o	=	aggregate output [c.u./ t.u.]
Π	=	profit = I [c.u./ t.u.], i.e. all profits are invested, and there is no external source of investment at this stage.
W	=	wages = C [c.u./ t.u.], i.e. all wages are consumed, and there is no household savings sink (or borrowing source) at this stage.

Furthermore, w and $\pi = 1 - w$ are wages and profit share of output, respectively. They are considered constant in this model. We also have money stock in the two sectors M_F and M_H.

The mathematical block diagram of this system is shown in figure 10:

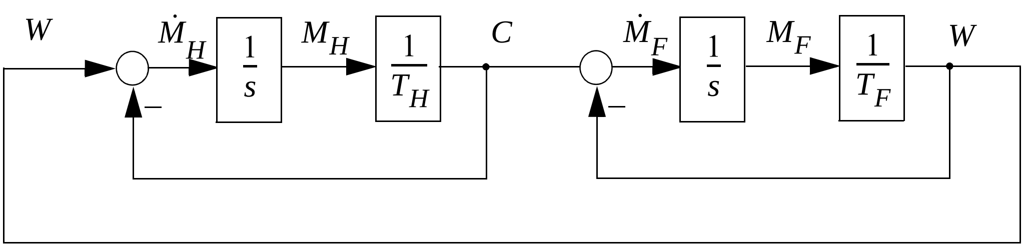

Figure 10

If we reduce the two inner loop subdiagrams, we get figure 11:

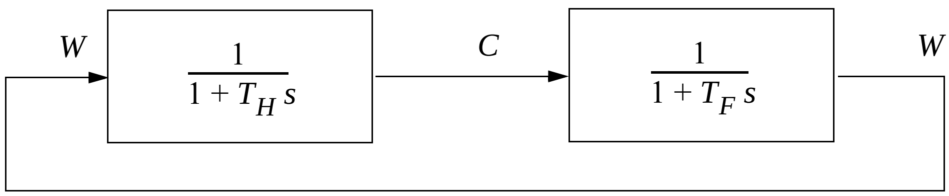

Figure 11

There is one important loop lacking in this block diagram, the profit = investment loop depicted in figure 9.We should, however, note that figure 11 is entirely correct in the sense that in a system defined as consisting of firms and households, the input to the firm sector is consumption only, and the output is wages only. Investment is a flow that is *internal* to the aggregate of firms as a whole. So how do we introduce profits, investment (and aggregate demand/output) into the block diagram representation? We do this by demanding that the two firm sector block diagrams shown in figure 12 are equivalent in an input-output sense:

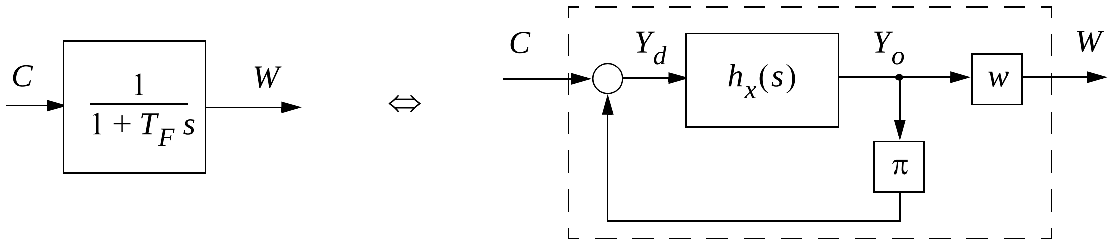

Figure 12

This gives an equation to find the unknown transfer function $h_x(s)$:

$$\frac{W}{C} = \frac{1}{1+T_F s} = \frac{h_x w}{1-\pi h_x} \tag{20}$$

Solving for h_x gives

$$h_x(s) = \frac{1}{1+wT_F s} \tag{21}$$

We observe that "extracting" the profit/investment loop leads to a reduced time lag for the modified firm sector. Figure 11 may now be transformed into the equivalent block diagram shown in figure 13:

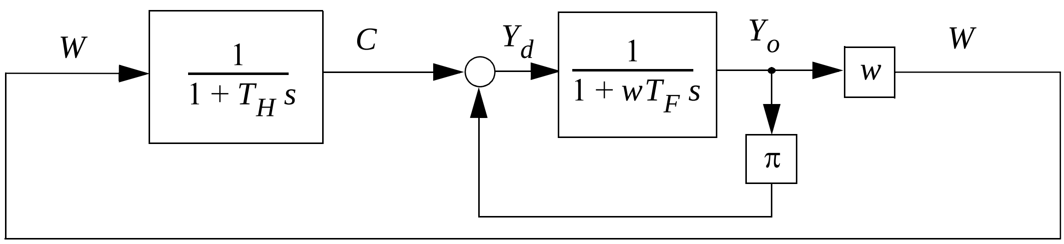

Figure 13

We may also expand figure 13 into an elementary block diagram corresponding to figure 10. The result is given in figure 14. (Note that we have here also substituted $\pi = 1 - w$.)

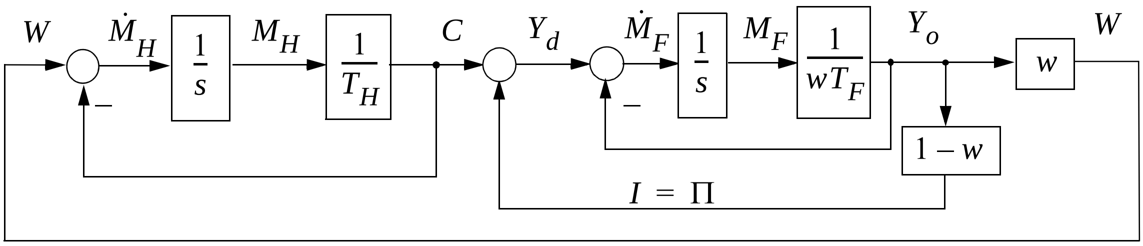

Figure 14

We have two integrators in this system. In other words, we have a system with two states; household and firm money stock. This system is autonomous (i.e. no exogenous inputs), and its time path is therefore decided solely by the initial distribution of the money stock between the two sectors. We will now use this example to illustrate the use of a modern simulation package, Simulink – and to find the equilibrium state of this system. The response of the system is shown in figure 15. Initial values are assumed to be $M_{H0} = 1400$ and $M_{F0} = 800$. System parameters are time lags $T_H = 2$ and $T_F = 20$ (weeks), and wages share of output is $w = 0.7$.

A Simulink block diagram corresponding to the one in figure 14 is shown in figure 15. This setup gives the responses shown in figure 16. (Note that syntax is somewhat different: Summation points are symbolized with rectangles with plus and minus signs, as opposed to circles used in the diagrams elsewhere in this paper.) We note how supply adjusts to demand in equilibrium. The plots also indicate that in equilibrium, money stocks are proportional to the respective time lags in the two sectors. This is easily seen by considering figure 10: in equilibrium we must have $C = W$. Since $W = M_F/T_F$ and $C = M_H/T_H$, this relationship follows.

The point of this paper, however, is to focus not on equilibrium, but dynamics. In this simple case we can find the algebraic solution for the system time path, which is

$$\begin{aligned} M_F(t) &= M_{F0}e^{-\alpha t} + \frac{M}{\alpha T_H}(1 - e^{-\alpha t}) \\ M_H(t) &= M_{H0}e^{-\alpha t} + \frac{M}{\alpha T_F}(1 - e^{-\alpha t}) \end{aligned} \quad , \text{ where } \alpha = \frac{T_F + T_H}{T_F T_H} \qquad (22)$$

Note that total money stock, M, is invariant, since there are no sources or sinks of money in this model.

The system is linear and therefore amenable to algebraic solution. In a more realistic model with non-linearities, algebraic solutions are very difficult to find, if they exist at all. In such cases, numerical simulation packages like Simulink are very useful.

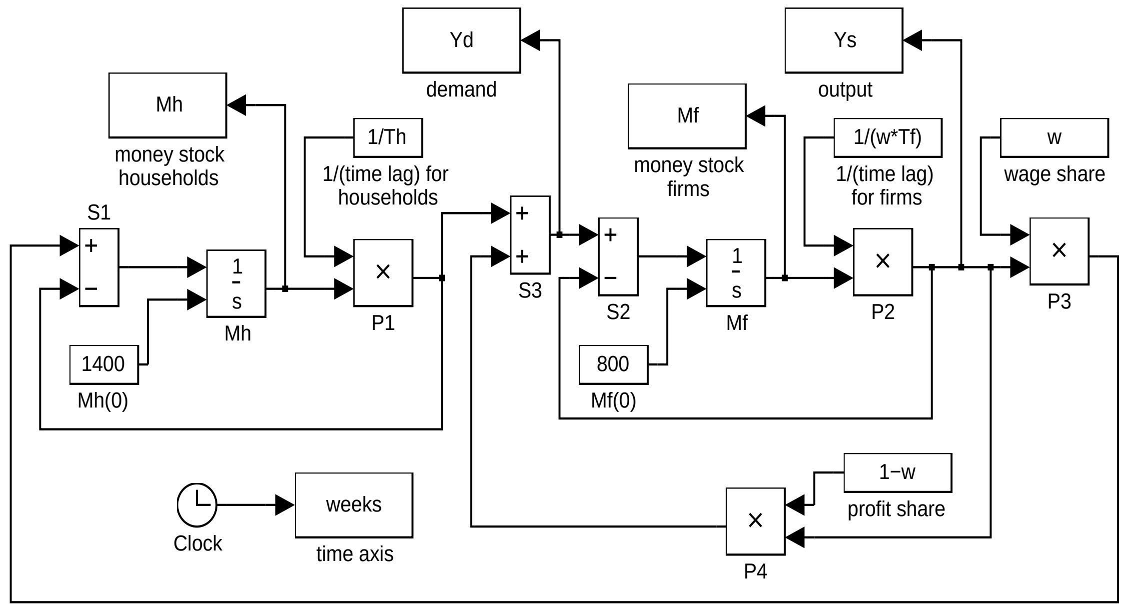

Figure 15

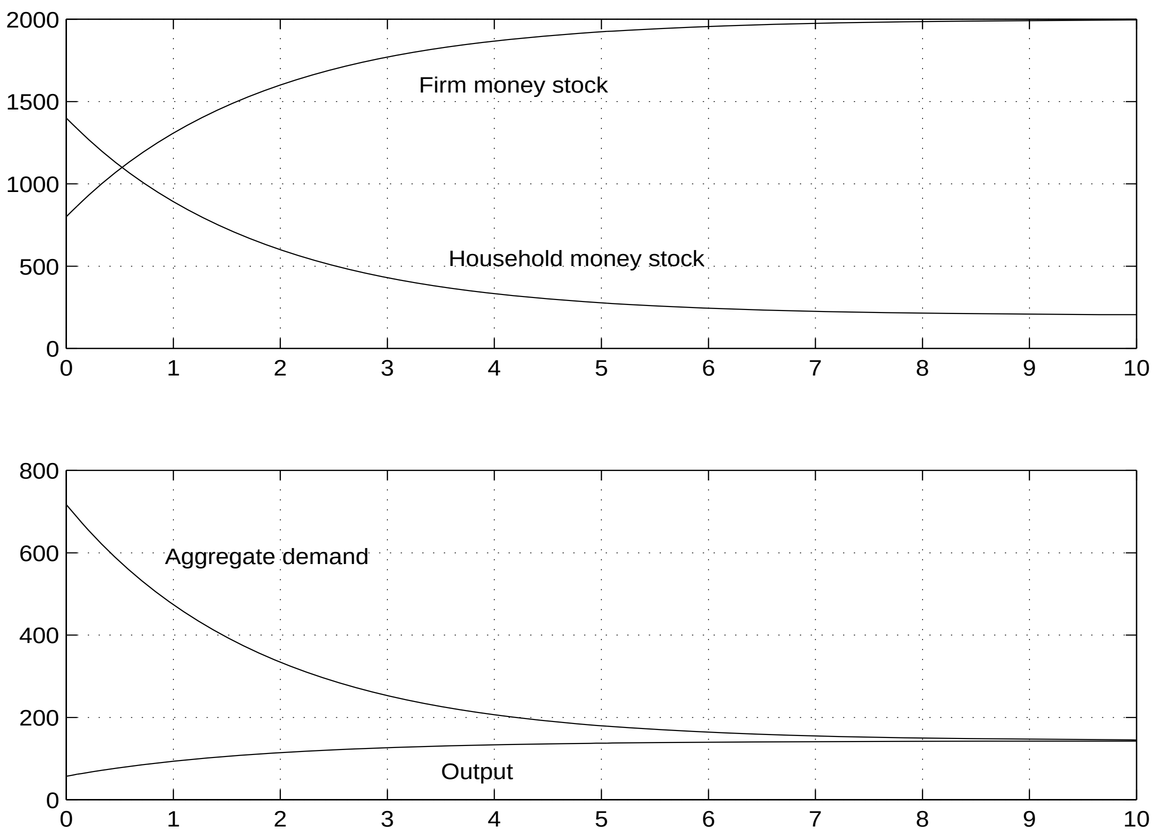

Figure 16

We will now introduce exogenous inputs to discuss the phenomenon of the multiplier. We assume the usual textbook model where all profits are paid to households together with wages, and where households consume a share *c* of their income and save the rest. See figure 17.

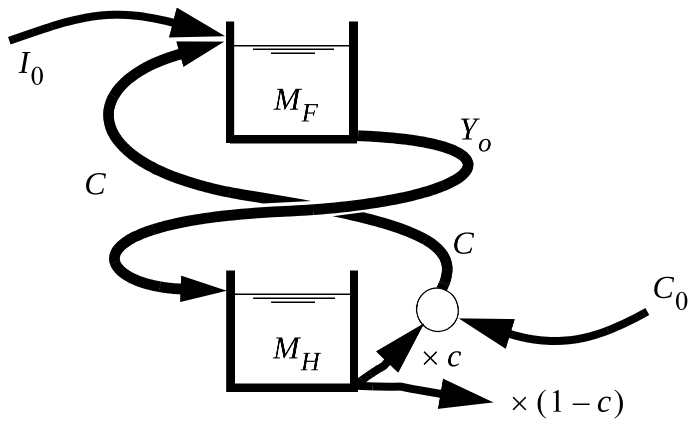

Figure 17

We have two (exogenous) input money flows, I_0 and C_0. The block diagram for the system corresponding to figure 17 is shown in figure 18.

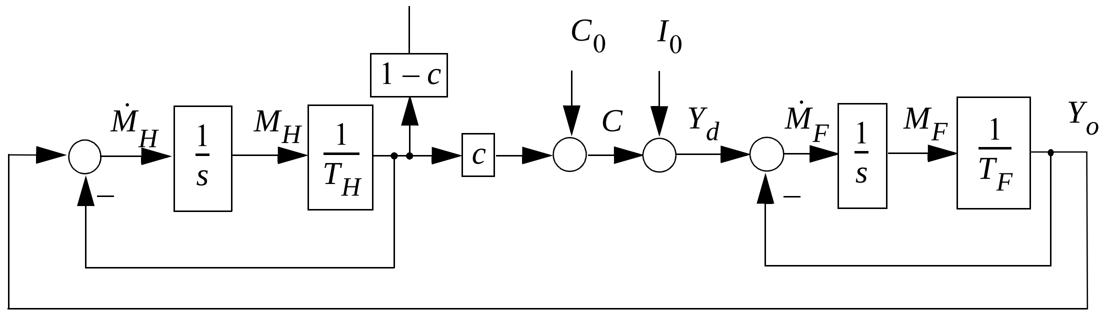

Figure 18

We see that a change in exogenous investment or consumption has the same effect. The transfer function from investment (or exogenous consumption) to output is, reducing the block diagram,

$$\frac{Y_o}{I_0}(s) = \frac{1 + T_H s}{T_F T_H s^2 + (T_F + T_H)s + 1 - c} \tag{23}$$

If we assume that investment changes as a step function with amplitude ΔI_0 at time $t = 0$, the Laplace transform of this step function is $\Delta I_0 / s$. We then have for the change in output,

$$\Delta Y_o(s) = \frac{1 + T_H s}{T_F T_H s^2 + (T_F + T_H)s + 1 - c} \cdot \frac{\Delta I_0}{s} \tag{24}$$

The final value theorem for Laplace transforms says that, for a time-dependent function $f(t)$ tending to a constant value as $t \to \infty$, we have

$$\lim_{t \to \infty} f(t) = \lim_{s \to o} s f(s) \tag{25}$$

Applying this to (24), we get

$$\lim_{t \to \infty} \Delta Y_s(t) = \lim_{s \to o} s[\Delta Y_s(s)] = \frac{\Delta I_0}{1-c} \tag{26}$$

which is the familiar expression for the multiplier. If all income is consumed ($c = 1$), the multiplier is infinite. The system is on the border of stability: One of two eigenvalues for the system (equivalently: poles in the transfer function) is in origo. Outside sustained injection of money wil increase circulation persistently between the two sectors, since no money is taken out of circulation by households saving part of income – output increase will never stop.

The final value theorem is a fast and convenient tool to find equilibrium outcomes (if any) for linear systems, but tells nothing about the transient (i.e. before equilibrium is reached) behavior of the system. We do not bring the algebraic solution here, but instead show the time path from a Simulink run, in figure 19. The system is initially in equilibrium when investment money flow is increased as a step function by $\Delta I_0 = 5$ at $t = 25$. The propensity to consume is assumed to be $c = 0{,}75$, i.e. we have a multiplier of 4.

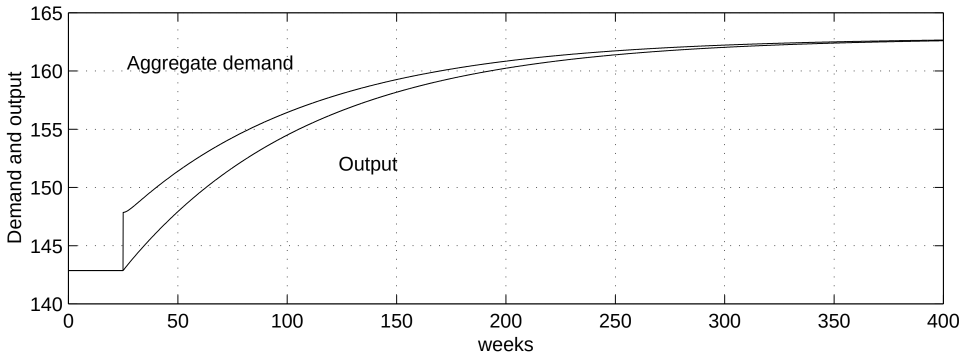

Figure 19

We observe that a 5 units increase in investment flow results in output asymptotically increasing by 20 units. Note the fairly long time lag of adjustment, which is ≈ 86 weeks.

4. The money stock inconsistency of the IS/LM model

Up to this point, our purpose has been to show how system-theoretic, block diagram-type tools are useful for macroeconomics, and to justify the first order time lag ("vessel") model as a main component in such models. We will now use this and the obvious dynamic extension of the static IS/LM model to demonstrate that IS/LM as such is fundamentally flawed. We are aware of the existence of other severe critiques of IS/LM. But the point here is that the brief analysis given below is sufficient in itself to completely invalidate it. It is not based on arguments and considerations that may be more or less convincing depending on which economics camp one identifies with – but simply on a gross mathematical inconsistency, which if true cannot be contested.

We start with the static IS/LM equilibrium equations, where aggregate demand must equal output, $Y_d = Y_o = Y$; and demand for money L must equal money stock M.

$$Y = C(Y) + I(r) + G_0 \tag{27}$$

$$M = L(Y, r) \tag{28}$$

We use a simple IS/LM variant, with exogenous net government spending G_0, and with investment being independent of output. This simplified choice makes no difference for the arguments to be made. The model corresponds to the one given in Ferguson and Lim (1998, pp 2 - 3). The relations for consumption, investment and liquidity demand are assumed linear in output and/or interest. Then we have

$$Y = C(Y) + I(r) + G_0 = C_0 + cY + I_0 - br + G_0 \tag{29}$$

$$M = L(Y, r) = kY - hr \tag{30}$$

Here c, b, k, h are constant parameters. We remind ourselves at this stage that this "comparative statics" model has as its premise that is a simplified representation; it is assumed to be the equilibrium solution to what in reality is a continuously varying dynamic system. Ferguson and Lim give the following dynamic extension of this model:

$$\dot{Y}_o = \alpha(Y_d - Y_o) = \alpha(C_0 + cY_o + I_0 - br + G_0 - Y_o) \tag{31}$$

$$\dot{r} = \beta(L - M) = \beta(kY_o - hr - M) \tag{32}$$

α, β are constant parameters. Verbally, these two differential equations say that the rate of change of output is proportional to the difference between aggregate demand and output, and that the rate of change of the interest rate is proportional to the difference between demand for liquidity and money stock[1].

The denominaton for the stock M is still [currency unit], while Y_d ,Y_o ,C_0 ,I_0 ,G_0 now get the denomination [currency unit / time unit] and become true flows – in contrast to their denomination in the comparative statics model, which is [currency unit].

1. One could reasonably argue that the transaction demand for money in (32) should instead be kY_d, but the choice is to follow Ferguson and Lim. And this choice does not have any impact on the argument to be made.

We represent equation (31) by a block diagram, figure 20 [1].

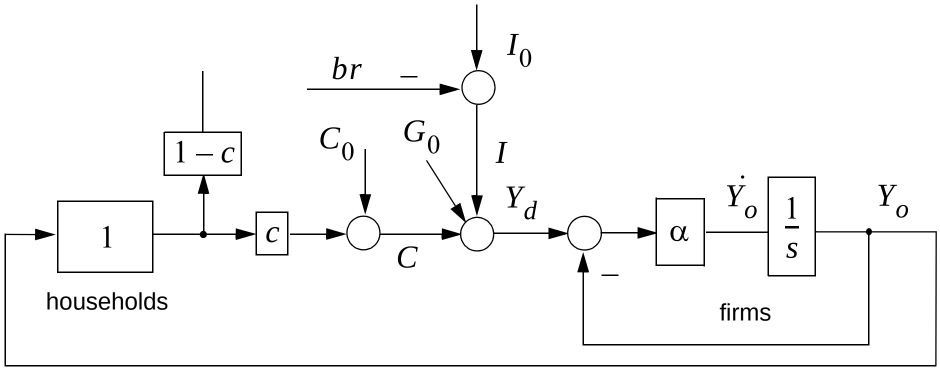

Figure 20

For the block diagram corresponding to the money market equation (32), see figure 21:

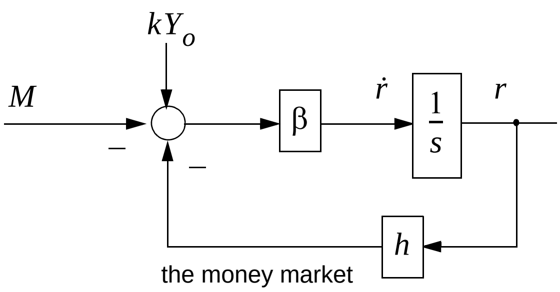

Figure 21

Before combining these two diagrams to one representing the whole system, we wish to reformulate equation (31). It may be written as

$$\dot{Y}_o/\alpha = Y_d - Y_o \text{, which must be } = \dot{M}_F \tag{33}$$

since $Y_d - Y_o$ is the net nominal money flow into the firm sector. By this we have incorporated firm money stock M_F in the model. Equation (33) explains the slightly reformulated but equivalent "firm" substructure in figure 22 below, which – except for this modification – is a result of a straightforward connection of the two sub-diagrams for the real economy and the money market.

(The modification (33) may alternatively be explained by exploiting a rule for block diagram manipulation: Interchanging the sequence of blocks on a path (in this case the two blocks containing α and $1/s$) does not change the transfer function along that path.)

1. Note that this dynamic model implies that the household sector has instantaneous dynamics, signified by the block with unity gain. Comparing with figure 18, this corresponds to the time lag in the household sector tending to zero, $T_H \to 0$. This assumption may be acceptable, since the time lag of the firm sector is so much larger due to a high share of between-firm transactions, as discussed in subsection 2.2. One should, however, be aware that this assumption implies that money stock in the household sector is zero: there is no buffer there, only a through flow.

By this modification we have accounted for the dynamics of the firm sector money stock M_F, *which in fact must be identical to the entire money stock of the economy*, since households are implicitly assumed to have no money stock, and the financial sector only appears indirectly via exogenous flows in this model.[1]

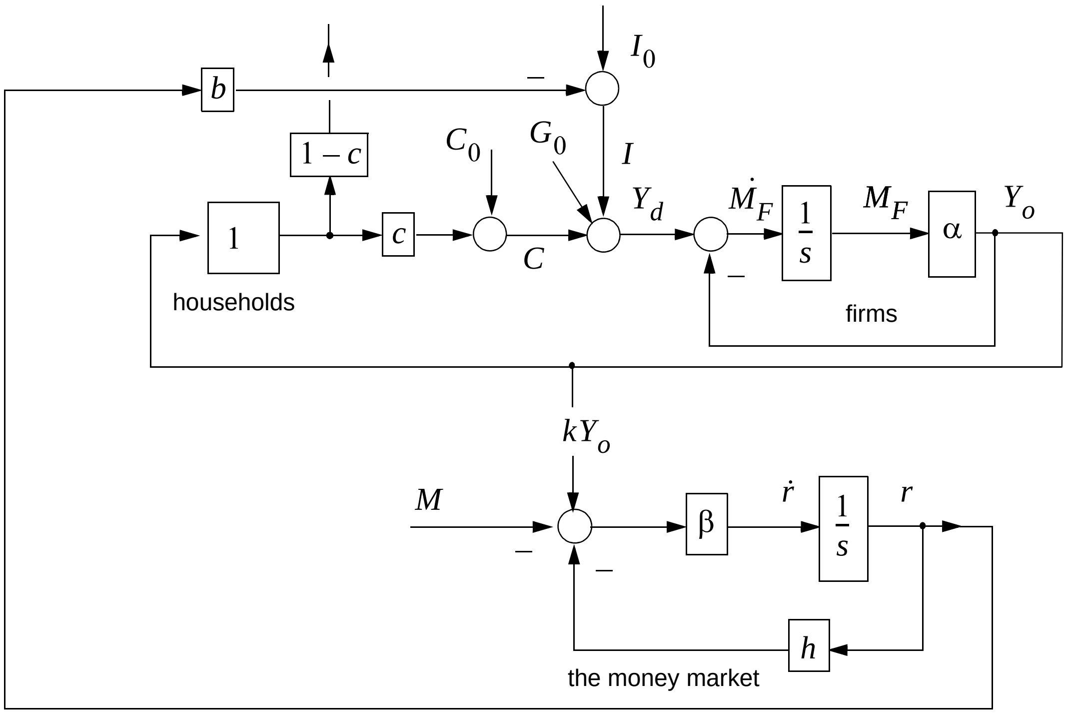

Figure 22

By now the inconsistency of the IS/LM model may be clearly observed: While money stock in reality is endogenous (M_F) and a system state, it is at the same time assumed to be an exogenous (input) variable M. What makes this inconsistency go unnoticed, is that the actual presence of money stock (M_F) within the the Y_d to Y_o dynamics, *disappears* in the (comparative) statics framework.

The correct model, in its most simplified version, should then be as shown in figure 23:

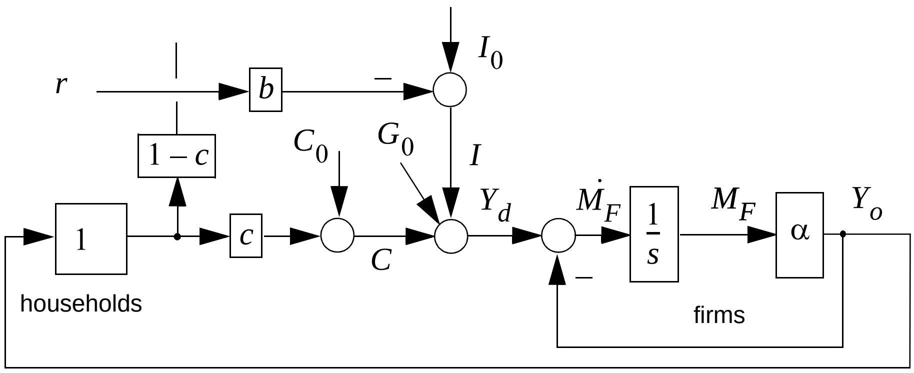

Figure 23

1. Note how the loose ends in the model due to a lack of a financial sector stands out in the block diagram formulation. The savings flow proportional to $1-c$ in the upper left just drains out of the system, and the flows C_0, I_0, G_0 enter the system from “somewhere”, together with money stock *M*. But this is another critique, which we don’t need to pursue here.

The model reduces to one dimension only. And *r* becomes a controlled input variable, not a system state, while *M* is no longer a controlled input variable but a system state.

5. Conclusion

If we dynamise the static IS/LM model *on the terms of its adherents* (neoclassical synthesicists), it rigorously follows that their view of money stock being an exogenous variable together with government spending (G_0), has to be substituted by the interest rate and government spending as control variables. They should then logically transit to the (Post) Keynesian position on the role of the interest rate. And all economics schools should simply abandon the IS/LM model.

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Fundamental financial accumulation dynamics

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Abstract

Any economic system with interest on money lent has the potential to gradually develop a level of debt that leads to crisis. Parameters and simple models for the dynamics of financial accumulation are proposed and explored. It turns out that concepts from linear control systems theory, and continuous-time representation, is quite useful for this exercise. It is argued that the problem of "exploding" debt is grave and largely ignored.

Keywords: accumulation, instability, lending, compound interest, dynamics

JEL classification: B50, C02, C60, C67

1 Introduction

This paper discusses the basic mathematical conditions for financial accumulation. The model consists of a "moneylender" who re-lends part of financial inflows from debt service on existing loans so that future financial income will be larger. At the other end is an agent who is in debt but still borrows what the moneylender offers. The two units may be thought of as macroeconomic aggregates, so that we have a society which is polarised between a group of lenders and a group of borrowers (called "sectors" in the following). The term "bank" will be used in between for the moneylender, but this is not a bank in the modern interpretation of the term (regulated by the Basel accords and thus able to create net credit money (Andresen, 2008)) but an entity that only re-lends received money that is left over after the lender has paid his expenses including wages. In this sense the lender corresponds to the "moneylender" of antiquity, among other places criticised in the Old Testament. Since ancient times there has been awareness of the instability inherent in a system where agents re-lend part of their income from loans. This is the rationale for periodic debt forgiveness ("jubilee") as proscribed in the Bible. The reader is referred to Appendix B for some quotations, and to (Hudson, 2009).

This paper discusses these dynamics using concepts from control systems theory. Time is continuous, and money flows are assumed to be smoothly varying in time, even if the actual money "flows" between agents occur as time-discrete events. This assumption is considered acceptable on the time scale (years and decades) we are considering. More on this below, and in Appendix A.

2 The model

A sector receives a money flow contributing to the sector's stock of money. The inflow and the current stock of money basically decides the sector's outgoing flow to other parts of the economy – its spending. But there are also outflows that are not decided by the sector (or agent) in question, but imposed on it from other parts of the economy. Such flows will be termed *non-discretionary.* Taxes or debt service are examples of non-discretionary flows.Taxes are dynamically unimportant since that type of payment occuring at some moment implies no future related flows. Debt service flows, however, have interesting dynamics that unfold over time: an initial one-shot input (received loan) leads to a stream of future events (debt service outflows).

A loan may in continuous time be considered an impulse (a delta function) input to a unit, and then the opposite-sign debt service flow becomes the impulse reponse (more on this in Appendix A) of what we will term a *debt service subsystem (*from now on abbreviated "DSS"*).* This impulse response is a non-discretionary flow. The model may be explained via the block diagram in figure 1. In the lower part

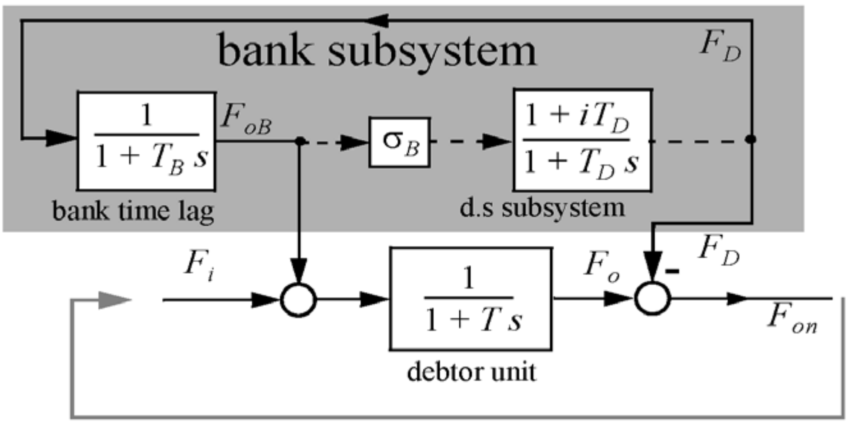

Figure 1: A bank subsystem with recycling of loans

of the figure is a debtor unit[1], which may be a single agent or a sector. If the unit is the entire aggregate of firms and households, F_{on} (= net outflow after debt service F_D) is recycled to the input as the flow F_i, indicated by the shaded arrow lowermost in the figure. In this case the outflow F_o corresponds to the country's GDP.

[1]The variable s in the diagram is a differentiation operator, so that that the lower block $\frac{1}{1+Ts}$ corresponds to the linear differential equation $T\frac{d}{dt}F_o(t) + F_o(t) = F_i(t) + F_{oB}(t) \iff \frac{d}{dt}F_o(t) = -\frac{1}{T}F_o(t) + \frac{1}{T}\left[F_i(t) + F_{oB}(t)\right]$.

F_D is different from other flows in the figure in the sense that it is the result of a *rule* (the loan contract). This rule imposes – it is non-discretionary – a flow F_D on the indebted sector, which is subtracted from the gross debtor outflow F_o, and inserted as an inflow to the bank unit (the aggregate of all lenders). Note direction of arrows. By this the accounting remains correct: money removed from one flow is input somewhere else. To indicate the presence of rule-based interaction as opposed to a sector's own-decided outflow, the corresponding lines are dotted in the figure.

We have here assumed a scenario where the flow of new loans is a strict feedback from what banks receive in debt service on current loans. This is pure lender-controlled financial accumulation. (New loans may instead be mood-dependent and not directly decided by what inflows banks receive, but this is not considered in this paper.)

We will use the term "‘bank" here in a quite generic sense: any type of unit that has any type of financial claim (here called a "loan") on another unit/sector as long as the claim obliges the debtor to furnish a future stream of returns. The interest rate is i and duration of loans is T_D . As mentioned earlier, debt service is modeled as a continuous flow, while in the real world debt service occurs as time discrete packets. In our continuous-time setting this could have been precisely accounted for by a train of delta functions, but this is not necessary, following the above argument about the long time scale for the dynamics to be discussed, and also the low-pass filter property of the sectors in the system.

The model in figure 1 has a great advantage: It allows for calculating the dynamics of an aggregate economy where current debt service is used continuously to extend new loans, and where both the effect of interest rate and loan duration is accounted for. This is in contrast to much of literature of the Post Keynesian and Circuitist economic schools, where one often – due to the inferior tools used – has to abstract from interest and also assume that loan extension and repayment takes place in distinct and concluded "rounds", see for instance (Lavoie, 1992) pp. 151 – 157, (Graziani, 1996), and (Fontana, 2000). This topic is treated more extensively in Appendix A.

The (aggregate) "bank" in the figure is modeled as a first order linear system with unity gain, assuming that the flow received by the bank is output again with some lag T_B.These first-order linear dynamics implies that the money held by the bank[2] is $M_B = F_{oB}T_B$. Thus M_B is the state variable of the bank subsystem. The outflow F_{oB} consists of both the bank's paying for expenses, and its new loans flow which is its financially reinvested share σ_B of F_{oB}, $0 < \sigma_B < 1$. We will from now on call σ_B the *financial reinvestment coefficient*, abbreviated *FRC*. The real economy (debtor unit) is – like the bank – modeled as a first order linear dynamic system with unity gain.

It now remains to explain the DSS in the figure. The transfer function is

$$G_D(s) = \frac{1 + iT_D}{1 + T_D s} \tag{1}$$

which may be discussed by introducing the equivalent structure shown to the right in figure 2. Now debt

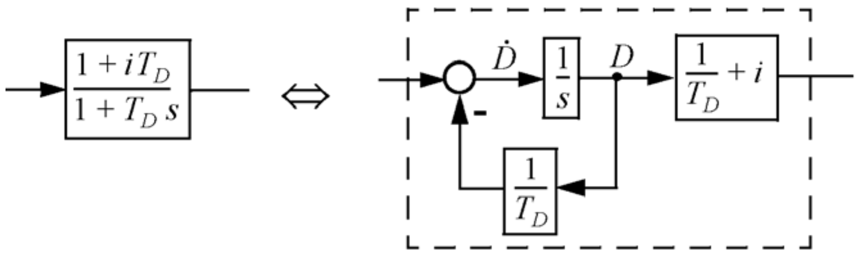

Figure 2: Equivalent debt service block

D is visible in the right subsystem. This DSS, with inflows and outflows as in figure 1, corresponds to the equations

$$\dot{D} = \sigma_B F_{oB} - \frac{1}{T_D} D \tag{2a}$$

$$F_D = (i + \frac{1}{T_D}) D \tag{2b}$$

[2] Some readers may object that the concept of banks "holding money" is meaningless, since banks may be considered to create money when lending, and destroy money when loans are repaid. This is the Post Keynesian position, which this author supports. But it is for purposes of simplified presentation convenient to assume that the bank works like a non-bank financial institution ("moneylender"), in the sense that it does not net create money. This also allows us to include other types of accumulating units in our "extended bank concept".

This scheme (from now on called the “exponential debt service” scheme) is unconventional, since both the principal and interest flow components are proportional to remaining debt. This differs from for instance an annuity scheme where the sum of principal and interest is constant, or a bond-type scheme where principal is only paid (in its entirety) when the loan matures. The advantage of the scheme (2) is that it allows for analysis using eigenvalues, and finding algebraic solutions – while annuity or bond-type dynamics involve time delays and are therefore algebraically less tractable. And it will be demonstrated in subsection 2.3 that differences in total system behaviour are unimportant in regard to which scheme is assumed. Figure 3 shows the debt service flows for the exponential debt service scheme compared to the

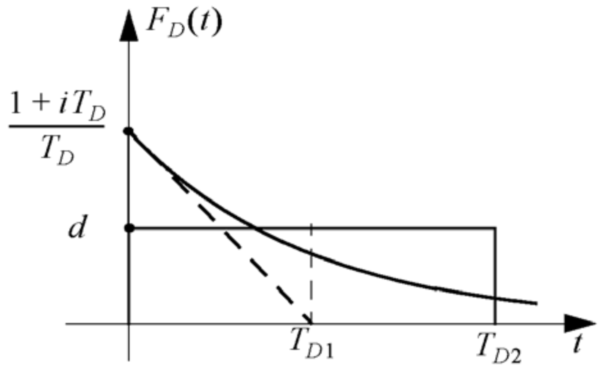

Figure 3: Debt service for annuity and exponential schemes

annuity-type scheme. If we consider a loan of 1 \$ extended at $t = 0$, these debt service flows will be the impulse responses of the debt service subsystems. For approximate equivalence, we suggest that both types of DSS should have the same mean lag. This means that loan durations differ, with $T_{D2} = 2T_{D1}$ (this multiplicative factor will be somewhat adjusted in subsection 2.3). Mathematically, the duration of the exponential debt service scheme is infinite, but we define it to be T_{D1}, since this is the mean lag of the graph. The areas under the graphs correspond to the accumulated debt service sums. They are > 1, so the DSS does not have an impulse response with unit area (it would have had that if i was 0, since then one had to pay back only what was initially borrowed). The value of the constant parameter d in the figure, which gives the annuity debt service flow, is derived below.

2.1 The annuity-type debt service subsystem

We assume that a loan of 1 \$ is extended at $t = 0$, and demand that the discounted value of a received constant flow d between 0 and T_D shall be equal to 1:

$$d \int_0^{T_D} e^{-it} dt = 1, \text{ which gives } d = \frac{i}{1 - e^{-iT_D}} \tag{3}$$

If the loan is a perpetuity i.e. $T_D = \infty$. (3) then gives $d = i$ as expected. For the special case $i = 0$, L’Hopital’s rule, or the integral in (3), gives $d = 1/T_D$, also as expected.

We may now construct a subsystem for this annuity type DSS that has a rectangular impulse response with amplitude d. It is shown in figure 4. The subsystem works like this: A new loan (an impulse) is

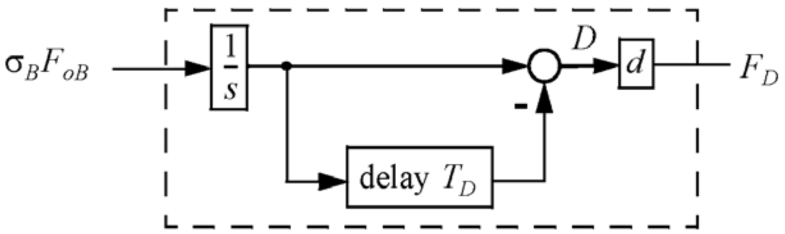

Figure 4: Debt service subsystem with annuity scheme

received, and the integrator makes the value in the upper branch jump to the size of the loan and stay there. After a duration of T_D, the lower branch jumps to the same level and is subtracted from the upper branch value. This ensures that debt service for that particular loan stops when the loan terminates. We have a rectangular response with amplitude equal to the size of the loan, multiplied with the factor d to give the correct debt service outflow from the DSS.

This DSS contains a time delay, and closed algebraic solutions of systems containing time delays is generally not possible. But the system is still linear. Therefore a continuous flow of new loans will, by convolution with the DSS impulse response, still give the precise debt service outflow. In other words: the effect of continuous recirculation of loans in a macroeconomic model may be correctly accounted for also in the annuity case. And we will see below that in this special case stability may be checked algebraically in spite of eigenvalues not being available.

2.2 When may debt "explode"?

A widely covered topic in literature and a persistent political-economic, moral and religious issue since ancient times is the mechanism of lenders accumulating financial claims on the rest of society by re-lending income from current loans. This danger is recognised for instance in the Bible, where a "jubilee" is proscribed every 50ieth year to reset outstanding debt to zero (see Appendix B).

Obviously, a persistent re-lending of debt servoce flows *may* lead to financial debt/asset polarisation in a society. The structure in figure 1 allows us to check the conditions for this occuring. Debt/asset polarisation corresponds to instability of this linear system. If we initially confine ourselves to a system with an exponential debt service scheme, stability may be checked by considering system eigenvalues. By inspection of figure 1, we see that system dynamics is decided entirely by the shaded "bank" part of the structure. The dynamics of the lower "debtor" part does not feed back to the bank part and is therefore decided solely by what happens there. The characteristic equation for the bank part is

$$(1 + T_B\lambda)(1 + T_D\lambda) - \sigma_B(1 + iT_D) = a_2\lambda^2 + a_1\lambda + a_0 = 0 \tag{4}$$

A necessary (and for a second order system like this, also sufficient) condition for the system's eigenvalues to be negative (i.e. stable system) is that all coefficients a_k in the characteristic polynomial have the same sign. a_1 and a_2 are always positive, while $a_0 = 1 - \sigma_B(1 + iT_D)$ may be < 0 for certain parameter values. Then one eigenvalue is in the right half plane. We have instability (= debt growth = financial accumulation). The condition $a_0 < 0$ corresponds to:

$$\sigma_B > \frac{1}{(1 + iT_D)}, \text{ or equivalently:} \tag{5a}$$

$$i\sigma_B > \frac{1 - \sigma_B}{T_D}, \text{ or} \tag{5b}$$

$$iT_D > \frac{1 - \sigma_B}{\sigma_B} \tag{5c}$$

We note that T_B is not part of the instability condition. If the condition (5) is fulfilled, debt growth is exponential (after an initial transient period due to the other, stable eigenvalue). Loan duration T_D may be in the order of – say – a decade. The bank time lag T_B should realistically be in the weeks/months range. So we may assume $T_B \ll T_D$. This means that the bank time lag subsystem in figure 1 may reasonably be substituted by unity. If we also ignore the debtor subsystem which has no impact on dynamic properties as already mentioned, the simplified remaining system needed to discuss debt build-up dynamics becomes as shown in the block diagram to the left in figure 5. To the right we

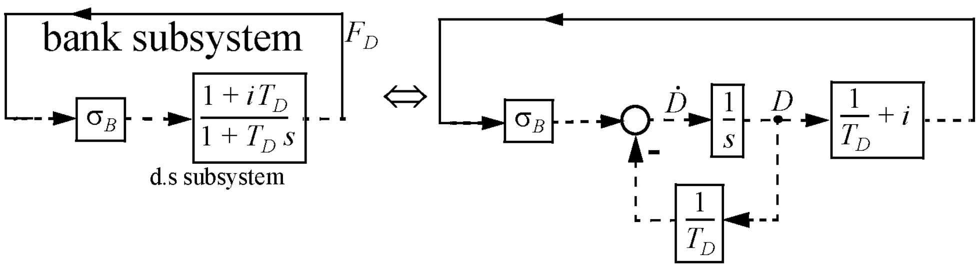

Figure 5: Simplified accumulation system

have inserted the equivalent DSS from figure 2 so that the sole system state, D, is shown. This block diagram corresponds to the autonomous first order linear differential equation

$$\dot{D} = \left(-\frac{1}{T_D} + \sigma_B(\frac{1}{T_D} + i)\right) D = \lambda D \tag{6}$$

which has the solution $D = D_0 e^{\lambda t}$, where D_0 is initial debt. We have exponential growth for $\lambda > 0$; which is condition (5).We will now discuss the roles of the three parameters T_D, i, σ_B:

From (5c) we observe that a percentual increase in σ_B has a stronger effect towards accumulation than a similar increase in i. This may seem counter-intuitive to many, since the focus in this type of discourse is usually the impact of i.

For $\sigma_B = 1$, i.e. all financial income is re-lent, (6) becomes $\dot{D} = iD$, the "classic" equation for accumulation through compound interest, which will then take place for any $i > 0$. An expression of the fascination with – and alarm against – this phenomenon is the table in figure 6 which is a facsimile from

1 Danach ergeben sich vom Jahr 0 bis 1990 bei jährlichem Zinszuschlag folgende ausgewählte Kontostände:

Jahr	Rechnungs-einheit DM	Rechnungs-einheit kg Gold am 9. 1. 90 18 500,– DM/kg	Rechnungs-einheit goldene Erdkugeln (5,973 E + 24 kg)
0	0,01		
95	1,03		
100	1,31		
142	10,20		
189	101,10		
236	1001,55		
296	18708,22	1	
438	19 094 706	1026	
1466	1,16 E+29	6,22 E+24	1
1749	1,148 E+35		1 Mill.
1890	1,116 E+38		1 Mrd.
1990	1,468 E+40	8,026 E+35	134 Mrd.

Heinrich Haußmann: *Der Josephspfennig*. Fürth 1990.

Figure 6: The dramatic dynamics of exponential growth

(Kennedy, 1991). One pfennig (0.01 Deutsche Mark – this was written before the advent of the Euro) deposited in year 0 at 5% interest is by 1990 worth 134 billion massive spheres of gold, each the size of the Earth.

Admittedly, 5% is in real terms a fairly high (real) interest rate, but the table still illustrates the dramatic dynamics of exponential (financial) growth[3].

Another implication of (5) is that cet. par., a large T_D means steeper debt growth. If the loans are perpetuitities ($T_D = \infty$), we have debt growth regardless of the size of σ_B and i, with

$$\dot{D} = i\sigma_B D \tag{7}$$

We get the same result if we assume that all repaid money is lent again, and the lender's costs and consumption are paid out of received interest exclusively, through a share $1 - \sigma_B$ of the interest flow. Then debt growth will occur for any $\sigma_B > 0$, as indicated by (7).

2.3 Accumulation with annuity-type debt service

We now want to check conditions for accumulation (instability) when the DSS is not of the (for simplification purposes) unconventional exponential type as in figure 2, but of the annuity type, shown in

[3] Allegedly also commented like this by Albert Einstein: "the most powerful force in the universe is compound interest." Ironically, this quotation is mostly used today not in the spirit of its critical originator: it is touted to market financial investment.

figure 4. We also in this case choose to ignore the bank time lag subsystem, which is set to unity. The transfer function for the annuity DSS is

$$G_1(s) = \frac{d}{s}(1 - e^{-T_D s}) \tag{8}$$

where d is given by (3), and T_D is the duration of the loan. When we close the loop, we don't get a characteristic polynomial but an irrational expression, due to the term e^{-Ts}. Therefore we cannot check instability via eigenvalues. But since the system is still linear, we may use the Nyquist stability criterion. The loop transfer function $G(s)$ is

$$G(s) = -\sigma_B \frac{d}{s}(1 - e^{-T_D s}) \tag{9}$$

(a minus sign has to be inserted because the criterion is based on the feedback being negative, while the feedback is positive in our case.) The frequency response, given by setting $s = j\omega$ in $G(s)$, is displayed in the form of a polar plot in figure 7. When ω takes on values from $-\infty$ via 0 to ∞, we get a corresponding

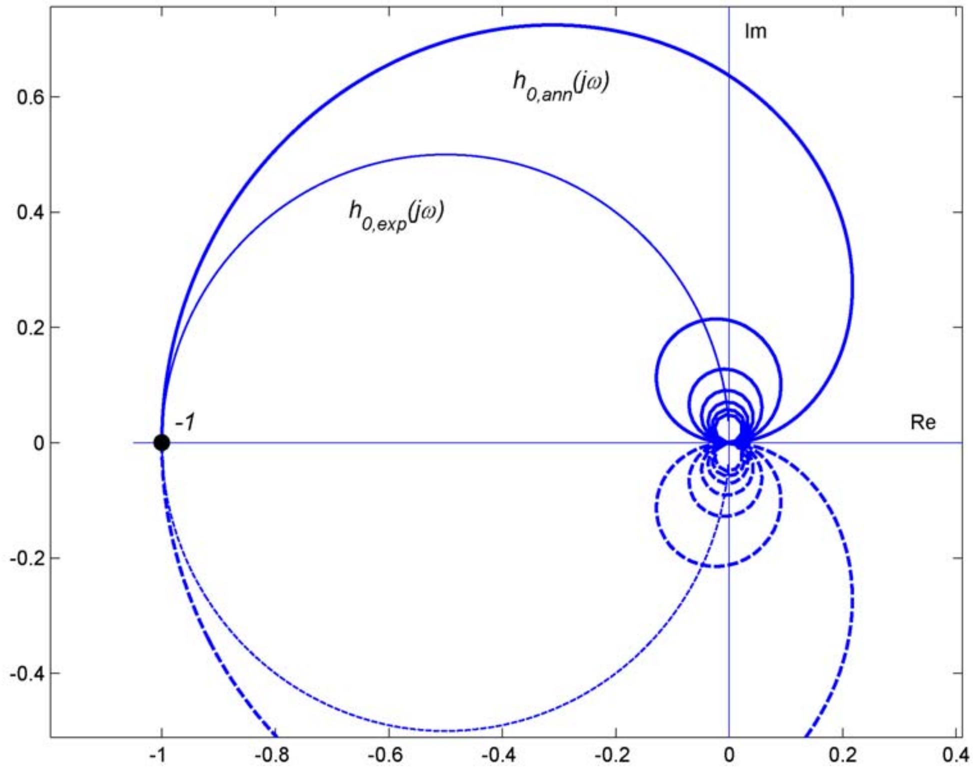

Figure 7: Polar diagrams of $G(s)$ for stability check; annuity and exponential DSS.

closed graph for the frequency response $G(j\omega)$ as displayed in the figure. The dotted half of the graph corresponds to $G(j\omega)$ for $\omega < 0$. $G(s)$ is open-loop stable since the impulse response goes to zero with increasing t. Then the Nyquist criterion simply says that the closed-loop system is stable when the leftmost part of the graph crosses the negative real axis to the right of the point -1.The figure also shows the corresponding graph when the DSS is of the exponential type (where we have already used eigenvalues to check instability). The graph with this DSS is simply a circle, indicated with a thin line. In the figure, the choice of parameters σ_B, i, T_D is such that both graphs go precisely through -1, which means that the two corresponding closed-loop systems are on the border of (in)stability. The chosen parameter values correspond to the two dots in figure 8 below.

While the Nyquist criterion as a general rule can only be applied based on a graph, in this special case we may employ it algebraically. If we consider (9) with $s = j\omega$, we see from angle and absolute value that the leftmost crossing of the negative real axis must take place for $\omega = 0$. We have

$$G(j0) = \lim_{\omega \to 0} \left(-\sigma_B \frac{d}{j\omega}(1 - e^{-j\omega T_D}) \right) = (\text{real}) = -\sigma_B T_D d \tag{10}$$

We substitute (3) for d. The Nyquist criterion, and (10) then gives the condition for financial accumulation:

$$iT_D > \frac{1 - e^{-iT_D}}{\sigma_B} \tag{11}$$

This may be compared to (5c) for the exponential DSS. A better comparison is achieved if we plot borderline stability graphs for both types of DSS, for different sets of parameters σ_B, i, T_D. This is done in figure 8, with i on the x axis, T_D on the y axis, for four different values of σ_B. The graphs for the

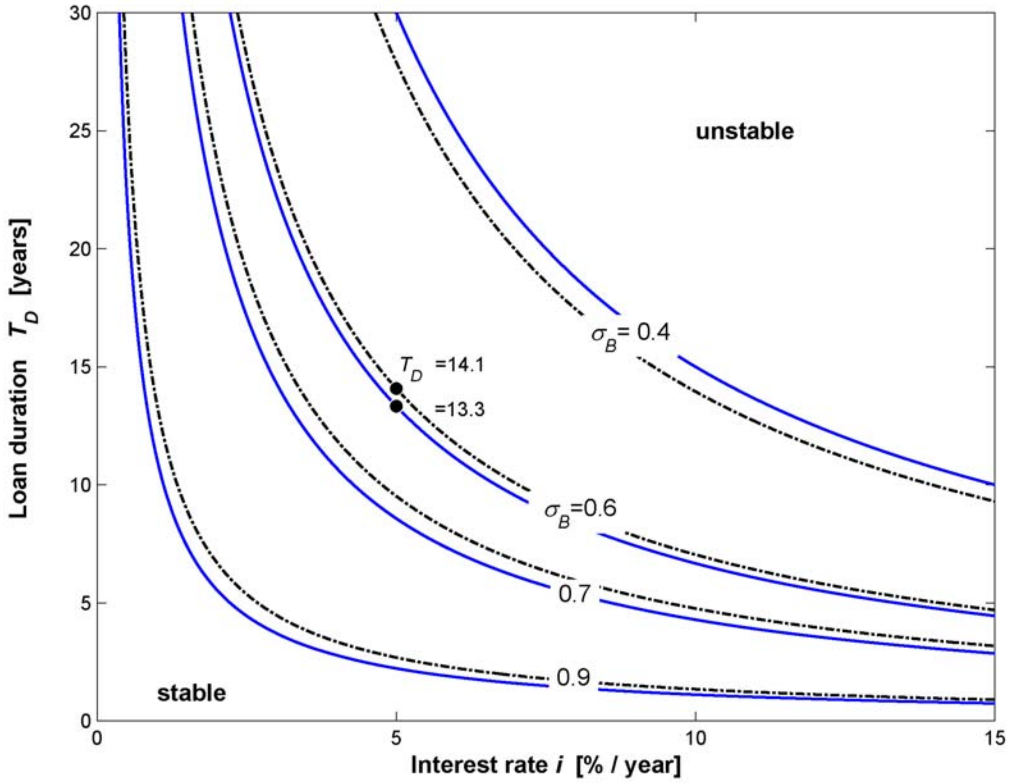

Figure 8: Regions of debt growth ("instability") for values of σ_B, i, T_D

exponential DSS case are solid, while the annuity case graphs are dash-dotted. From the graphs we observe as expected that cet.par., high interest rates or long loan durations give instability (i.e. debt growth, financial accumulation), for both types of DSS. And as already pointed out, an FRC closer to 1 gives debt growth, cet. par. We observe that the graphs for both types of DSS lie fairly close and have similar shapes (all graphs are hyperbolae). This gives support to the notion that the exponential DSS may be used for studying debt growth dynamics instead of the less algebraically tractable annuity DSS.

In the figure, loan duration T_{D2} for the annuity DSS has been adjusted in relation to T_{D1} for the exponential case, following the argument in conjunction with figure 3. In the figure, the T_D on the y axis $= T_{D1}$. By experimenting it was established that $T_{D2} = 1.6T_{D1}$, not $T_{D2} = 2T_{D1}$ as suggested in figure 3, gave the best coincidence for the graphs over a reasonable range of values of σ_B. This adjustment does not, however, invalidate the use of the exponential DSS instead of annuity DSS, since the stability properties of both are so similar.

As an example of how stability information may be extracted from the figure, it is seen that at an interest rate of $5\%/y$ and $\sigma_B = 0.6$, a loan duration $T_{D1} = T_D > 13.3$ will give accumulation when the DSS is exponential, and loan duration $T_{D2} > 1.6T_D = 1.6 \cdot 14.1 = 22.6$ gives accumulation for the annuity DSS case.

2.4 Firms with no income during a start-up period

If we confine ourselves to loans being given to firms (abstracting from household borrowing), the model presupposes that money flows to these firms from day one in the form of demand for consumption and investment goods. Then the firm sector must (be able to) deliver a corresponding flow of products in the opposite direction. How then account for the situation where a firm receives a loan, but for a fair amount of time will not have any further monetary inflow since it has no products or services to deliver during its build-up phase?

Essentially, the solution is to modify the time profile of debt service, i.e. the impulse response of the debt service subsystem (DSS). If a new loan is extended at $t = 0$, the impulse response of the DSS is now set to zero for an initial period T (perhaps in the order of a year). The firm is exempt from debt service in this period. After $t = T$, debt service starts and follows the same profile(s) as already discussed, but after the original loan has first been amplified by a factor e^{iT} since compound interest must be added before debt service starts. Conditions for accumulation with this modified debt service profile changes somewhat, but the changes are not important for the analysis and quite simple. We will modify the exponential debt service scheme in eq. (1) so that it has the above properties (we could have done the same with the annuity scheme, but it does not make any significant difference for our analysis). The modified transfer function is

$$G_{D'}(s) = e^{iT} e^{-Ts} \frac{1 + iT_D}{1 + T_D s} \tag{12}$$

The term e^{iT} accounts for amplifying the debt, and e^{-Ts}accounts for the time delay before debt service starts. Since $G_{D'}(s)$ is irrational due to the term e^{-Ts}, we use the Nyquist criterion to check stability. Following a similar argument as that leading to (9), we now get

$$G(s) = -\sigma_B e^{iT} e^{-Ts} \frac{1 + iT_D}{1 + T_D s} \tag{13}$$

Again we may confine ourselves to considering (13) for $s = j\omega$ with $\omega = 0$. We have

$$G(j0) = \left[-\sigma_B e^{iT} e^{-j\omega T} \frac{1 + iT_D}{1 + T_D j\omega} \right]_{\omega=0} = (\text{real}) = -\sigma_B e^{iT} (1 + iT_D) \tag{14}$$

The system is unstable (i.e. accumulation occurs) for $-\sigma_B e^{iT} (1 + iT_D) < -1$. This corresponds to conditions for accumulation resembling those in (5):

$$\sigma_B e^{iT} > \frac{1}{(1 + iT_D)}, \text{ or equivalently:} \tag{15a}$$

$$i\sigma_B e^{iT} > \frac{1}{T_D}(1 - \sigma_B e^{iT}), \text{ or} \tag{15b}$$

$$iT_D > \frac{(1 - \sigma_B e^{iT})}{\sigma_B e^{iT}} \tag{15c}$$

As expected, relieving firms of debt service for an initial period with the loan growing correspondingly, moves the system somewhat closer to the instability border for the same set of the three parameters interest, loan duration and banks' financial re-investment coefficient. Comparing (15) to (5), we see that stability-wise, a model with debt relief in an initial period, is equivalent to amplifying the FRC to $\sigma_B = \sigma_B e^{iT}$ in the original model (1).

With debt service relief in an initial period and the extreme special case $\sigma_B e^{iT} > 1 \iff \sigma_B > e^{-iT}$, conditions (15) tell us that accumulation will always occur.

3 Final remarks

An economic system with lenders recycling financial income as new loans will as a rule be unstable – as warned against since ancient times. For all financial investors (lenders) strive to accumulate. To the degree they succeed, we get increased asset/debt polarisation between lenders and borrowers. Such polarisation occurs since only successful accumulators survive through the market's Darwinian selection process. Thus slow motion debt explosions will be the rule and not the exception. The reason that this is not much recognised or discussed, is probably the time scale for the dynamics involved (several decades), and that the growth path of an exponential function isn't very noticeable until the dramatic late stage.

It also possible that the reason for lack of recognition of the basic accumulation mechanism is – paradoxically – that it is so trivially obvious, if one bothers to think about it. Even the ancient Mesopotamians recognised it. The theory's antique origin, its close relation with religion, and its simplicity all contribute to explain why fringe groups and "crackpots" embrace it. But one should be very careful about dismissing a theory just because it is loved by the fringe. One then has a case of a baby being thrown out with the bathwater. This seems to be the case by parts of the economics profession.

Seen from a control systems perspective however (which ought also to be shared by economists), these runaway long-term dynamics are extremely harmful, and some macroeconomic control mechanism(s) should be implemented.

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Appendices

A Why time-continuous models?

Any model is only an approximation to the real phenomena it tries to represent. Most dynamic economic models are time-discrete. Before the advent of today's sophisticated simulation software, discrete-time models were easier to solve (for example with Excel spreadsheets), which partly explains the discrete-time bias. Another (but erroneous) justification for time-discrete models is that transactions between agents or sectors occur at discrete instances in time, and nothing happens in between. But a time-discrete model presupposes regularly spaced events, while real-world transactions occur with uneven intervals. A precise and elegant way of accounting for such unevenly spaced events is using time-continuous models, but representing the discrete events with delta (impulse) functions: If a unit of money is passed at time $t = t_1$ to an agent or a sector, this mathematically corresponds to an impulse function, commonly symbolised with $\delta(t - t_1)$. This function is a mathematical idealisation: it may be defined as the limit of a rectangular-shaped time function,

$$\delta(t) = \lim_{\varepsilon \longrightarrow 0} \delta_\varepsilon(t), \text{ with } \delta_\varepsilon(t) = \begin{cases} 1/\varepsilon, |t| \leq \varepsilon/2 \\ 0, |t| > \varepsilon/2 \end{cases} . \tag{16}$$

$\delta(t)$ has infinite amplitude and zero duration, but such that its area is unity. $\delta(t)$ is (as approximated by $\delta_\varepsilon(t)$) depicted to the left in figure 9. In an economic model in continuous time, the impulse function allows a correct representation of time-discrete transactions: an amount of money Q passed to a sector or an agent at time t_1 is represented by the function $Q\delta(t - t_1)$. The denomination of this function is money *flow* [$/y], while the area under the function has denomination money *amount* [$]. The *impulse response* $h(t)$ of a unit (in our case an economic agent, a sector or the entire macroeconomic system) is defined as the output signal[4] resulting from one $ input at $t = 0$. The impulse response of a first order linear dynamic system with the input $F_i(t) = \delta(t)$ is

$$F_o(t) = h(t) = \begin{cases} \frac{1}{T} e^{-\frac{t}{T}}, t \geq 0 \\ 0, t < 0 \end{cases} \tag{17}$$

$h(t)$ is shown to the right in figure 9. It is a flow with denomination [$/y]. The area under $h(t)$ is

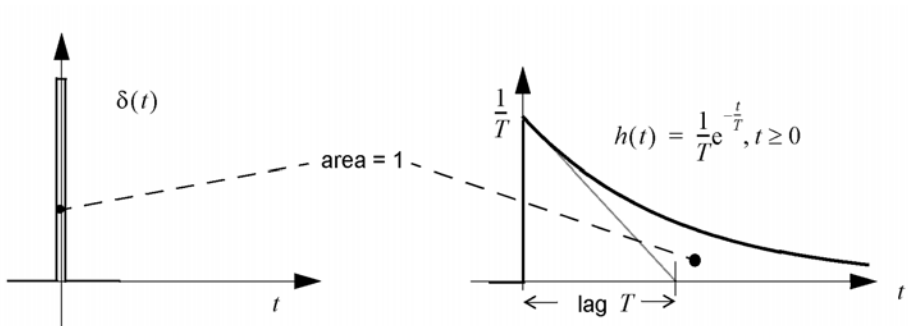

Figure 9: Impulse function (left) and impulse response (right)

unity[5]. This is as expected, since money is neither created nor destroyed when passing by a unit. The mean time lag of $h(t)$ is

$$\int_0^\infty t h(t) dt = \int_0^\infty t \frac{1}{T} e^{-\frac{t}{T}} dt = T \tag{18}$$

(The mean time lag may be estimated by inspection of the graph for $h(t)$, because T is the value of t at the intersection between the tangent of $h(t)$ at $t = 0$ and the time axis, as indicated to the right in figure 9.)

A further argument in favour of choosing the continuous-time framework is that a train of irregularly spaced impulses (which in fact is the *precise* representation of transactions in continuous time) is very

[4]The symbol $h(t)$ is reserved in the control/signals (and) systems literature to signify the output response to an impulse function, as distinct from responses to other input functions.

[5]This property is also expressed by the unit's transfer function having a static gain of unity.

well approximated by a continuous flow when the incidence of transactions is high. This is portrayed in figure 10. When we are working with aggregates of many agents like firms and all households,

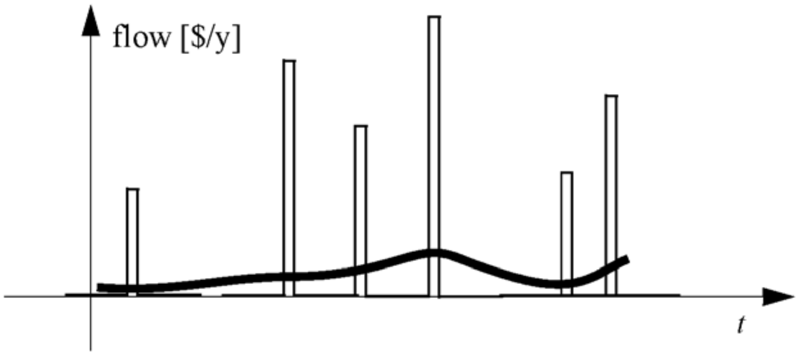

Figure 10: sum of areas under impulses = area under curve [$]

"transaction impulses" between aggregates occur so frequently that continuous flow representation is quite satisfactory. The dynamics of a sector with many units is sluggish related to the incidence of transactions. A subsystem's time constant T expresses this sluggishness (or "inertia"). Interpreted in the frequency domain it is a low pass filter with cutoff frequency $1/T$. Sharp fluctuations in the input will be smoothed out after having passed through. So the output will be similar whether the input is (faithfully) described as a chain of sharp spikes as shown in figure 10 or approximated by the corresponding smooth graph in the same figure.

A further argument for continuous-time representation is that a system may have a large spread in time constants, which is difficult to account for – and also observe by inspection of equations/block diagrams – in time-discrete models. The systems under consideration here exhibit a broad dynamic range from weeks to decades.

Finally, an important advantage with continuous-time representation is that that the response in figure 9 is *dispersed in time*, a property which obviously is present in real-world economic systems: If an amount of money is received by some sector at some instance, the amount will be spread out in time when it is spent. Parts of it will follow a very convoluted path in the sense that it will be used by many agents for transactions *within* the sector, before being paid *out of* the sector[6]. The same holds for money being received by a single agent within a sector at a certain moment; it will not all be spent at once but spread out over time. The first-order continuous time lag model accounts for the dispersed character of the response in a simple, but sufficient manner[7]. The dispersion-in-time property, which holds for all input-output relationships for agents and sectors, *invalidates the approach of analysing monetary circuit dynamics by assuming that these unfold in concluded "periods"*, which is a common assumption in the Post Keynesian/Circuitist literature as mentioned earlier.

B Biblical quotes

> *"When your brother Israelite is reduced to poverty and cannot support himself in the community, you shall assist him as you would an alien and a stranger, and he shall live with you. You shall not charge him interest on a loan, either by deducting it in advance from the capital sum, or by adding it on repayment"* – Leviticus 25:35-36

> *"If you advance money to any poor man amongst my people, you shall not act like a money-lender: you must not exact interest in advance from him"* – Exodus 22:25

> *"You shall not charge interest on anything you lend to a fellow- countryman, money or food or anything else on which interest can be charged. You may charge interest on a loan to foreigner but not on a loan to a fellow countryman..."* – Deuteronomy 23:19-20

[6] The topic of increase in lag for a defined (sub)system due to money circulating within the defined sector/subsystem before leaving it, is comprehensively treated in (Andresen, 1998).

[7] A pioneer in recognising and using this in macroeconomic modeling and simulation, was A.W. Phillips, in a seminal 1954 paper (Phillips, 1954).

"O lord, who may lodge in thy tabernacle? ...... The man .... who does not put his money out to usury ....." – Psalms 15

"He never lends either at discount or at interest. He shuns injustice and deals fairly between man and man" – Ezekiel 18:8-9

"..on the Day of Atonement, You shall send the ram's horn round. You shall send it through all the land to sound a blast, and so you shall hallow the fiftieth year and proclaim liberation in the land for all its inhabitants. You shall make this your year of jubilee. Every man of you shall return to his patrimony, every man to his family......In this year of the jubilee you shall return, every one of you, to his patrimony... if the man cannot afford to buy back the property, it shall remain in the hands of the purchaser till the year of the jubilee. It shall then revert to the original owner, and he shall return to his patrimony.... When your brother is reduced to poverty and sells himself to you, you shall not use him to work for you as a slave. His status shall be that of a hired man and a stranger lodging with you; he shall work for you until the year of the jubilee. He shall then leave your service, with his children, and go back to his family and to his ancestral property..." – Leviticus 25, excerpts

Basel Rules, endogenous Money Growth, Financial Accumulation and Debt Crisis

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Abstract

The rationale for a Basel-type bank regulation regime is to avoid insolvency in the bank sector. But it has the side effect of endogenous credit money growth. The growth rate turns out to be inversely proportional to the required minimum capital/asset ratio. For base money to keep up with this growth, the government should run a persistent deficit. Endogenous credit money growing at the same rate as debt from licensed banks contributes to avoid debt crises, as opposed to non-bank lending where debt grows but not money stock, something that is dangerous in the long run. Finally, the phenomenon of banks selling loans onwards is examined. It is shown that this doesn't only decrease the bank's risk, it may also imply steeper asset growth for the selling bank.

1 Introduction

Today's international regulation regimes for banks are less of the reserve requirement type and more based on requirements on banks to be robust against insolvency, by demanding that a bank's claims on others must exceed others' claims on the bank by some reasonable margin. Banks are required to stay above a given lower bound for their capital/asset ratio, also incorporating some risk weighing of different types of assets. Since the only (acknowledged) rationale for this banking regulation regime is robustness against insolvency, it is of interest to examine whether there are side effects, and whether these are benign or not. This paper shows that a Basel-type regime implies the important side effect of endogenous credit money growth. It turns out that the growth rate is inversely proportional to the required minimum capital/asset ratio.

The model to be discussed is very simple, as indicated by the assumptions made. Hopefully, it still embodies the properties needed for the analysis to be of value. In the first stage, all banks are aggregated into one unit. This aggregate of banks (called "the Bank" with a capital B) has one type of asset, which is the aggregate of loans to households and non-bank firms. Its liabilities are the aggregate of deposits. We initially abstract from the presence of a government, a central bank and high-powered money (reserves). We also until further notice ignore the Basel rules for risk weighting of different types of assets. We assume that there is no currency in circulation, so that money stock is simply the aggregate of deposits. Until further notice we assume that all lending is done by banks.

In later stages we introduce a central bank, reserves and risk-weighting, and the systemic impact of highly-geared non-bank financial institutions. Finally the phenomenon of banks selling loans onwards is examined.

2 A generic bank model without a Central Bank

The model is defined in continuous time. "\$" is used as a symbol for one unit of generic money. Brackets are used to signify denomination. Denomination for money flows is then [\$/*y*] (where "*y*" means "year"), and for stocks it is [\$]. Empty brackets [ ] signify a dimensionless entity. All monetary entities are in nominal terms. We define the following variables and parameters:

A(t), L(t) = assets, liabilities [$]. Note that L = money stock, as stated above

k = the required minimum capital/asset ratio [ ]

i_A = interest rate on assets (= loans) [1/y].

i_L= interest rate on liabilities (= deposits = credit money) [1/y]; $i_L < i_A$

i = "equivalent net interest rate" (explained below) [1/y]

r = loan repayment rate [1/y]. r is defined such that the loan repayment flow is proportional to the loan; we have $rA(t)$. This is unconventional, since repayment schemes are usually of the bond- or annuity type. But for our analysis this is acceptable.

$\lambda =$ loss rate [1/y]; a flow $\lambda A(t)$ is written off due to borrowers defaulting on their loans

$\beta =$ share of net interest income that is left for banks after they have paid their expenses including wages [ ]; $0 < \beta < 1$

l = flow of new loans [$/$y$]

We assume that banks lend as much as they are allowed to, i.e. they (manage to) stay at the lower limit k. This presupposes that the general mood among lenders and borrowers is not very pessimistic (when both sides or one side hold back). Then we have

$$k = \frac{A-L}{A}, \text{ or } A - L = kA, \text{ or } L = (1-k)A \tag{1}$$

(A variable's dependency on time t is here and in the following mostly implied and not indicated.)

The differential equation for asset change is

$$\dot{A} = l - \lambda A - rA \tag{2}$$

(We use dot notation for time derivatives; $\dot{A}$ is the same as $\frac{dA}{dt}$.)

The differential equation for liability change is

$$\dot{L} = l - rA - \beta(i_A A - i_L L) \tag{3}$$

Note that net Bank income $\beta(i_A A - i_L L)$ appears with a minus sign in $\dot{L}$, not with a plus sign in $\dot{A}$: net income to the aggregate of banks appears in the form of reduced deposits. Using the rightmost equation in (1), the last term in (3) becomes

$$-\beta[i_A - i_L(1-k)]A = -\beta i A, \text{ where } i = i_A - i_L(1-k), \tag{4}$$

Here *i* may be termed an "equivalent net interest rate". In *i* we now also include all types of fees on borrowers and depositors. These fees are assumed proportional to *A* and *L*, and may therefore be considered to represent an extra interest-like income for the Bank.

We substitute (4) in (3), and substitute for *L* with the rightmost variant of (1). This gives

$$\dot{A}\,(1-k) = l - rA - \beta iA \tag{5}$$

We subtract (5) from (2) and divide the result by *k* on both sides. This gives

$$\dot{A}(t) = \frac{\beta i - \lambda}{k}\,A(t), \tag{6}$$

which has the solution

$$A(t) = A_0 e^{gt}, \tag{7}$$

where we have introduced the aggregate assets growth rate

$$g = \frac{\beta i - \lambda}{k}, \tag{8}$$

and A_0 is the value of the Bank's assets at $t = 0$.

We note that g increases with the equivalent net interest rate and the Bank's profit share of income β, which is not surprising. A more interesting result is *that the growth rate is inversely proportional to the capital/asset ratio.* This growth rate also applies to the money stock *L*, since we have $L = (1-k)A$ from (1), and may differentiate this on both sides:

$$\dot{L}(t) = (1-k)gA_0e^{gt} = \frac{(1-k)\beta i - (1-k)\lambda}{k}\,A_0e^{gt} \tag{9}$$

We observe that endogenous credit[i] money growth will occur for $k < 1$. This is (as far as this author knows) a non-recognised side effect of a Basel-type regime.

Using (3), (4), (7) and (8), the Bank's net lending flow $l - rA$ is

$$l - rA = \dot{L} + \beta(i_A A - i_L L) = (1-k)gA + \beta iA = \frac{\beta i - (1-k)\lambda}{k}\,A_0e^{gt}, \tag{10}$$

which we will return to further below. Comparing (9) and (10), we note that the net lending flow is somewhat larger than the money creation flow $\dot{L}$, which is reasonable since the Bank also lends its own profit flow, and this is not accompanied by net creation of money.

The Bank's profit flow is

$$\beta i A(t) = \beta i A_0 e^{gt} \tag{11}$$

That this flow grows steeper the lower the capital/asset ratio is, explains banks' wish to operate at the limit k.

We try a set of numerical values to check out g:

$$i_A = 0.07, i_L = 0.03, \beta = 0.2, \lambda = 0.005 \text{ and } k = 0.08 \tag{12}$$

This gives g = 4.35% per year, which is within a reasonable magnitude. Note from (8) that g is very sensitive to λ; we need $\beta i > \lambda$ for assets to grow at all.

3 Including a Central Bank and a government

We now introduce a central bank (CB) and reserves. It is assumed that banks' deposits with the CB fluctuate with government spending and taxation, and grow due to interest paid for these deposits. Government bonds are in their entirety assumed to be held by banks, and considered to be equivalent with interest-bearing deposits at the CB.

Any CB where the country in question has its own national currency (as opposed to for instance the Euro zone), is constitutionally an arm of the government — the "independence" of CB's that has become the rule in later years is a political construct that may be reversed by the government or national assembly. Thus a government's "debt" that builds up with its CB through deficit spending in excess of the income from selling bonds, is only an accounting and legal convention. In line with this, the government is in this paper considered to be able to spend freely (and thus net create money) by debiting its account at the CB. A possible real-economic impact of this type of net HPM creation is of course inflation, but that is no more an issue than the possible inflationary effect of banks' exponential net money creation, established in the previous section.

We distinguish between risk weight of reserves (zero) and all other assets in the Basel rule (these are for simplicity assigned a 100 % risk weight). We now define:

R(t) = reserves = the Bank's deposit with the CB = high-powered money (HPM) [$]. We assume that $R > 0$. The Bank's total financial assets are now $A + R$, where A = loans as before.

i_R = interest rate on HPM to banks from the CB. This is an exogenous monetary control variable for the system [1/*y*].

$\gamma(t)$ = government net spending (= deficit) flow. It may be negative, corresponding to a surplus budget. γ is an exogenous fiscal control variable for the system [$/*y*].

The derivation of the growth equation may now be done along the same lines as in the previous section. Applying the Basel rule that risk weights shall only apply in the denominator and that reserves R are assigned zero weight, we get

$$k = \frac{A+R-L}{A+0\,.R}, \text{ or } A + R - L = kA, \text{ or } L = (1-k)A + R \qquad (13)$$

The differential equation for non-reserve asset change is

$$\dot{A} = l - \lambda A - rA \qquad (14)$$

The differential equation for change in the Bank's reserve part of assets is

$$\dot{R} = i_R R + \gamma \qquad (15)$$

The differential equation for liability change now becomes

$$\dot{L} = l - rA - \beta(i_A A - i_L L) + \gamma, \qquad (16)$$

where the second last term in (16) may, using (13), be written as

$$-\beta[i_A - i_L(1-k)]A + \beta i_L R = -\beta iA + \beta i_L R, \text{where } i = i_A - i_L(1-k) \text{ as before.} \qquad (17)$$

Using (17), (16) becomes

$$\dot{L} = l - rA - \beta iA + \beta i_L R + \gamma, \qquad (18)$$

We substitute for $\dot{L}$ in (18), using the rightmost variant of (13), and also substitute (15) for $\dot{R}$. This gives

$$\dot{A}(1-k) + i_R R + \gamma = l - rA - \beta iA + \beta i_L R + \gamma \qquad (19)$$

where γ cancels out on both sides and $i_R R$ may be moved to the right side:

$$\dot{A}(1-k) = l - rA - \beta iA + \beta i_L R - i_R R \qquad (20)$$

We subtract (20) from (14) and divide the result by k on both sides. This gives

$$\dot{A}(t) = gA(t) - \frac{\beta i_L - i_R}{k} R(t), \text{ where } g = \frac{\beta i - \lambda}{k} \qquad (21)$$

Compare this to equation (6). The growth equation has a similar structure, but is influenced by the additional variable R, whose growth is decided by the two control variables γ and i_R in (15). We have established that A and L will grow exponentially. For the system to uphold the balance between monetary aggregates, R must grow at the same rate. If R is depleted, banks will increasingly lack reserves for their transactions with each other, with the government and with the public (for notes and coins). This means that γ must be positive, which corresponds to persistent government deficit spending — not through bond sales but through net HPM creation. More precisely, γ and i_R should be such that they, via (15), achieve $R = \theta A$, where the parameter θ is somewhere in the range $0 < \theta < 1$. Then (21) becomes

$$\dot{A}(t) = gA(t), \text{ with } g = \frac{\beta i - \lambda - \theta(\beta i_L - i_R)}{k} \tag{22}$$

Note that the assumption here is that the government allows the Bank to decide debt growth only constrained by a Basel-type C/A-ratio requirement, and then accommodates by ensuring a similar reserve growth. One might instead take the position that government should decide the rate of debt growth. This might be easier to achieve in a 100% reserve system:

3.1 A special case: a 100% reserve system

We will now consider the case where reserves have to mirror deposits 100%. This is the famous proposal put forward by, among others, Irving Fisher during the Great Depression. It has been persistently (re)launched to this day by individuals or groups that are more or less considered to belong to the economics "fringe", and has (in this author's opinion: undeservedly) not been considered worth serious discussion by the academic mainstream.

In our model, 100% reserves correspond to $L = R$. Since reserves are not weighted in the denominator of the capital/asset ratio as defined in the Basel rules, this corresponds to $k = 1$:

$$k = \frac{A+0}{A} = 1 \tag{23}$$

Using (17) and $k = 1$, (21) becomes

$$\dot{A}(t) = (\beta i_A - \lambda)A(t) - (\beta i_L - i_R)R(t) \tag{24}$$

The right term in (13) differentiated becomes simply

$$\dot{L} = \dot{R} \tag{25}$$

Equation (22) now becomes

$$\dot{A}(t) = gA(t), \text{with } g = \beta i_A - \lambda - \theta(\beta i_L - i_R) \tag{26}$$

We observe that A growth will be much slower, cet. par. The value of k has a very strong impact. But k may obviously not be changed abruptly, as one may do with interest rates (see also conclusions, section 6).

If we choose to pay banks zero interest on their reserves; $i_R = 0$, all new money is created via government spending — no new money is created via bank lending. Thus the expression often used by the proponents of a 100% reserve system: *"money is spent, not lent, into existence"*.

4 Debt crisis?

The aggregate debt service burden on firms and households also grows at the rate g with the above model,

$$(i+r)A(t) = (i+r)A_0 e^{gt} \tag{27}$$

If $\gamma = 0$, cf. (13), money stock L grows at the same rate. If the government runs a persistent deficit, $\gamma > 0$, money stock grows steeper than A. Introducing money velocity $v[1/y]$, we have

$$Y = Lv \tag{28}$$

Assuming that v is fairly constant[ii] , nominal GDP, Y [\$/y], also grows pari passu with, or steeper than, A. Then nominal debt (burden) growth is lower than output growth. Abstracting from possible inflation issues, $\gamma \geq 0$ gives a long-term trajectory that is robust against debt crisis.

But debt crises occur in the real world. The model should be modified to account for this. We will focus on the phenomenon of debt persistently increasing more than GDP, which actually has occurred in OECD countries, and which is possibly the most fundamental (and more basic than the housing bubble and new complicated financial instruments) cause of today's financial crisis. See figure 1.

We introduce an alternative type of lender which has the property that *its assets may accumulate without creation of net credit money:* assume a "Moneylender" operating in parallel with the Bank. We define the Moneylender as an aggregate, consisting of — among others — investment banks and funds, which borrow money and then lend (re-invest) this money at higher interest/return rates.

The "Bank" and "Moneylender" are both aggregates. Transactions within each aggregate net to zero. But the Moneylender have an aggregate deposit with the Bank. The Moneylender does not create deposits (money), since the borrower's increased deposit is cancelled by the Moneylender's decreased deposit — as opposed to the Bank, which creates net deposits, cf. (9). When loans are issued by the Moneylender, debt grows but money does not.

We develop the model as follows:

$A(t), L(t) =$ assets, liabilities as already discussed. L is the aggregate liabilities of the Moneylender, but not money.

i_A = interest rate on assets (= loans), similar to the Bank.

i_L = interest rate on liabilities, i.e. on the Moneylender's debt, $i_L < i_A$.

λ = the Moneylender's loss rate; a flow $\lambda A(t)$ is written off due to borrowers defaulting, as in the Bank model.

Other parameters have the same meaning as in the Bank case. The capital/asset ratio k is also here kept constant, so that the Moneylender's liabilities grow proportionally with assets. We have $L = (1-k)A$, from (1). This decides the net borrowing of the Moneylender, assuming that agents want to invest that amount with it at the interest rate i_L. Since there is no minimum k for the Moneylender mandated by the government, the only thing that keeps it from operating with a k very close to zero, is its evaluation of risk. A common concept used is gearing, which we will call φ.

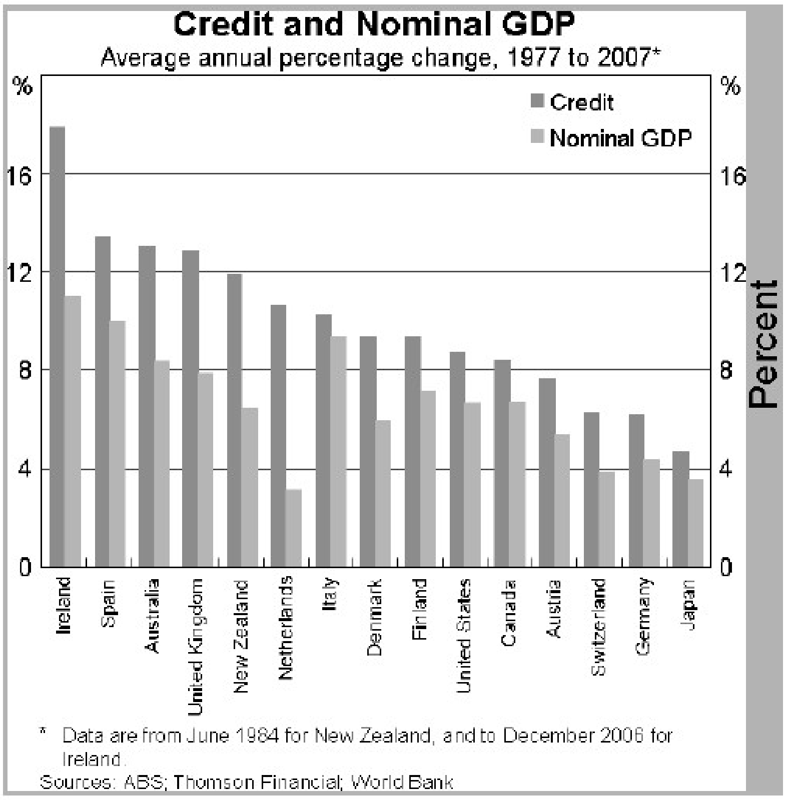

Figure 1: Debt outruns GDP in OECD countries *(courtesy: Reserve Bank of Australia)*

We have $$\varphi = \frac{L}{A-L} = \frac{(1-k)A}{A-(1-k)A} = \frac{1-k}{k}, \text{ or } k = \frac{1}{1+\varphi} \tag{29}$$

We note that for small k, φ is essentially its inverse. A moneylender with a gearing of 30 (not uncommon) has a capital/asset ratio of 3.2%.

We will now develop the growth model for the Moneylender. We assume the Moneylender holds no money, so that any money borrowed or received (as part of interest income or repayment) is immediately lent[iii]. The Moneylender's profit flow is, similar to the Bank case,

$$\beta[i_A - i_L(1-k)]A = \beta iA, \text{where } i = i_A - i_L(1-k) \tag{30}$$

The lending flow from the Moneylender is the sum of its profit flow and its net borrowing flow which it passes on as new loans ("net" in the sense: difference between borrowing flow and repayment flow to creditors). The profit flow plus the net borrowing flow minus losses on extended loans must equal the assets increase rate, $\dot{A}$. We have

$$\beta i A + \dot{L} - \lambda A = \beta i A + (1-k)\dot{A} - \lambda A = \dot{A} \tag{31}$$

leading to

$$\dot{A}(t) = \frac{\beta i - \lambda}{k} A(t), \tag{32}$$

which is similar to (6).

k may be chosen very small as mentioned above. By this the Moneylender, following (8), will have a steeper asset and profit growth than the Bank, except when or if the loss rate λ increases significantly. But in the long pre-crisis phase λ is small. Then the Moneylender will persistently increase its share of the financial market in relation to the Bank. This is a possible explanation for the disproportionate growth of aggregate debt related to credit money and GDP, and lays the ground for a grave crisis, even if it may take decades to develop.

5 Banks selling loans onwards

We have discussed the case where the Moneylender borrows to extend loans. Another common phenomenon in today's financial environment is when a bank sells an existing loan to a moneylender. We shall develop a simple model for this, and discuss what sort of incentives there are for such activity.

We disaggregate the Bank model, so that from now on we are considering an individual bank. This is necessary since a reserve (HPM) increase to one bank is accompanied by a corresponding decrease for another bank — abstracting from government net spending or taxation, and from CB open market operations. The question to be examined is whether a bank — abiding by a Basel-type capital/asset requirement — may achieve a comparable growth in profit flows by holding a certain share of its assets in the form of lower-yield reserves. We assume that such reserves are acquired by selling loans. We have the following entities:

$R(t)$ = the bank's reserves = the bank's deposit with the CB

$\tilde{A}(t)$ = the bank's total financial assets $= A(t) + R(t)$, where $A(t)$ = all other financial assets than reserves

$\tilde{k} =$ an "equivalent" minimum capital/asset ratio [ ], explained below, $\tilde{k} < k$

i_R = interest rate on HPM to banks from CB [1/*y*]

l_1 = flow of new bank loans sold on to the Moneylender [ ]; $0 < l_1 < l$

$\mu =$ share of notional loan value [ ], received for a loan sold on to the Moneylender, fees included, $0 \ll \mu \leq 1$. This implies that the Moneylender may get a rebate even when bank fees are included.

ρ : the bank is assumed to follow a strategy of keeping a constant ratio $R/A = \frac{\rho}{1-\rho}$, see (33) below. ρ is the HPM share of the bank's total financial assets $\tilde{A}$.

Using (13) and the above definitions, we have

$$k = \frac{\tilde{A}(t)-L}{A+0.R} = \frac{A+R-L}{A} = \frac{A\left(1+\frac{\rho}{1-\rho}\right)-L}{A}, \text{or } \frac{A-L}{A} = k - \frac{\rho}{1-p} = \tilde{k} \qquad (33)$$

Compare this with (1). We have a decrease in k to a lower minimum capital/asset ratio $\tilde{k}$, as long as we confine ourselves to the regulatory requirements on the non-reserve part of assets A. $\tilde{k}$ will be used as an intermediate parameter which will be dispensed with later on. From (33) we have

$$L = \left(1 - \tilde{k}\right)A \qquad (34)$$

We will now do a similar, but somewhat more complex, derivation as that leading to (7). The differential equation for non-reserve-asset change is now

$$\dot{A} = (l - l_1) - \lambda A - rA \qquad (35)$$

The differential equation for liability change is

$$\dot{L} = l - rA - \beta iA \qquad (36)$$

where we have again introduced an i,

$$\beta iA = \beta(i_A A - i_L L) = \beta\left[i_A - i_L\left(1 - \tilde{k}\right)\right]A \qquad (37)$$

For reserves R we have

$$\dot{R} = \mu l_1 + \beta i_R R = \frac{\rho}{1-\rho}\dot{A} = \mu l_1 + \beta i_R \frac{\rho}{1-\rho}A \qquad (38)$$

which may be solved for l_1,

$$l_1 = \frac{\rho}{(1-\rho)\mu}\left(\dot{A} - \beta i_R A\right) \tag{39}$$

We substitute (34) into (36), and subtract both sides of the result from (35). This gives

$$\tilde{k}\dot{A} = (\beta i - \lambda)A - l_1 \tag{40}$$

We substitute (39) for l_1 and $k - \frac{\rho}{1-\rho}$ for $\tilde{k}$, and solve for $\dot{A}$,

$$\dot{A} = \frac{\beta\left(i + \frac{\rho}{(1-\rho)\mu} i_R\right) - \lambda}{k + \frac{\rho}{1-\rho}\left(\frac{1-\mu}{\mu}\right)} A \tag{41}$$

which has the solution

$$A(t) = A_0 e^{g_2 t} \tag{42}$$

where we have again introduced an aggregate assets growth rate

$$g_2 = \frac{\beta\left(i + \frac{\rho}{(1-\rho)\mu} i_R\right) - \lambda}{k + \frac{p}{1-\rho}\left(\frac{1-\mu}{\mu}\right)} \tag{43}$$

A_0 is the value of the bank's non-reserve assets at $t = 0$. Note that l_1 has been eliminated in the process and thus plays the role of intermediate variable. This is a consequence of the model and the assumptions made about the other parameters. l_1 may be calculated by substituting (42) into (39).

Total assets have the same growth rate and — using (33) — follow the growth equation

$$\tilde{A}(t) = A_0\left(1 + \frac{\rho}{1-\rho}\right) e^{g_2 t} \quad = A_0 \frac{1}{1-\rho} e^{g_2 t} \tag{44}$$

Equation (43) is fairly complicated, so we will discuss it based on the graphs in figure 2 next page. We use the parameter values (12), with additional parameters chosen as

$i_R = 0.03$, ρ in the interval [0.01 0.5], μ in the interval [0.3 0.99]. (45)

We observe from the graphs that above a certain and relatively low value of μ (the value may be shown to be

$$\mu = 1 - \frac{k\beta i_R}{\beta i - \lambda}, \tag{46}$$

which is 0.84 using the values (12)) the asset growth for the bank is now steeper than for the case with no selling of loans onwards; we have $g_2 > g$. Furthermore, this bank gains a higher-quality asset portfolio, since a share of its assets carry no risk. In a situation with increasing risk (i.e. λ is on the rise, or expected to rise), this is an added incentive.

We conclude that a bank, due to the rule of zero weighting of reserves in the capital/asset ratio denominator, is given a special incentive to sell its extended loans onwards. In the aggregate however, all banks cannot do this due to reserves being a limited resource. For the Moneylender, the incentives are that loans are offered in a finished package and a possible loan purchase rebate, which is here expressed by the parameter μ possibly being < 1. Thus both parties have incentives.

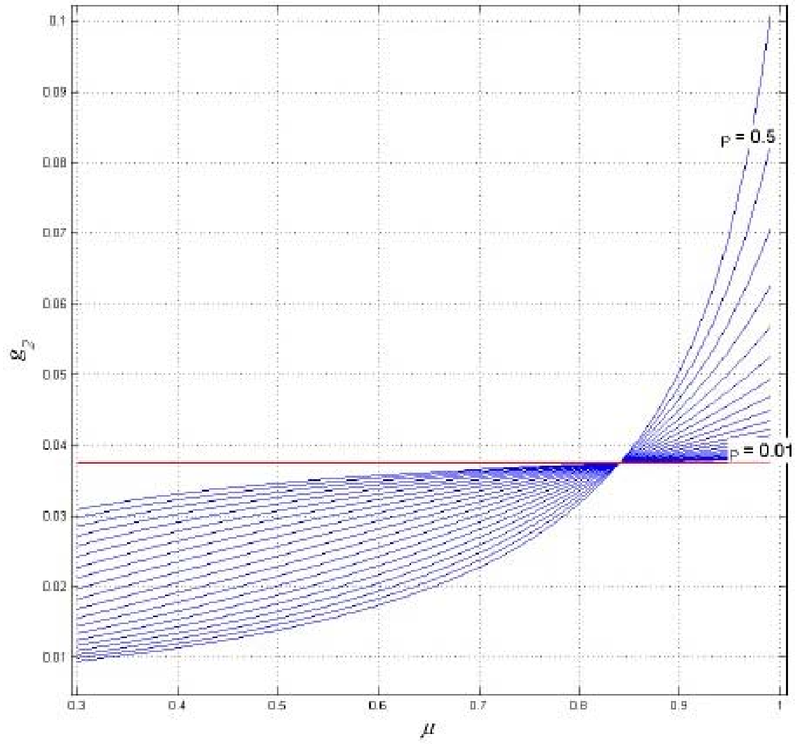

Figure 2: Asset growth rate g_2 as a function of μ and ρ. The horizontal line corresponds to g in (8)

6 Conclusions

We have, based on a few generic and simplified models, tried to chart the basic mechanics of debt and money growth. A clear understanding of this is necessary to enable meaningful discussion of policy proposals related to stabilising financial systems.

We draw the following conclusions:

- Under today's Basel regime, credit money grows endogenously, at an exponential rate. This crucial property should be recognised.

- Debt build-up at a steeper rate than GDP should be avoided, and curbed through regulatory measures.
- Debt creation by non-bank financial institutions is dangerous in the long run since credit money is not created along with such loans.
- The required minimum capital/asset ratio for banks may be used to control debt growth and could be increased from today's low value. But it can only be changed gradually — it cannot be employed as a short-term regulating instrument.
- Fiscal policy in the form of persistent deficit spending is necessary in an economy where nominal GDP grows. The deficit flow may be varied around its growing exponential reference path and by this also function as a short-term regulating instrument.
- The banks should not receive any interest on their reserves. With $i_R = 0$, a strategy of controlling credit money and debt growth through deficit spending has a stronger effect.
- The 100% reserve requirement proposal should be seriously discussed. Then the endogenous bank debt and credit money growth rate will be approximately twelve times lower than in today's situation. Such a system will give the government a very potent input via its deficit spending, for regulating money and debt growth.
- The extending of bank loans and then selling them to non-bank financial institutions should be curbed.

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[i] We distinguish between credit money which is created through bank lending, and Central Bank Money (reserves, base money). See next section.

[ii] While v is difficult to pin down and fluctuates within some band, it is by this author considered a crucial behavioural variable for agents in the economy. It is an expression of confidence and will decrease sharply when the economy is impacted by a negative shock, like a stock market crash or a financial crisis. This leads to decreased spending, lending and investment and may set in motion a dangerous downwards spiral where v decreases further.

[iii] This assumption is acceptable for the discussion here. Introducing a buffer stock of money in the Moneylender model does not change long-range accumulation dynamics.

A long-range Financialisation Mechanism

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Abstract

One prominent characteristic of the decades-long run-up to today's global financial crisis, is the increasing relative size of debt and the financial sector in countries' economies. A mechanism explaining one aspect of this, due to financial accumulation through non-financial capitalists' lending, is explored. The exercise also leads to the conclusion that in the aggregate, financial accumulation by capitalists through the alternative option of real-economic investment, is not feasible.

1 Introduction

We wish to examine an aspect of debt growth dynamics of a macroeconomy, on a decades-long time scale. In this scenario, non-financial firm (often abbreviated "NFF" from now on) owners/capitalists have a choice between using the non-consumed part of their profits to extend loans, or for investment. The conjecture is that in the very long run, lending will be more profitable than investing for NFF capitalists, and therefore preferred. For simplicity, we assume that they lend in the form of buying bonds from (other) non-financial firms. Thus, in the aggregate the NFF "capitalist" lends to "himself" (we use the male sex for this creature). When bonds are paid back, the principal is re-lent after a short time lag. Capitalists consume some of their profits, workers spend all their wages.

Another part of financial activity, lending from licensed banks, which have the privilege of increasing credit money as part of their lending, is assumed to have occured to a certain level, and then frozen. Thus the stock of circulating bank-created money is assumed constant, and by implication also debt to licensed banks. But the related interest burden on NFF's is ignored here. The presence of non-bank financial institutions is also abstracted from, except that they and licensed banks manage the lending flows from NFF capitalists (owners). The fees to banks for this are are assumed only to be used for banks' expenses and wages, and not for bank lending. To sum up these assumptions, the only financial accumulation allowed in the system, is through NFF capitalist activity. This is obviously an unrealistic scenario in the sense that the main factors for a long-term financial crisis are ignored. But the point of the exercise is to check whether there is also an incentive for NFF capitalists to gradually behave more like lenders than investors in the long run. We will return briefly to the the role of banks in the conclusion.

Further assumptions are that we do not account for losses on capitalists' bonds and on their investment. And there are no stock market or housing bubbles in this model, only the long-term financial accumulation process (such bubbles have faster dynamics and are seen as excursions on top of the long-term debt-growth path.The term "bubbles" should not be used for the dynamics to be examined here).

We wish to answer two main questions:

Is it, as time goes, more attractive to recycle profits to owners of non-financial firms as loans instead of investing them – and how may this unfold, also when part of these profits stem from NFF activity?

– If the answer is yes to the first question, we have a possible explanation for one of the mechanisms behind the financialisation process.

2 The model

Readers are recommended to check out the block diagram in figure 1 in the appendix. This type of model representation is much used in the control systems community, but may be easily understood also by academics with other backgrounds. It is quite useful when one has become somewhat familiar with it.

This paper however, is written mainly for an economist audience[1], and therefore the model will be developed based on equations only. We will see that it boils down to a simple four-state linear dynamical system, where it furthermore turns out that three states may be ignored at the time scale (decades) that we are considering, resulting in a first order system.

The model is defined in continuous time. "\$"is used as a symbol for the monetary unit. Brackets are used to signify denomination. Denomination for money flows is then [\$/y] (where "y" means "year"), and for stocks it is [\$]. Empty brackets [ ] signify a dimensionless entity. All monetary entities are in nominal terms. We define the following variables and parameters:

T_F = First order time lag for the aggregate of non-financial firms [y].

T_K = First order time lag for the aggregate of capitalists, who own the NFFs [y].

T_W = First order time lag for the aggregate of (non-saving) workers/households [y].

M = total money stock in circulation, here assumed constant [\$].

$Y_d(t)$ = aggregate demand for NFF products and services [\$/y].

$Y_{dn}(t)$ = aggregate demand remaining after NFFs' debt service [\$/y].

$Y_o(t)$ = aggregate NFF output [\$/y].

$M_F(t), M_K(t), M_W(t)$ = money stock in circulation [\$]: held by firms, capitalists and workers, respectively. We have $M = M_F + M_K + M_W$.

$D(t)$ = NFF capitalists' financial assets = bonds [\$].

i = interest rate on bonds [$1/y$].

r = loan (bond) repayment rate [$1/y$]. r is defined such that the loan repayment flow is proportional to the aggregate loan, we have $rD(t)$. This is unconventional, since bond repayment schemes are usually that the entire principal is paid at maturity. But in the aggregate, this is acceptable.

σ = share of interest income that is left for capitalists after fees for managing lending and payment flows [ ]; $0 << \sigma < 1$.

$F(t)$ = flow of fees for managing the lending flows [\$/y].

π = share of NFF output that capitalists receive [ ]; $0 < \pi < 1$. The workers' share is then $1 - \pi$.

s_K = share of capitalists' profit flow that is not used for consumption [ ]; $0 < s_K < 1$.

s_{KL} = share of capitalists' non-consumption flow that is used for buying bonds [ ]; $0 < s_{KL} < 1$. The share $1 - s_{KL}$ is then used for investment.

$L(t)$ = flow of new loans [\$/y], includes re-lending of re-paid bonds.

$I(t)$ = flow of new investment [\$/y]

$W(t)$ = workers' wages flow [\$/y]; workers are assumed to use their entire wages for consumption $= C_W(t)$, after a lag T_W.

$C(t)$ = aggregate consumption flow [\$/y].

$C_K(t)$ = capitalists' aggregate consumption flow [\$/y], we have $C = C_W + C_K$.

$\Pi_R(t)$ = profit flow from NFF activity to capitalists (= NFF owners) [\$/y].

$\Pi_L(t)$ = profit flow from interest paid to NFF owners on their bonds [\$/y].

[1]Thanks to Carl Chiarella for advice on making this paper (hopefully) more readable to economists.

$\Pi(t) = \Pi_R(t) + \Pi_L(t)$ = aggregate profit flow [\$/y].

$\Pi_o(t)$ = lagged profit flow emerging from aggregate of capitalists, used for sconsumption or saved [\$/y].

We start the presentation with the input/output dynamics of three defined aggregates: non-financial firms, capitalists (who own these firms), and workers. These dynamics may be explained via the firm aggregate which has time lag T_F – properties for capitalist and worker aggregates are similar, except for time lags being T_K and T_W. We assume that all lags are of the first-order type, corresponding to a differential equation (using the aggregate of firms)[2]:

$$T_F \dot{Y}_o = -Y_o + Y_{dn} \tag{1}$$

The money held at any time[3] by the aggregate of firms, must satisfy

$$\dot{M}_F = -Y_o + Y_{dn} \ , \tag{2}$$

so that

$$Y_o = \frac{M_F}{T_F} \ (= M_F v_F) \tag{3}$$

where v_F is firm money velocity [$1/y$] (but we will not use v_F in the following). With such input-output dynamics, a stepwise change in the input flow gives an output response that adjusts asymptotically to the input in the form of a stable exponential with a lag T_F. This type of subsystem (= aggregate) is of order 1. As mentioned above, the capitalist and worker subsystems have the same properties. Equations for the capitalist aggregate are:

$$T_K \dot{\Pi}_o = -\Pi_o + \Pi \tag{4}$$

with

$$\Pi_o = \frac{M_K}{T_K}, \tag{5}$$

and for workers:

$$T_W \dot{C}_W = -C_W + W \tag{6}$$

with

$$C_W = \frac{M_W}{T_W} \tag{7}$$

In addition to these three differential equations, we have one for debt growth:

$$\dot{D} = s_K s_{KL} \Pi_o \tag{8}$$

The total system is therefore of order 4. The lags T_F, T_K, T_W will be of the magnitude weeks or at most months. On the decades-long time scale we are considering for the financial accumulation process, we may therefore ignore the effect of T_F, T_K, T_W for growth dynamics. Then

the outputs for the three aggregates may be considered equal to their inputs. (9)

We will exploit this further below. But first we need to complete the system by listing the remaining (non-differential-) equations.

The profit flowing to capitalists is:

$$\Pi = \Pi_L + \Pi_R = \sigma i D + \pi Y_o \tag{10}$$

For wages to workers we have their share of output:

$$W = (1 - \pi) Y_o \tag{11}$$

[2] We use dot notation for time derivatives. And a variable's dependency on time t is here and in the following mostly implied and not indicated.

[3] Obviously circulating money stock must reside somewhere at any time. And for money velocity not to be infinite, money has to stay with an aggregate for a finite time. This is accounted for by the time lag in the first order differential equation representation.

For demand to firms after firms' servicing debt we have:

$$Y_{dn} = Y_d - (i + r)D \tag{12}$$

And demand before debt service is

$$Y_d = C + I + L + F \tag{13}$$

where

$$C = C_K + C_W = (1 - s_K)\Pi_o + C_W, \tag{14}$$

and

$$I = s_K(1 - s_{KL})\Pi_o, \tag{15}$$

and

$$L = rD + \dot{D}, \tag{16}$$

and

$$F = (1 - \sigma)iD \tag{17}$$

This completes the set of equations describing the system.

We now wish to find the solution for the NFF profit flow $\Pi_R(t)$. The conjecture is that it will shrink. We have

$$M = M_F + M_K + M_W, \tag{18}$$

so that any increase in money stock held by one aggregate must mean a corresponding decrease in the sum for the two other aggregates. By the assumption (9) that flow in may be considered equal to the flow out, we have for the aggregate NFF capitalist that the flow in is also equal to money held divided by the time lag. Using this, (10) and (5), we get:

$$\Pi = \Pi_L + \Pi_R = \sigma iD + \pi Y = \Pi_o = \frac{M_K}{T_K}, \tag{19}$$

where we have now, in accordance with (9), introduced the term $Y = Y_o = Y_{dn}$.

For the worker/household sector (11) and (7) give

$$W = (1 - \pi)\, Y = C_W = \frac{M_W}{T_W} \tag{20}$$

Using (9), and (3), (19), (20) to substitute for M_F, M_K, M_W in (18) we get

$$M = T_F Y + T_K(\sigma iD + \pi Y) + T_W\,(1 - \pi)\, Y \tag{21}$$

We solve for Y:

$$Y = \frac{M - T_K \sigma iD}{T_F + \pi T_K + (1 - \pi)\, T_W} \tag{22}$$

Our total system's dynamics are, with the simplification (9) and similar for the aggregates of capitalists and workers, described by just one differential equation. From (8) and (19) we have:

$$\dot{D} = s_K s_{KL} \Pi_o = s_K s_{KL}(i\sigma D + \pi Y) \tag{23}$$

In (23), we substitute (22) for Y:

$$\dot{D} = s_K s_{KL}(i\sigma D + \pi \frac{M - T_K \sigma iD}{T_F + \pi T_K + (1 - \pi)\, T_W}) = a + bD, \tag{24}$$

where

$$b = s_K s_{KL} i\sigma(1 - \frac{\pi T_K}{T_F + \pi T_K + (1 - \pi)\, T_W}) = s_K s_{KL} i\sigma \frac{T_F + (1 - \pi)\, T_W}{T_F + \pi T_K + (1 - \pi)\, T_W} \tag{25}$$

and

$$a = s_K s_{KL} \frac{\pi M}{T_F + \pi T_K + (1 - \pi)\, T_W} \tag{26}$$

We note the interesting property that the growth rate b will be higher the faster money passes through the capitalist aggregate, i.e. when T_K is small.

If we assume an initial value $D(0) = 0$, the solution to (24) is

$$D(t) = \frac{a}{b}\left(\mathrm{e}^{bt} - 1\right) = \frac{\pi M}{i\sigma\left[T_F + (1-\pi)\,T_W\right]}\left(\mathrm{e}^{bt} - 1\right), \tag{27}$$

which is exponential growth (minus a constant).

All other flows in the system may now be deduced based on (27). Eq. (10) gives the financial profit flow Π_L to the aggregate NFF capitalist:

$$\Pi_L = \sigma i D \tag{28}$$

Since Π_L is proportional to D, it also grows exponentially.

Using (19), (22) and (27), we have

$$\Pi_R = \pi Y = \pi \frac{M - T_K \sigma i \frac{\pi M}{i\sigma[T_F + (1-\pi)T_W]}\left(\mathrm{e}^{bt} - 1\right)}{T_F + \pi T_K + (1-\pi)\,T_W} = \bar{a} - \bar{b}\left(\mathrm{e}^{bt} - 1\right), \tag{29}$$

where

$$\bar{b} = \frac{\pi M T_K}{\left[T_F + \pi T_K + (1-\pi)\,T_W\right]\left[T_F + (1-\pi)\,T_W\right]} \tag{30}$$

and

$$\bar{a} = \frac{\pi M}{T_F + \pi T_K + (1-\pi)\,T_W} \tag{31}$$

Thus, while Π_L grows exponentially, Π_R falls at a similar rate – an unstable negative exponential path. The same is the case with Y and W.[4]

3 Conclusion

We have shown that even a simple, best-case linear model without accumulation by banks and non-bank financial institutions (NBFIs), gives an unsustainable dynamic, at least in the very long run, and that profits from NFF capitalists' lending, Π_L, crowd out profits from investing, Π_R.

If we also allow the aggregate NFF capitalist to change the weighting of his two outgoing flows, based on observing a long-term trend of his financial profits crowding out his NFF profits, this corresponds to letting s_{KL} increase with

$$\frac{\Pi_L}{\Pi_R + \Pi_L}, \tag{32}$$

making the crowding-out mechanism stronger.

Additionally, in a more realistic setting, with banks and NBFIs also in the system – visibly accumulating through their own lending – one should expect the NFF capitalists to get an additional incentive to increase s_{KL}. Increasing s_{KL} – for whichever reason – will make the debt growth process steeper than exponential, and even less sustainable.

Another conclusion follows from the above exercise is if we set $s_{KL} = 0$, i.e NFF capitalists use all their saving for non-financial investment, and nothing to extend loans. Then there is no exponential debt growth. This means that the NFF capitalists *cannot accumulate financially in the aggregate.* Investment is a zero-sum game for them. This seems to be a fairly dramatic conclusion ....

[4] This does not happen in the real world, however. The reason for this is that bank lending, abstracted from in this paper, increases M, which is held constant here. So we have race between long-run exponential debt build-up, which is unsustainable, and exponential money growth on the same time scale, which, ignoring the question of inflation, ameliorates the situation. Bank money creation, however, is accompanied by bank loan exponential build-up, which adds to the debt burden discussed here.

Appendix: A block diagram representation

The model earlier described through a set of equations, is shown in figure 1 as a block diagram. It may seem complex at a first glance, but it gives a better overview of the interactions in the system than a set of equations only.

The rules for interpreting this diagram are as follows:

1. The variable exiting a rectangular block, as long as the block only contains a coefficient, is the product of the variable entering the block and the expression within the block. Thus we have $\Pi_L = \sigma(iD)$.
2. If the block contains a "time lag", this signifies a first-order linear dynamic, as explained earlier.
3. If it contains an integrator symbol, this means that the output is the integral of the input.
4. A small dot upon a line signifies a *branching point.* This means that a variable is used as an input to two other parts of the system. Example: we have Y_o being used both for profits and wages, $\Pi_R = \pi Y_o$ and $W = (1 - \pi)Y_o$.
5. A small circle at an intersection of lines signifies a *summation point*: the variable associated with an arrow leaving a circle is the sum of variables associated with arrows entering the circle. Thus we have $\Pi = \Pi_R + \Pi_L$. An arrowhead with a minus sign associated with it, means that the corresponding variable is to be subtracted in the summation.

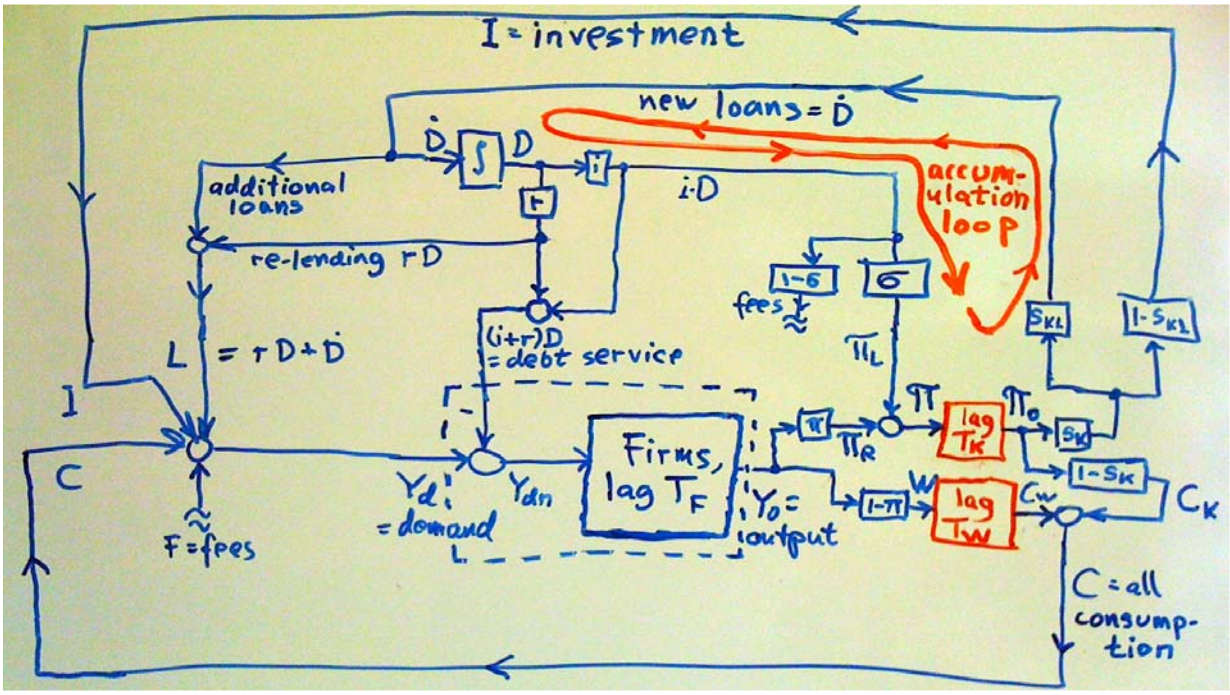

Figure 1:

Capitalists can enjoy a persistent profit flow in an economy with no injection of fresh money

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Abstract

There is a large and elaborate literature in economics about the (in)feasibily of capitalists in the aggregate enjoying a stable profit. Many conditions have been put forward for this to be feasible, for instance that extra ("fresh") money must be persistently added to the system, typically in the form of bank credit. This brief note argues that this is not necessary, and that this is very simple to conclude by using a continuous time linear model of a closed economic circuit. The paper also explains Marx' m-c-m' puzzle. Furthermore it argues that a constant – not falling – profit rate is feasible, and that this profit rate is independent of capitalists' share of output.

1 Introduction

In this brief note it will be argued that capitalists' profits and profit rate can be achieved and held stably at some value above zero in an economy, without the need of injection of additional money. The model to be presented is very simple. There is no foreign sector, no government and no banks, only households and non-financial firms in a circuit where the circulating amount of money is constant. All investment is done by capitalists recycling part of their profits. It will be seen that even such a simple model is sufficient to make the main arguments.

Output is shared between workers and capitalists. There are two lags in the model, one for firms and one for households. (Capitalists could have had a lag too, but this is not necessary for our argument.) Workers consume all their wages. Capitalists receive profits, consume a share of this, and invest the rest.

2 The model

Readers are recommended to check out the block diagram below, figure 1. All needed information is contained in that diagram.The model, however, will initially be presented based on equations. We will see that it boils down to a three-state linear dynamical system.

We define the following variables and parameters; denominations are indicated in brackets:

T_F = first order time lag for the aggregate of non-financial firms [y].

T_W = first order time lag for the aggregate of (non-saving) workers/households [y].

$Y_O(t)$ = aggregate income to be shared between workers and capitalists [$\$/y$].

$Y_D(t)$ = aggregate demand to firms [$\$/y$].

$K(t)$ = capitalists' accumulated capital [\$].

d = depreciation rate on K [$1/y$].

r = profit rate [$1/y$].

π = share of aggregate income that capitalists receive [ ]; $0 < \pi < 1$.
The workers' share is then $1 - \pi$.

s = share of capitalists' profit flow that is invested, not used for consumption [ ]; $0 < s < 1$.

$I(t)$ = flow of investment [\$/y]

$W(t)$ = workers' wages flow [\$/y];
workers are assumed to use their entire wages for consumption = $C_W(t)$, with a lag T_W.

$\Pi(t)$ = profit flow to capitalists [\$/y].

$C_K(t)$ = capitalists' aggregate consumption flow [\$/y].

$C(t)$ = total consumption flow [\$/y], we have $C = C_W + C_K$.

We start the presentation with the input/output dynamics of two defined aggregates: firms and worker households. These dynamics may be explained via the firm aggregate which has time lag T_F – properties for the worker aggregates is similar, except for the time lag being T_W. We assume that all lags are of the first-order type, corresponding to a differential equation (using the aggregate of firms):

$$T_F \dot{Y}_o = -Y_O + Y_D \tag{1}$$

The money held at any time[1] by the aggregate of firms, must satisfy

$$\dot{M}_F = -Y_O + Y_D \ , \tag{2}$$

so that

$$Y_O = \frac{M_F}{T_F} \ (= M_F v_F) \tag{3}$$

where v_F is firm money velocity [1/y] (but we will not use v_F in the following). With such input-output dynamics, a stepwise change in the input flow gives an output response that adjusts asymptotically to the input in the form of a stable exponential with a lag T_F. This type of subsystem (= aggregate) is of order 1. The worker household subsystem has the same properties. Equations are:

$$T_W \dot{C}_W = -C_W + W \tag{4}$$

with

$$C_W = \frac{M_W}{T_W} \tag{5}$$

The last differential equation is for capital accumulation and depreciation:

$$\dot{K} = -dK + I \tag{6}$$

which is not part of the circuit but only a measure of success seen from the capitalists' position.

To complete the model, we need some additional (non-differential-) equations. The profit flowing to capitalists is:

$$\Pi = \pi Y_O \tag{7}$$

For workers' wages we have their share of output:

$$W = (1 - \pi)\, Y_O \tag{8}$$

For demand to firms we have:

$$Y_D = C + I \tag{9}$$

where

$$C = C_K + C_W = (1 - s)\Pi + C_W, \tag{10}$$

and

$$I = s\Pi, \tag{11}$$

This completes the set of equations describing the system.

The model until now described through a set of equations, is shown in figure 1 as a block diagram:

[1] Obviously circulating money stock must reside somewhere at any time. And for money velocity not to be infinite, money has to stay with an aggregate for a finite time. This is accounted for by the time lag in the first order differential equation representation.

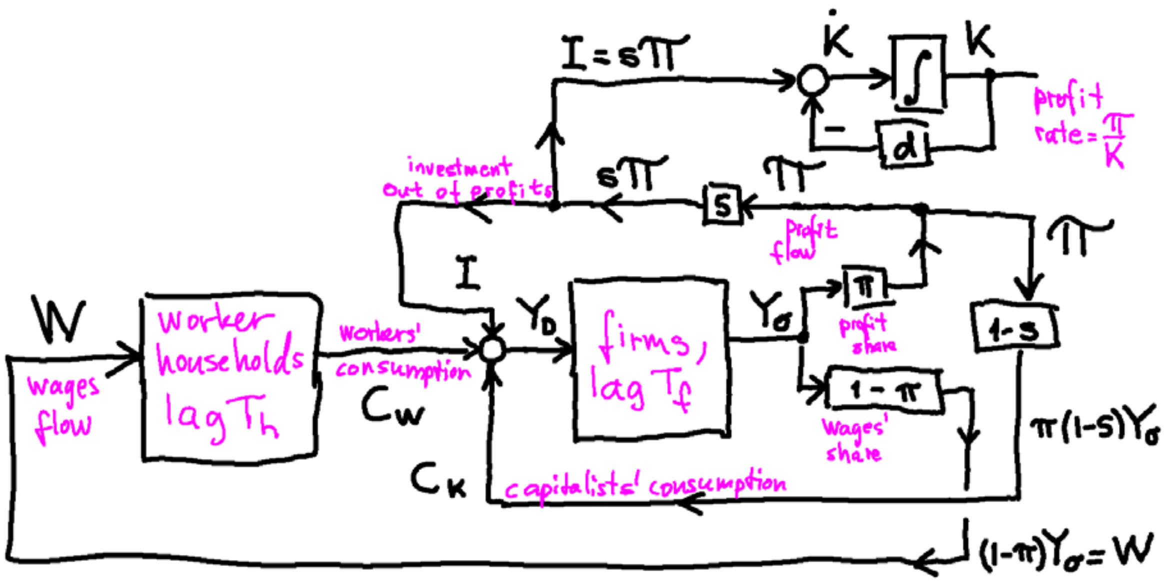

Figure 1: Block diagram for monetary circuit with capitalists and workers.

3 The argument

We now wish to check whether there exists a stationary system state where the aggregate profit flow is positive and constant, and what the condition(s) for this are. The reader is forewarned that the exercise is exceedingly simple, compared to the amount of research and discussion that has been done about this issue. This begs the question of whether I have completely misunderstood or overlooked something. Anyway, here goes:

Since the system is in equilibrium (= stationary), all derivatives are zero. From (2) we get the trivial result

$$Y_O = Y_D = Y \tag{12}$$

Capitalists invest the flow $I = s\Pi = s\pi Y$, cf. (11) and (7). They extract the flow $\Pi = \pi Y$. In Marxian terms, as long as $s < 1$, i.e. capitalists consume some of their profits, $m' > m$! It can't be simpler, and may be compared to the incredibly convoluted discussions among marxists and Marxian economists about this.

Turning now to the profit rate, and setting the left side of (6) to zero, we get

$$dK = I = s\Pi = s\pi Y \Longrightarrow K = \frac{s\pi Y}{d} \tag{13}$$

leading to the equilibrium profit rate

$$r = \frac{\Pi}{K} = \frac{\pi Y d}{s\pi Y} = \frac{d}{s} \tag{14}$$

We have the interesting result that capitalists' profit rate in equilibrium is *not* dependent upon their profit share π of output. And the higher capitalists' consumption out of profits is (i.e. small s), the higher the profit rate. Capitalists also decide the profit rate in the sense that it is higher with a higher depreciation rate.

So, may these questions be resolved *that* simply?
If not, why?

The fundamental mechanism behind the global financial crisis

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Abstract

Any economic system with interest on money lent has the potential to gradually develop a level of debt that leads to crisis. It is argued that the problem of "slowly exploding" debt is grave but (until recent events) largely ignored. The paper employs tools from System Dynamics.

Keywords: accumulation, instability, lending, compound interest dynamics

JEL classification: B50, C02, C60, C67

Figure 1 shows money circulation with firms and households in a macroeconomic setting. Households receive income flows of wages (W) and profits (Π) (it is assumed that firms are owned by part of households). And households use their income to consume (C) or invest (I). Most households only consume and don't invest, and they only receive wages and no profits (but the share of households being stock owners does not have any bearing on the issues discussed in this paper). The four entities W, Π, C, I are *flow* variables $[\$/y]$. Firms also buy from each other as indicated by the money flow arrows within the firms aggregate. At any time any firm and household have some stock of money[$] as a buffer for its participation in the circulatory system. Summing up all the individual stocks, we get the aggregate amount of money in circulation among firms and households, which we will call M.

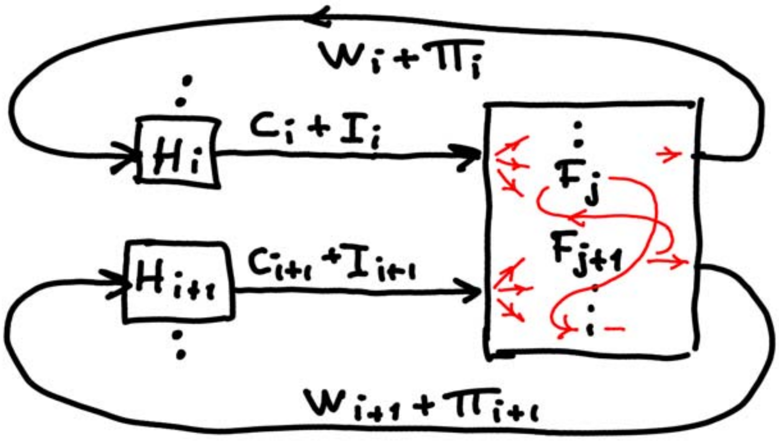

Figure 1: Money circulation with households and firms. No banks yet.

1 A model of *financial accumulation.*

Any economic system with interest on lent money has the potential to gradually develop a level of debt that leads to crisis. The model consists of a financial sector which re-lends part of financial inflows from debt service on existing loans so that the aggregate of loans will grow and future financial income will be correspondingly larger. At the other end is the rest of the economy introduced above – the "real economy": households and non-financial firms, where the aggregate flow Y is now the sum of all $C + I$ flows. The real economy is in debt, but still borrows (and in later stages have to borrow) what the banks offer. The units are macroeconomic aggregates, so that we have a society which is increasingly polarised between a group of lenders and a group of borrowers. See figure 2. The positive feedback from debt service to new loans is indicated with plus signs. *(Note that this is not strictly the syntax of a causal diagram, it is only used at this stage for explanatory purposes.)*

Wages and expenses paid by the financial sector may, seen from the financial sector's side, be considered a "leakage" back to the real economy that weakens the accumulation process. Note that the money flowing to banks as interest and repayment is in its entirety returned to the real economy, after some lag. Abstracting from the effects of the lag in the financial sector, this means that money will not disappear from the real economy; all of it will be cycled back. Due to accumulation however, it will to an increasing degree come back with strings attached – appearing as added debt. So we have a growth of the positive feedback flows to and back again from the financial sector. They may grow faster than the aggregate of transaction flows $Y(t)$ within the real economy (Y = Gross Domestic product = GDP = consumption + investment = $C + I$).

We have the relation

$$Y(t) = M(t)v(t) \tag{1}$$

where $v[1/y]$ is money velocity, the number of times a dollar turns over per year. The other variables with denominations to be used in the model are $M[\$]$, $Y[\$/y]$ and debt $D[\$]$. Parameters needed are interest $i[1/y]$ and repayment rate $d[1/y]$. We also assume a first order time lag T_b between banks receiving debt service, and extending new loans/paying expenses and wages.

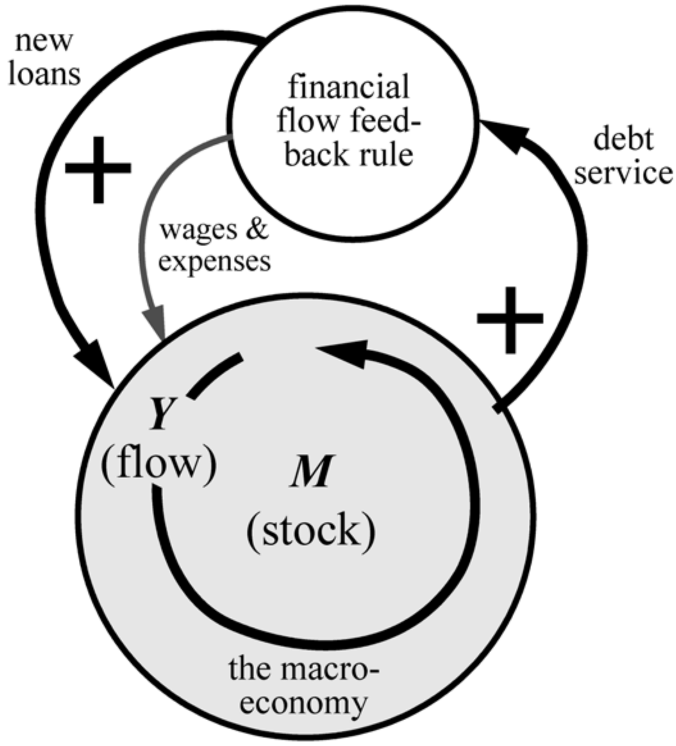

Figure 2:

Money velocity v may be considered a constant parameter, but it will become a variable and fall dramatically during the late debt crisis stage, when all agents hold back in their spending and banks hold back in their lending.

Debt service is a *non-discretionary* flow (you can't decide the size of the payment flow, it is decided by the loan contract and you are obliged by it) while the Y flows are *discretionary*, at least within some fairly flexible bounds (you have to eat, but you may postpone the purchase of a new TV or holiday). When non-discretionary flows become dominating, the economy as a whole becomes less resilient and more fragile. The frequency of insolvencies increases.

An economic system with lenders recycling financial income as new loans will as a rule become polarised betwen lenders and borrowers, as warned against since ancient times. For all financial investors (lenders) strive to accumulate. To the degree they succeed, we get increased asset/debt polarisation. Such polarisation occurs since only successful accumulators survive through the market's Darwinian selection process. Thus slow motion debt explosions will be the rule and not the exception – debt crises occur in the real world. During the last thirty years debt has persistently increased more than GDP's worldwide, see figure 3. *This is possibly the most fundamental (more basic than the US housing bubble and new complicated financial instruments) cause of today's global financial crisis.*

The reason that such processes are not much recognised or discussed, is probably the very long time scale for the dynamics involved (several decades), and that the growth path of an exponential function is not very noticeable until the dramatic late stage.

It also possible that the reason for lack of recognition of the basic accumulation mechanism is – paradoxically – that it is so trivially obvious if one bothers to think about it. Even antique societies recognised it. The insight's ancient origin, its close relation with religion (prohibition against interest on loans in old Christianity and current Islam), and its simplicity, all contribute to explain why fringe groups and "eccentrics" embrace it. But one should be very careful about dismissing a theory just because it is supported by the fringe. One may then have a case of a baby being thrown out with the bathwater. This seems to be done by parts of the economics profession.

Seen from a control systems perspective however (which ought also to be shared by economists), these runaway long-term dynamics are extremely harmful, and some macroeconomic control mechanism(s) should be implemented. A control strategy for a country could be to keep the debt/GDP ratio (or the debt service/GDP ratio) within some reasonably low bounds.

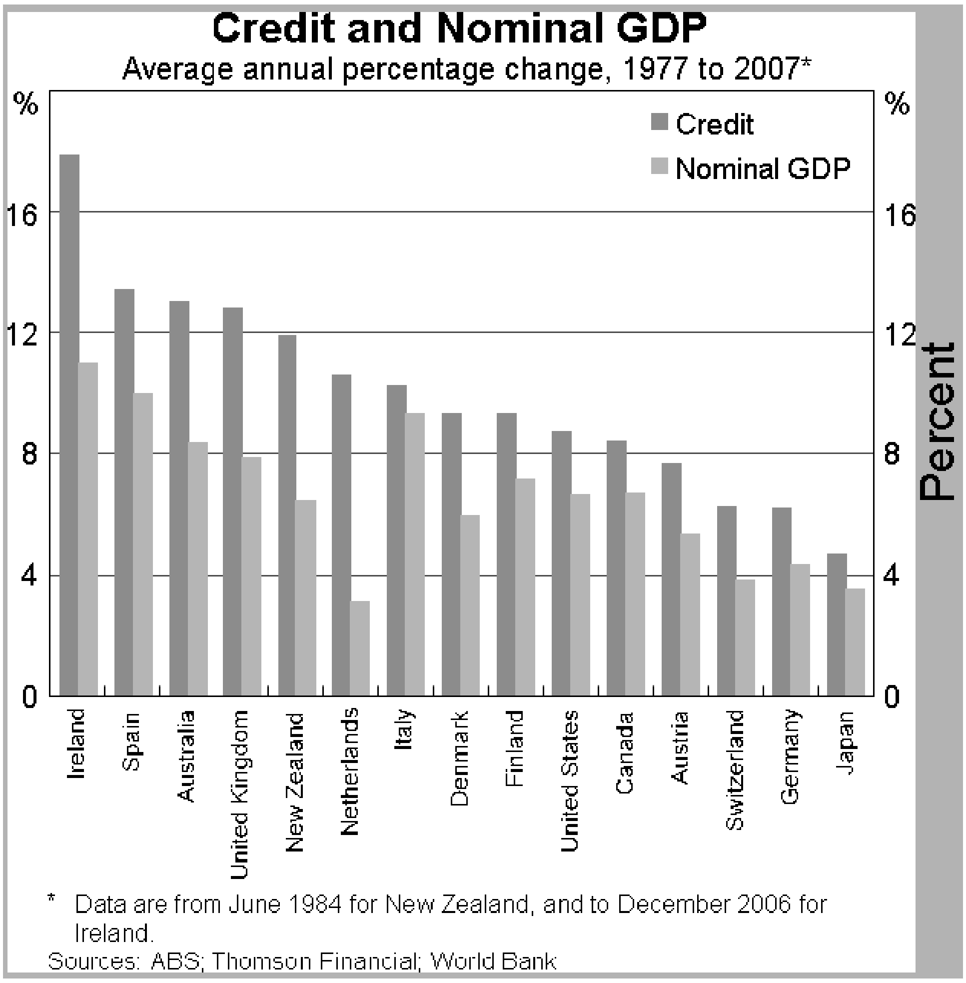

Figure 3: Debt outruns GDP in OECD countries *(courtesy: Reserve Bank of Australia)*

2 A system dynamics model

2.1 Private debt

We will build the model in five stages. All entities are in nominal, not deflated (= inflation-corrected) values. Some very simplifying assumptions are made, but (hopefully) without losing the essence of what is to be argued. The loan interest rate i and the repayment rate r are held constant. Depositors receive no interest on their bank deposits M. The aggregate "bank" is not assumed to be able to net create money when lending, even if that is allowed to some degree in actual modern economies. The aggregate bank may for explanatory purposes be thought of as a classic "moneylender" receiving physical currency from the debtors, storing it temporarily in a vault (M_b, which plays the role of a buffer), and then lending out the money again, the share left over after bank expenses and wages are paid.

Figure 4 shows the stage 1 model Three differential equations are shown in the upper left corner, corresponding to the three stocks in the stock-flow diagram. Two new parameters are introduced: $0 < \beta < 1$ is the share of interest income iD that is available for new lending after expenses are paid. T_b is the time lag of the bank buffer. Note that the total amount of money in the system is not only M, as used in the introduction above, but $M + M_b$. The total amount is constant, so that any increase in the bank buffer M_b implies a similar decrease in the amount M available for consumption and investment transactions. A further analysis of the system which will not be done here, gives the result that for any pair i and $\beta > 0$, debt D will grow exponentially. This may be shown algebraically, or via simulations. The last approach is necessary when one wants to account for the effects of dangerous debt levels in an economy, more on this in later stages.

The next stage is shown in figure 5. Here we have added the real part of the economy, with the aggregate of transaction flows $Y = Mv$. We have also added a measure of the burden of debt, the fraction $(i+r)D/Y$. We note that both the banking time lag and money velcoty are assumed exogenous

$$\frac{dD}{dt} = \dot{D} = -rD + \frac{M_b}{T_b}\left(\frac{i\beta + r}{i+r}\right)$$
$$\frac{dM_b}{dt} = \dot{M}_b = (i+r)D - \frac{M_b}{T_b}$$
$$\frac{dM}{dt} = \dot{M} = \frac{M_b}{T_b} - (i+r)D = \left[-\dot{M}_b\right]$$

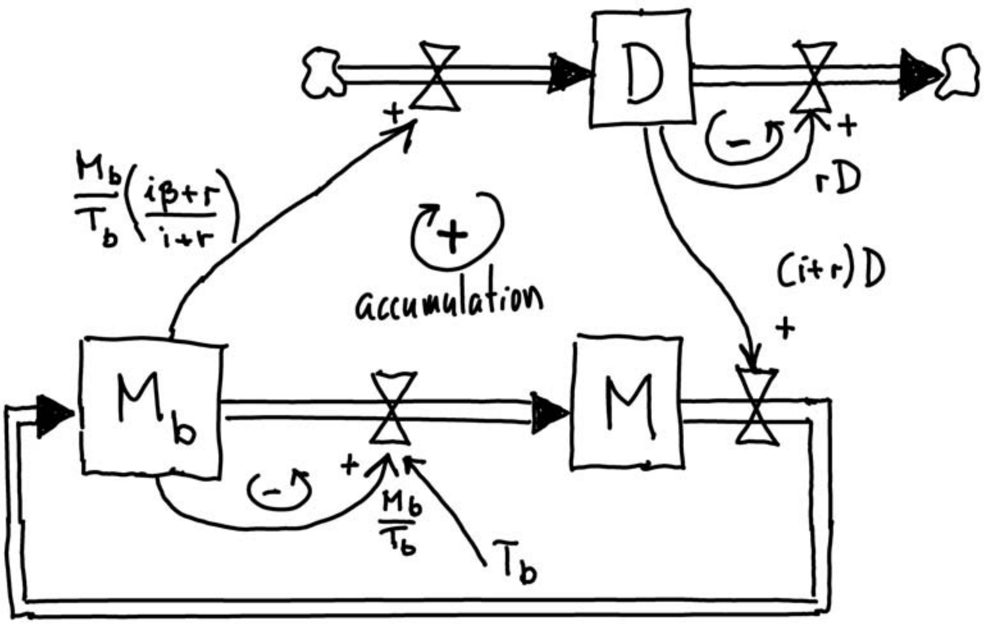

Figure 4:

and constant at this stage. This model has pure exponential debt growth as in stage 1, since there are no new closed loops influencing the stocks.

We now go to stage 3, figure 6. We have introduced a loss rate $\lambda[1/y]$, which expresses the yearly share of D lost due to bankruptcies and insolvencies. The larger the debt burden, the higher is λ. Debt is reduced. We get a balancing loop as indicated.

Then to stage 4, figure 7. Money velocity v has now become an endogenous variable. It is influenced by a new intermediate variable, "optimism", which again is influenced both by the loss rate and the current debt burden. We assume delays (or preferably 1st order information lags) at the optimism inputs, since the mood in society needs time to change. Now we get a postive feedback loop that may be dangerous: people and firms hold on to their money (reducing v) which contributes to even more reduction in optimism.

Finally, to stage 5, figure 8. Another dangerous feedback loop which exacerbates a debt-induced cris, is that banks spend and lend less when optimism is low. Then the buffer lag T_b is increased, banks hold longer on to their incoming debt service flow. In the first decades the only influential feedback loop is "accumulation". The other two reinforcing loops come into play in the end phase when the debt burden is high. Like in many countries and regions these days.

This is structurally the complete model. What remains to be able to do simulations, are some reasonable parameter values and nonlinear functional relationships.

$$\frac{dD}{dt} = \dot{D} = -rD + \frac{M_b}{T_b}\left(\frac{i\beta+r}{i+r}\right)$$

$$\frac{dM_b}{dt} = \dot{M}_b = (i+r)D - \frac{M_b}{T_b}$$

$$\frac{dM}{dt} = \dot{M} = \frac{M_b}{T_b} - (i+r)D = \boxed{-\dot{M}_b}$$

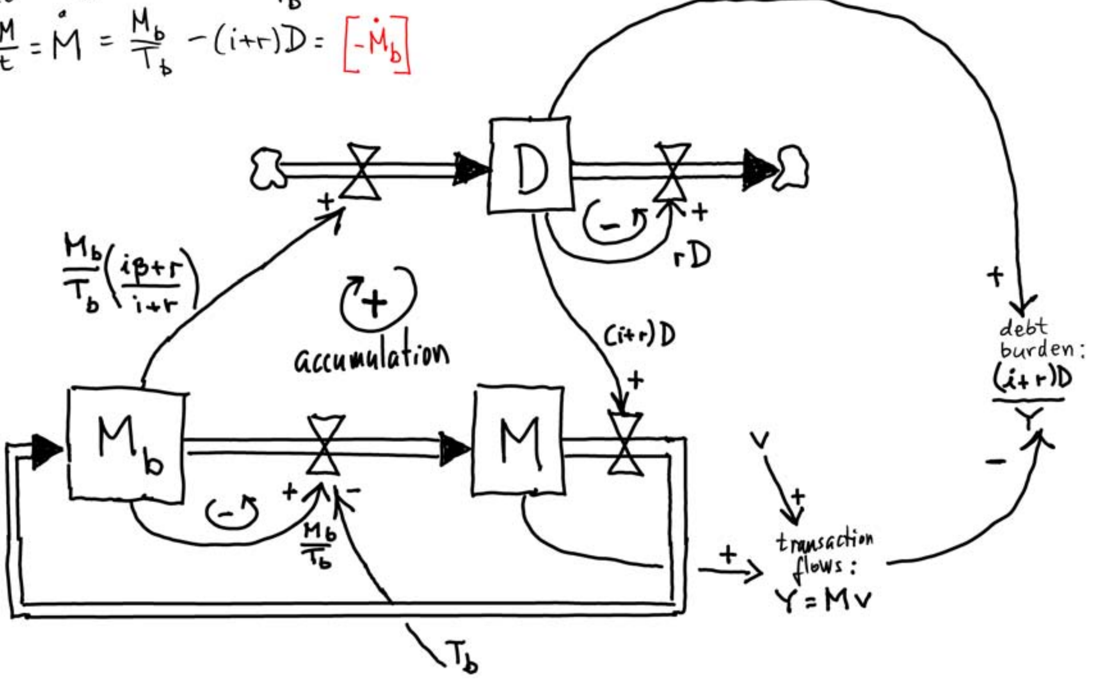

Figure 5:

$$\frac{dD}{dt} = \dot{D} = -rD + \frac{M_b}{T_b}\left(\frac{i\beta+r}{i+r}\right) - \lambda D$$

$$\frac{dM_b}{dt} = \dot{M}_b = (i+r)D - \frac{M_b}{T_b}$$

$$\frac{dM}{dt} = \dot{M} = \frac{M_b}{T_b} - (i+r)D$$

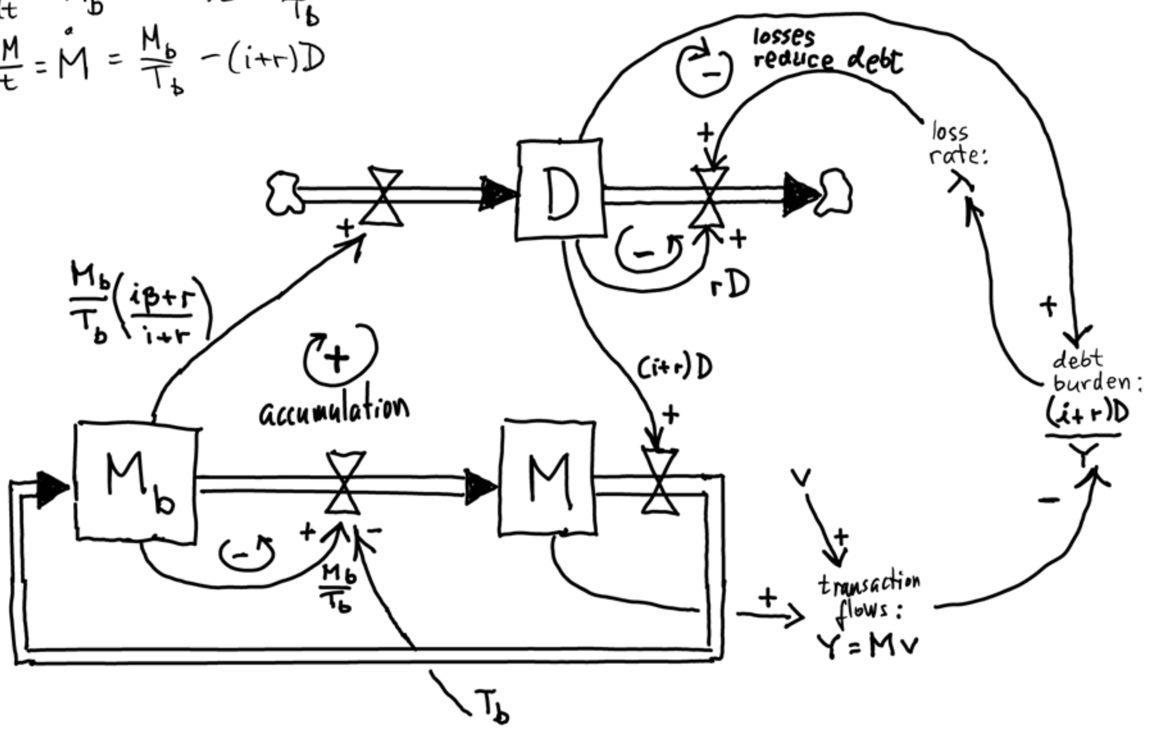

Figure 6:

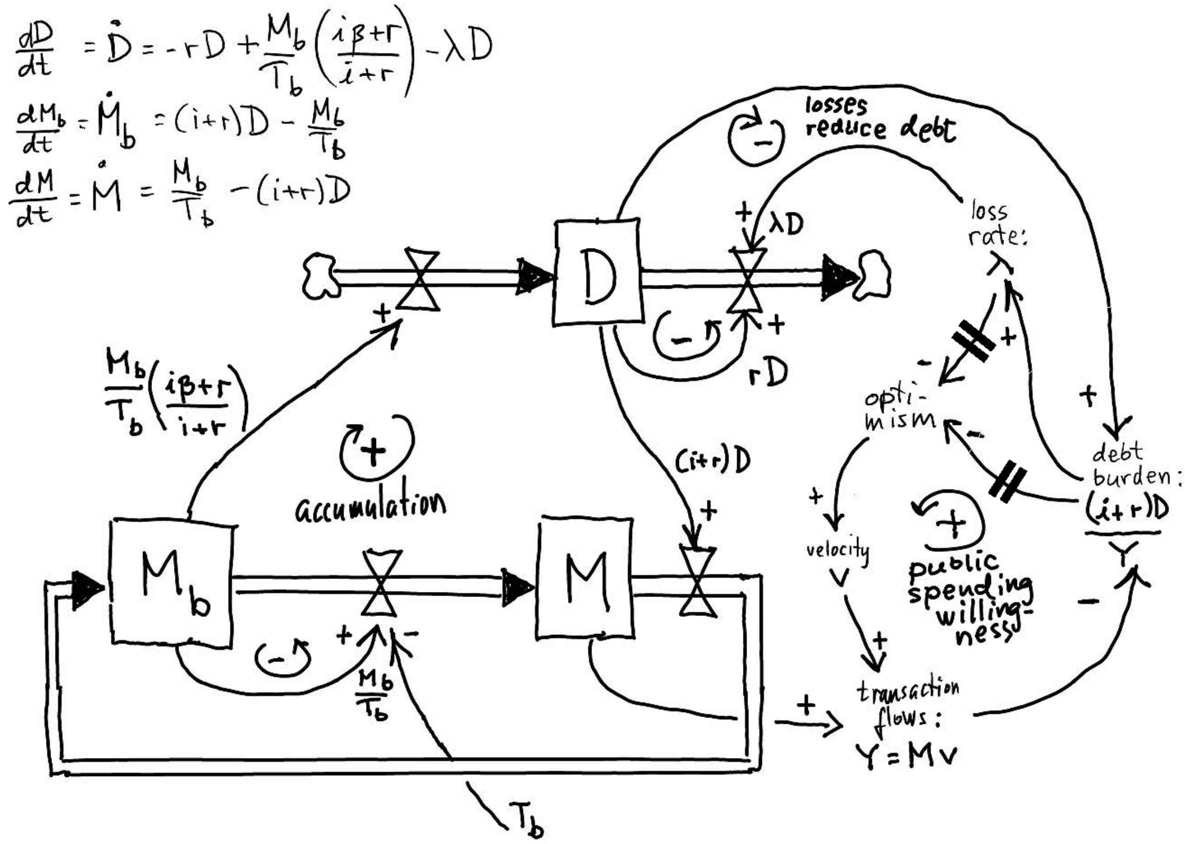

Figure 7:

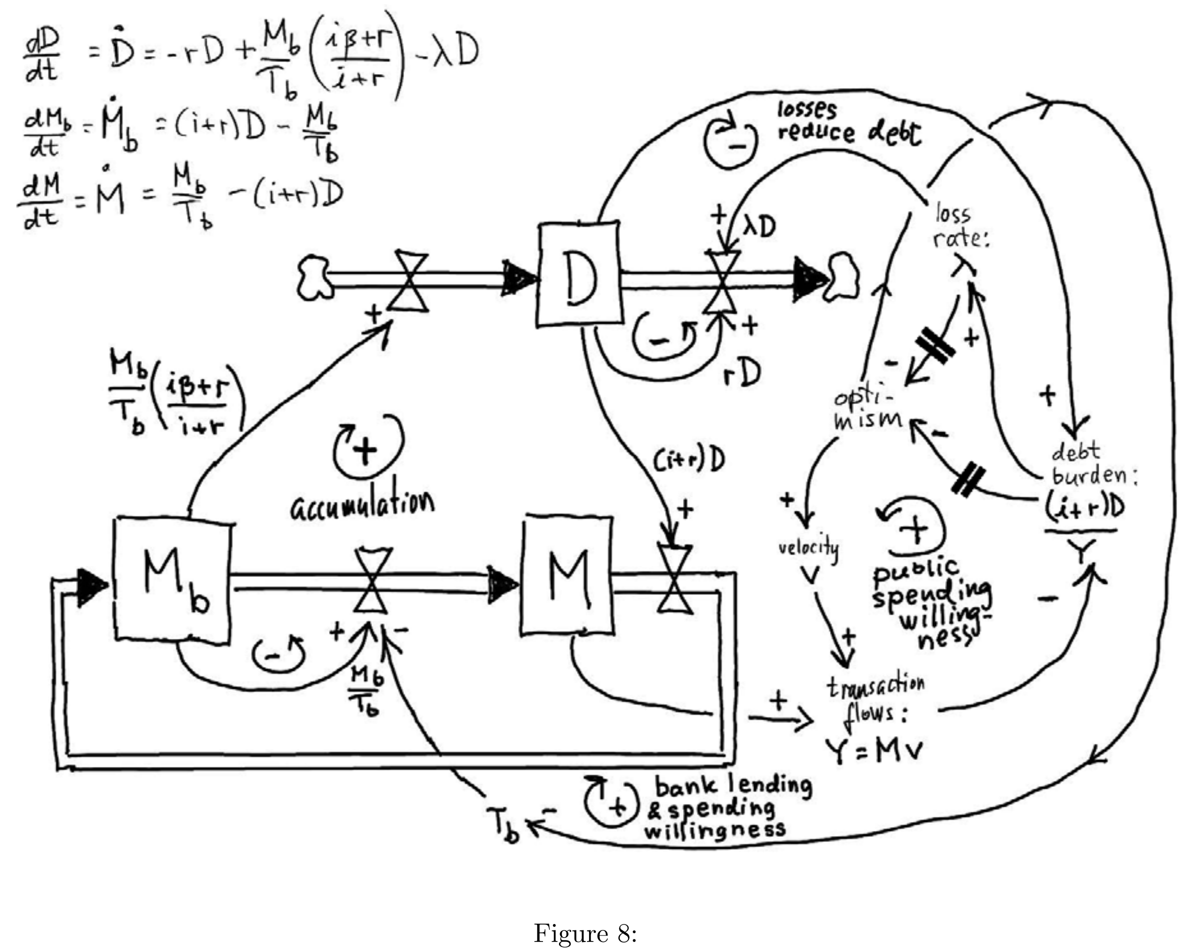

Figure 8:

Die Parallelwährung:
Optionen, Chancen, Risiken.

BVMW
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A PARALLEL EMERGENCY CURRENCY VIA THE MOBILE PHONE NETWORK

„Worldly wisdom teaches that it is better for the reputation to fail conventionally than to succeed unconventionally" – John Maynard Keynes

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Übersicht:
Dieser Beitrag beschreibt die Einführung einer nationalen Parallwährung als bargeldloses Währungssystem, welches über das Mobilfunknetz realisiert wird. Durch diese sogenannte "Notfallwährung" kann die jeweilige Regierung den Prozess kostengünstig steuern. Es wird die Option eines kontrollierten Übergangs zu nationalen Währungen geboten, aber auch der Verbleib in der Eurozone ist möglich. Die Vorteile einer elektronisch verwalteten Währung werden erläutert, aber es werden auch mögliche Probleme wie mangelndes Vetrauen in die Währung, Inflation oder Kapitalflucht diskutiert.

1. Introduction

I have written about parallel emergency currencies on several earlier occasions (Andresen, 2010). The situation in most eurozone countries has worsened considerably since then. An indebted government has to extract euros out of the non-government economy to service its debt, by taxing more than it spends. The foreign-indebted private sector also extracts euros, sending these to creditors. The only way to (theoretically) counter these two „bloodletting" flows from a domestic economy is to increase net exports to a level that surpasses the sum of these two outgoing flows. This is exceedingly difficult, especially after debt service burdens have increased steeply due to risk-caused increases in interest levels on new euro loans, and because of idle production capacity due to the crisis.

Debt could be partly written off and/or the debt service rates could be ameliorated, but to the degree the creditors refuse this, the domestic economy will be increasingly starved for money. Firms and individuals simply do not have enough of the instrument for the conducting of regular economic activity. This again leads to lower government income due to reduced tax payments and larger social outlays. The crisis is also amplified by increasing pessimism among individuals and firms: to the degree they possess euros, they hold back in spending, hiring and investment – and/or they move their money out of the country. All this contributes to further pessimism. We have an unstable downward spiral.

Politically, both the EU elite and the elites in the crisis countries are strong supporters of the euro. There is also a majority in the general populace for staying with the euro – mostly based on fear of what will happen if one reverts to a national currency. The mainstream advice seems to be to stick with the euro and hope for an internal devaluation of wages and prices to enhance the crisis country's competitiveness so much that future net exports will enable it to service its debts. This is a painful and slow process for the population (at best lasting several years). Furthermore, the outcome is doubtful, especially since many trading partners are trying the same recipe.

A way out could be to furnish both households and firms with an additional universally accepted countrywide means of exchange, so that the large amount of unemployed may get jobs, and firms' spare capacity may be utilised. A euro-debt crisis country has a large output gap, and such a gap could be much diminished, without giving rise to significant inflation effects. Utilisation (and very fast activation) of this idle capacity may be achieved by nationally issued „electronic parallel money". This will quickly reduce unemployment and enable people and firms to exchange goods and services. It will also increase confidence, put a brake on the downward spiral, and even – as it will be argued below – enhance the circulation and net national acquisition of euros.

This proposed parallel medium of exchange will from now on be termed „emergency currency", abbreviated „EC". A unit of this currency will also be called „EC".

2. The proposal – how does it work?

Transactions are done via mobile phone/SMS and automatically received and accounted for on a server with ample capacity at the country's Central Bank. Such a mobile banking system may be implemented through one of the technically proven schemes already in succesful operation in some developing countries (Hughes and Lonie, 2007). There are no physical/paper EC's in circulation. The government has an EC account at the Central Bank. This account is debited whenever the government pays wages or pensions, or buys goods or services. All citizens and domestic firms have accounts there too, also interested foreign entities (but we will expect EC's to circulate only domestically in an initial phase). By this EC's are created ex nihilo, „printed" by the Central Bank.

The government pays employees, pensioners and suppliers both in EC's and euros. The proportions may be adjusted based on how the process develops. Taxes are also collected in a mix of the two currencies. The government-issued EC will be fiat money, and will have some intrinsic value since it may be used by the public to settle tax obligations as argued by Modern Monetary Theory (Wray , 2006). An EC will therefore be initially accepted to a fair degree as a means of payment by an agent – individual or firm – that is obliged to pay taxes.

Employees and firms offering goods and services will gradually – as the scheme gets more popular – decide to accept a certain share of EC's as payment, while the rest must still be in euros. While the government pays wages and taxes in a government-decided mix of the two currencies, the mix in private sector transactions may be decided freely by the involved parties, and will differ between trades. Both the government mix and private sector mix will necessarily have to be adjusted with time and circumstances. Employers and employees may negotiate the share of wages being paid in EC's, based on how things develop.

An additional positive effect of introducing EC's is this: *By enabling activation of idle labour and production capacity, exports increase. Thus, even if this extra activity is mediated (partly) with EC's, this enhances the ability of the country to service its debt burden in euros.*

Another positive effect is that pessimism is reduced. This will decrease the liquidity preference of individuals and firms that possess euros but have been holding back in their spending. Money velocity in euros will become greater: *for a given amount of euro stock held by agents, the aggregate euro flow will increase, i.e. we get increased money velocity.*

3. More advantages of mobile phone EC's, versus bills and coins

1. The system can be implemented fast, and adjustments that turn out to be needed can be implemented in software, therefore very easily and cheaply.
2. The system is very cheap to run, compared to a system with notes and coins. And forgery is impossible.
3. There is no confusion with bills and coins (i.e. euros) that are being used in parallel.
4. This is a 100% reserve system. All deposits are high-powered (or base) money, residing at the CB. No deposit insurance is needed. Money cannot be lost, and this is clear to the public -- thus no bank runs.
5. A black economy in EC's is nearly impossible. The same with tax evasion. Intelligent software can monitor transactions 24/7, and flag human operators when suspicious patterns emerge. Knowledge of this implies a credible threat, so that agents to a large degree will abstain.
6. EC's cannot be used for capital flight, since they only reside at the CB.
7. Also, some more futuristic advantages merit mention: negative interest on money held (demurrage) may be easily implemented, to speed up circulation if that is needed.

8. A new possible control tool with the opposite effect is made possible by EC's only existing as accounts at the CB: A tiny but adjustable transfer tax between any accounts. This would be incredibly more effective to damp an overheated economy, than a CB interest rate hike. It can stop too much spending in its tracks. As far as I know, this is a feasible tool that has not been considered in the large economics literature on inflation control.

4. Discussion

The discussion will be done by addressing some counterarguments against, or expected questions about, the EC proposal.

The question of „confidence"

The EC is a *fiat currency,* not purely based on faith. From the outset it will enjoy a certain minimum of confidence since it is legal tender, issued by the government. And a basic confidence is ensured because it may be used to pay (a share of) taxes, as already mentioned.

One may in spite of this expect that initial confidence will be low, not the least because of widespread distrust in authorities that until now haven't done much to ameliorate the effects of the crisis.

To discuss the prospects of an EC, it might be useful to define two entities, *„trust"* and *„need".* Even if trust is low at the outset, need is very high: people and firms will have the choice of trying out an EC that is paid out to individuals and offered to firms, or not using or accepting it at all. So some initial use of the EC should be expected becase of the alternative of no work or no sale is considered even worse. Need will ensure some EC circulation, even if trust is low. With time, a positive feedback process will emerge: agents observe that transactions with EC's are happening, and this will increase trust, leading to more acceptance of EC's, and so on.

Inflation in EC's?

Assume that the government declares at the outset that the exchange rate EC to euro ought to be unity, and that firms are asked not to set *prices* in EC's high, but instead safeguard themselves in the start-up period by setting the initial EC *share* of an item's price low. What the government recommends will of course not necessarily be followed by vendors, but many will try this as a starting point. We should expect that firms (and individuals) that offer products or services where the dominant input factors are domestic, will be most willing to try offering a significant share of EC's in what they accept as payment. At the other end we have products that are imported, and the domestic input factors are subordinate: cars and petrol are examples. Here one can expect that only with time will such sellers start accepting EC's, and the share will never become high. But there will be a mechanism at work in the right direction also there: when EC use has reached a significant level for other consumer items, for instance food (where domestic input factors are significant), import-based firms can negotiate a wage share being paid EC's and the rest in euros, hence allowing also such firms to accept a share of EC's in the items they sell.

An important aside to this is that the existence of circulating EC's will enhance domestic output. To some degree this will lead to import substitution, cet. par. increasing exports which is a good thing concerning the ability to service euro debt.

Regardless, however, of possible government declarations about how the parallel currency ought to be valued, one should expect the EC to lose value from parity with the euro. And floating the EC versus the euro must be accepted, there is no point in trying to uphold an artificially favourable exchange rate by this creating a black market. But as already argued: as long as the economy is far away from full employment and firms have significant idle capacity, inflation pressures are not strong.

Euro debt and euro capital flight

One may at this stage correctly protest that introducing an EC does not in itself solve the euro debt problem. It also does not solve the problem of richer citizens moving their euros out of the country to avoid taxes or in fear of losses due to collapse of domestic banks.

To the first objection, one may reply that without a parallel medium of exchange an economy is wholly dependent on euros to uphold domestic activity. This puts the country in a very weak position when negotiating debt writedowns and/or lower interest rates and longer repayment times on existing debt

Furthermore, by enabling the economy to run much closer to full capacity and employing a much larger share of the population, the ability to export increases, and by this the ability to service euro debt. The automatic stabilisers will also be at work, giving the government more tax income, and reducing its expenses for unemployed benefits and other social costs due to the crisis.

The problem of euro capital flight is not solved by introducing EC's, except that increased domestic economic confidence may after a while induce many agents to repatriate their euros. But this will probably not make a significant difference. Anyway, the issue of capital flight is there regardless of whether the EC proposal is implemented or not, and must be adressed somehow. And it has more serious effects without a parallel EC system in operation.

5. Summing up: far better than the bleak alternatives

A parallel electronic emergency currency will – with immediate effects – ameliorate the strongly and persistently lowered living standards for most people in crisis countries, which is the bleak and only future (lasting many years, possibly a decade or more) that the EU and crisis country governments have been able to come up with. By the proposed scheme it should be possible to activate the immense underused potential that the hard-hit eurozone countries have, unemployed or underemployed people, to give many a better life and the country a return to social stability. It will primarily stimulate domestic production. It will also give euro-indebted countries a much better position in their bargaining for partial debt relief or less heavy euro debt service burdens.[1]

Finally, it enables a gradual and controlled transition (back) to a national currency, if that is what is wanted. This proposal gives the national assembly in a crisis country the freedom to deliberate and make such a grave decision at any chosen time, and base it on experience with how the parallel currency and the economy have fared.

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Links last accessed on November 07, 2012

1 If, however, a government does not wish to implement an EC system, regardless of how bad the alternatives are, the option is today there for non-government entities or even an innovative mobile phone company, to create an „electronic exchange association" encompassing the entire country using a similar technical solution. For further information on this, the reader is referred to Andresen (2010).

Ten advantages and two possibilities with a parallel electronic currency

Last version, 6 September 2012

Trond Andresen

This note concerns how the thirties' excellent Fisher et al proposal of 100% money could be resurrected today by exploiting new technology – no bills and coins, only accounts at the Central Bank (or at a Government Facility – "GF" from now on -- established for that purpose) where transfers are made via mobile phones. Mobile phone transfers of (regular) money have a proven track record, f.inst. «M-Pesa» in Kenya.

I assume the case of a parallel gvt-issued domestic currency, needed by a eurocrisis country, a dollar-based non-US national economy, or a US state – in a situation with a strong need for enhanced economic activity[1].

All citizens and firms have «Mobile Dollar» (I use this name just for convenience, could be «Mobile Euro» or «Mobile Cash», whatever) accounts at the CB (or GF). All transactions go via mobile phone.

1. The system is very cheap to run, compared to a system with notes and coins. And forgery is impossible.

2. The system can ble implemented fast, and adjustments that turn out to be needed, can be implemented in software, therefore very easily and cheaply.

3. Mobile Dollars can be used to pay taxes. This is an incentive for firms and persons to have confidence in Mobile Dollars, following the MMT/Chartalist argument.

4. This is a 100% reserve system. All deposits are HPM (base money), at the CB (or GF). No deposit insurance needed. Money cannot be lost, and this is clear to the public. No bank runs.

5. A black economy in Mobile Dollars is close to impossible. The same with tax evasion. Intelligent software can monitor transactions 24/7, and flag human operators when suspicious patterns emerge. Knowledge of this implies a credible threat, so that agents to a large degree will abstain.

6. Mobile Dollars cannot be used for capital flight, since they only reside at the CB (or GF).

7. There is no confusion with bills and coins (i.e. USD or euros) when such are being used in parallel.

8. By enabling activation of idle labour and production capacity, exports increase. Thus, *even if this extra activity is mediated (partly) with Mobile Dollars, this enhances the ability of the country to service its (hopefully downwritten) debt in USDs/euros.*

9. Circulation in USDs/euros within the crisis country will also increase, due to reduced pessimism – less liquidity preference. For a given USD/euro *stock*, USD/euro *flows* will increase.

10. Without a parallel medium of exchange a dollar/euro-based economy is wholly dependent on USDs/euros to uphold domestic activity. This puts the country in a weak position when negotiating writedowns and lower interest rates on existing debt. The creditors know that the country is totally dependent on additional borrowing or rolling over of USD/euro debt. With Mobile Dollars constituting an alternative medium of exchange, the balance of power in negotations is shifted in favour of the indebted country.

11. Also, some more futuristic advantages (possibilities) merit mention: Negative interest on money held (demurrage) may be easily implemented, to speed up circulation if that is needed.

12. A new possible control tool with the opposite effect is feasible by money only existing as accounts at the CB (or GF): A tiny but adjustable *transfer tax between any accounts*. This would be incredibly more effective to damp an overheated economy, than today's blunt tool of a CB interest rate increase. It can stop too much spending in its tracks. As far as I know, this is a new possible tool that has not been considered in the large economics literature on inflation control.

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1. But nationally-issued regular money for all, only in electronic form and residing at the Central Bank, could be the best future system for all countries. This, however, is not a topic for this brief note.

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Issue no. 63, 25 March 2013

In this issue:

Improved macroeconomic control with electronic money and modern monetary theory

Trond Andresen [The Norwegian University of Science and Technology, Norway]

Abstract
This paper combines the concept of electronic money (no physical currency) with Modern Monetary Theory (MMT). It argues – based on an MMT understanding of macroeconomics – how electronic monetary systems offer a big step forward for macroeconomic control, among other things by giving a government new and potent steering tools. More specifically the paper discusses how one in an electronic money environment can easily curb an overheated economy primarily through control of money velocity – not money supply. This is a necessary topic to explore, even if the opposite is needed in today's global situation, to convince academics and decision makers that running necessary large and persistent government budget deficits in depressed economies, is not "irresponsible" and does not need to imply strong inflation in later economic boom situations.

Keywords: modern monetary theory, electronic money, 100% reserve currency, money velocity, inflation control, stock/flow system

JEL classification: B50, E42, E5, G21, G28, H62

1. Introduction

In this author's opinion, the best theoretical platform for the understanding of today's macroeconomies and what might be done to improve them, is *Modern Monetary Theory* (from now on: "MMT"). MMT – also labeled "neo-chartalism" – has since the onset of the debt crises around 2008 gained influence in the global discourse on macroeconomic theory and crisis solutions. Some central academic proponents of MMT are L. Randall Wray, Stephanie Kelton, Scott Fullwiler, James Galbraith, and Bill Mitchell. A comprehensive text explaining MMT is (Wray, 1998). This paper assumes that the reader is somewhat familiar with, and not unsympathetic to, MMT.

In the MMT framework, a government and the Central Bank (CB) is seen as one unit. The "independence" of CB's that is the rule in most countries is a political and legal construct, and may as such be reversed by a national assembly. Any CB is constitutionally, at least in some final instance, an arm of the government. This is generally accepted, not solely by MMT adherents. For a country *issuing its own currency* (this is a prequisite for MMT to be valid as a platform for policy), a government's "debt" that builds up with its CB through deficit spending in excess of the income from selling bonds, is only an accounting convention. A government does not need to "finance" its spending through tax income or to borrow by issuing government bonds – a government may spend (and thus net create money) by debiting its account at the CB. Such a government is not revenue constrained. It can never "run out of" its own issued currency, and can always pay any debt if this debt is in its own – not foreign – currency. The role of taxes in MMT is to drain money to control demand and limit possible inflation, and to redistribute income.

In the MMT view, money has value and enjoys confidence since it is the only accepted means to pay taxes, and since the state can enforce tax payment. It does not need to be backed by any asset.

MMT assumes flexible exchange rates. Rigidly binding one's currency to foreign currency(-ies), removes the advantages of MMT. One is then on a de facto “gold standard”, and this is incompatible with MMT.

The obvious and common objection to MMT is "it will be inflationary". Yes, inflation may be an issue. This is a reasonable objection and will therefore be discussed below. That said, inflation is a possibility under *any* macroeconomic regime if nominal aggregate demand is near or surpasses some capacity limit. The possibility of inflation is not *in itself* an argument against MMT. Through taxation and other methods inflationary pressures can effectively be taken care of within an MMT paradigm. How to achieve this is one of the main topics of this paper.

As discussed, a government may use the option of injecting new fiat money (*base money, High-Powered Money* – from now on "HPM") into circulation. But in today's system we mostly have net creation of money through bank lending. This *credit* money – as opposed to HPM – grows *endogenously*. Endogenous bank-created money growth is a consequence of what banks do to maximise their profits without breaching Basel capital adequacy rules (Andresen, 2010). Control of money supply from the CB via banks, as told in the monetarist and mainstream economics money multiplier story, is not possible. Therefore one should instead give the government a monopoly on money creation, so that all money is HPM: new money should be spent, not lent by the banks, into the economy. This fits well with the MMT view, and has for many decades been, and still is, supported by many economists and economic reformers. The most famous proponent of 100% money is probably Irving Fisher (1936). His and other economists' "Chicago Plan" has recently been re-evaluated with a very postive conclusion (Benes and Kumhof, 2012). When banks wish to lend in a 100% reserve scenario, they would have to borrow HPM at lower rates, and live off the rates difference. But they should not create money themselves. This will *ceteris paribus* make control of money creation and, therefore inflation, easier.

That said, control of money supply is not the central point in this paper – it will focus on control of another entity: *money velocity*. As this paper will show, control of velocity is much more effective, and it becomes feasible – for the first time in the history of money – with electronic money (i.e. no physical currency).

In a recession or even depression-like situation – the case in most countries today – the attraction of MMT is obvious: since a government with own-issued currency is not financially constrained, such a crisis can be remedied by running arbitrarily high fiscal deficits as long as needed, i.e. spending extra HPM into the economy to employ people and buy goods and services. A government issuing its own currency can always employ all the unemployed.

But there is a challenge to MMT that has hardly been discussed by its proponents: in the *opposite* scenario, if an economy is running close to full capacity or beyond (for instance after a crisis where a large amount of money was injected, remaining in circulation), and there are ensuing inflationary pressures: how can a government drain the system and curb money flows? This is a genuine problem, and is not easily solved in today's technical monetary environment. But there are solutions to this if all circulating money is electronic; transacted via the Internet and the mobile phone network, and residing only as accounts at a national depository facility.

Electronic money will mercilessly – sooner or later – take over simply due to technological progress. It offers a dramatic improvement in convenience and cost. Banks are already implementing it for that reason. The certain eradication of physical currency is only a question of time. The process is comparable to the advent of the digital camera, leading to the death of photographic film. Such processes cannot be reversed. Luckily, it turns out that fully electronic money systems are not only cheaper and more convenient, they also offer potent new opportunities for macroeconomic control.

2. A problem – injection and drainage asymmetry

There will be negligible opposition in a depressive situation if a government hires more people and buys more goods and services, with brand new HPM, created out of thin air at the CB – not even by borrowing. Such policy is possible with an MMT understanding of macroeconomics. In such a situation, people will gratefully accept this, in spite of alarms from deficit hawks and some financial pages pundits.

But when a government tries to drain money back later on in a boom, running a surplus over time by increased taxes, there will probably be strong popular resistance[1]. Furthermore, in a boom there will usually also be a widespread over-optimistic mood in the population, enhancing such resistance – which can take many forms: media campaigns, demonstrations, capital flight, tax avoidance, stashing away cash, voting for right-wing parties arguing for "small government" with low taxation.

MMT proponents have to address this issue, even if this is a hypothetical scenario diametrically opposite to today's. For it is difficult to convince the public, academics and decision makers today of the acceptability of large and persistent (over years) deficit spending, if one does not have a recipe for what to do in a later boom:

> It’s true that printing money isn’t at all inflationary *under current conditions*— that is, with the economy depressed and interest rates up against the zero lower bound. But eventually these conditions will end. At that point, to prevent a sharp rise in inflation the Fed will want to pull back much of the monetary base it created in response to the crisis, which means selling off the Federal debt it bought. So even though right now that debt is just a claim by one more or less governmental agency on another governmental agency, it will eventually turn into debt held by the public (Krugman, 2013).

3. Electronic money – the system

Today it is technically feasible to discard physical money completely – no bills and coins – and do all transactions by debit card, personal computers (both quite common in developed countries), and/or via the mobile phone network – not common, but on the rise. Mobile phone money transfers have a proven track record, for instance "M-Pesa" in Kenya (Hughes and Loonie, 2007). With electronic money (“EM”) all transactions are reflected in movements between accounts. But there are in the proposed implementation here, no deposits with private

[1] This may be considered analogous to the well-known downwards "stickiness" of wages and prices.

banks[2]. All accounts are at the Central Bank (or at a National Depository – "ND" from now on – established for that purpose).

All citizens and firms have EM accounts at the CB (or ND). The advantages are obvious and many:

1. The system is very cheap to run, compared to a system with bills and coins.
2. Adjustments that turn out to be needed, can be implemented in software, therefore very easily and cheaply. No cumbersome and expensive printing/stamping and distribution of bills and coins.
3. Forgery is impossible. So are robberies.
4. This is a 100% reserve system. All deposits are HPM (base money), at the CB (or ND). No deposit insurance needed. Money cannot be lost, and this is clear to the public. No bank runs.
5. EM is an extremely inclusive and convenient system, giving poor and rural sectors of an economy – where ATMs and bank branches may be far between and not all people have accounts – a tool for easy economic participation and exchange.
6. A black economy in EM is close to impossible. The same with tax evasion. Intelligent software can monitor transactions 24/7, and flag human operators when suspicious patterns emerge. Knowledge of this implies a credible threat, so that agents to a significant degree will abstain.
7. EM cannot be used for capital flight, since it only resides at the CB (or ND). All foreign transactions are logged and thus controllable, as suggested in the previous point.

Finally, two unconventional advantages/possibilities:

8. Negative interest on money held (demurrage) may be easily implemented, to speed up circulation if that is needed.
9. A new possible control tool with the opposite effect is feasible by money only existing as accounts at the CB (or ND): A tiny but adjustable *transfer tax between any accounts*. This would be incredibly more effective to damp an overheated economy, than today's blunt tool of a CB interest rate increase. It can stop too much spending in its tracks.

In the next section we will discuss how some of the above advantages enable the government to curb spending in an economically overheated scenario.

4. Spending control

4.1 Money velocity is a crucial factor

It is first necessary to make an important point about money supply and money flows. Demand in an economy is not decided by the aggregate money supply (a *stock*), but by the aggregate of money *flows* Y, where Y is GDP. In a continuous-time modeling framework, the denomination of Y is [$/year], as opposed to M [$]. In nominal terms we have

[2] But private and cooperative banks still have a role to play: to vet and lend to borrowers, using funds gotten by selling bonds, offering time deposits or borrowing from the CB.

$$Y(t) = M(t)v(t),$$

where M is aggregate money stock and v is average money velocity. This is the quantity equation, adhered to by monetarists, and (much for the same reason) derided by many other economists. In this author's opinion, the monetarists are wrong because they ignore v and focus solely on M. There are also mainstream economists who point to the insufficiency of using M as a control variable:

> In terms of the quantity theory of money, we may say that the velocity of circulation of money does not remain constant. "You can lead a horse to water, but you can't make him drink." You can force money on the system in exchange for government bonds, its close money substitute; but you can't make the money circulate against new goods and new jobs (Samuelson, 1948:354).

But many outside the current mainstream are also wrong – not because they (correctly) argue that M is not a sufficient control variable – but because they consider v of no importance:

> Unfortunately, most economists are brainwashed with the trivializing formula MV=PT. The idea is that more money (M) increases "prices" (P) – presumably consumer prices and wages. (One can ignore velocity, "V," which is merely a tautological residual.) "T" is "transactions," for GDP, sometimes called "O" for Output (Hudson, 2010).

This might be characterised as throwing the Mv baby out with the M bathwater. One economist who saw the importance of velocity, was Irving Fisher:

> Free money may turn out to be the best regulator of the velocity of circulation of money, which is the most confusing element in the stabilization of the price level. Applied correctly it could in fact haul us out of the crisis in a few weeks ... I am a humble servant of the merchant Gesell (Fisher, 1933:67).

Fisher argued for a parallel money in the depression-ridden U.S., and levying a holding fee (negative interest, *demurrage* – originally proposed by the German-Argentinian merchant and monetary theorist Silvio Gesell[3]) on this money to force agents to spend. Thus it would be possible to increase activity even for a small M, due to higher v. Fisher understood that v is not a "residual" as Hudson calls it, but an important behavioural variable, and that it would be low in a depression, and needed to be boosted. It is strange that this is not more recognised, since v is in a one-to-one relation to (inverse) liquidity preference, and liquidity preference is a concept that is widely accepted and used among macroeconomists – not the least by Post Keynesians, who are very much against the quantity theory.

4.2 Control with electronic money

[3] Gesell received a strong recommendation in the *General Theory* (Keynes , 1973:355)

In today's system with credit creation of money through bank lending, control of *M*, as emphasised in monetarist and mainstream economics, is not possible. For credit money – as opposed to HPM – grows endogenously as already mentioned. Giving the CB monopoly on money creation, so that all money is HPM, will make such control more feasible.

With electronic money one is able to not only enhance control of *M* , but also achieve control of *v*, which until now has been mostly ignored (in part because such control is very difficult in a system containing physical currency). While *M* cannot be changed significantly within a short time span (since it is a stock and needs time to change, and since draining M wil be a controversial extra tax), this may be done with *v* (since it is a behavioural variable, not a stock, and no liquid assets are taken from the holders). By having control of both *M* and (especially) *v*, one may exercise potent control[4] of their product, $Y = Mv$.

There are (theoretically) a quadruple of ways to do *Mv* control:

1. *A fee (negative interest, demurrage) on money held: M* decreases slowly, *v* increases strongly and immediately, therefore *Y* increases immediately. And the government can exploit shrinking *M* by creating a corresponding extra HPM flow and thus spend more. This is a bonus in a recession/depression.
2. *A fee on transferring money between accounts: M* falls slowly, *v* falls immediately, therefore *Y* decreases immediately.
3. *Positive interest on money held:* in checking accounts, the opposite of item 1. This is today's sole tool: *M* increases slowly, *v* may decrease a little but slowly, therefore *Y* hopefully decreasing, but this is very mood-dependent.
4. *A small reward for transferring money between accounts:* the opposite of item 2, *M* grows persistently and exponentially, *v* increases strongly, *Y* "explodes".

Item 4 is obviously absurd, since agents can then increase their money holdings just by transferring money back and forth. It will be ignored in the following. I will now discuss the new possibilities given by items 1 and 2, and especially item 2.

Negative interest on money held (item 1) works, as demonstrated by the Wörgl parallel local crisis currency in 1932 (Lietaer, 2010), where money velocity turned out to be 12 – 14 times the velocity of the Austrian schilling[5]. This was also an inspiration for Irving Fisher's (futile) attempts to get a similar solution implemented in the depression-ridden U.S. But the Wörgl technical demurrage solution was cumbersome: one had to buy a stamp every month and glue it to a bill, for the bill to uphold its validity. And with coins one cannot even do that. With electronic money however, it is exceedingly simple: every day a tiny proportion of the amount in a checking account is deducted. And this proportion may be easily adjusted as the state of the economy changes.

Now to item 2: a fee on transferring money between accounts. *As far as this author knows, this is a new concept* in the context of economics, and easily implemented in an electronic money framework. One could object that it resembles a value added tax, but the important

[4] Note that I at this stage abstract from *fiscal* control tools. These are important, although not for the purposes of this paper. I will return briefly to them.

[5] After one year's successful operation it was prohibited and shut down by the Austrian Central Bank.

difference is that the fee is on *all* transfers, not only for purchases from firms (one may of course have a VAT like today, in parallel with an account transfer fee). This property, combined with all money residing as checking accounts at the CB (or ND), makes avoiding the fee impossible and removes all need of human control. The size needed for such a fee to have an impact is difficult to decide ex ante, but a conjecture is that this measure will be quite potent, comparable to demurrage on money held. One could start with a very low (and therefore economically and politically harmless) level – say 0.1% – and monitor the impact. If the impact in a trial period is too small, increase the fee a little.

4.3 Fiscal policy with electronic money

From an MMT perspective, fiscal policy is more important than monetary policy. All money as electronic HPM in accounts at the CB (or ND) will make taxation and levying of fees easier. This will be the case both for collection, control and adjustment. Tax evasion and crime will be sharply reduced as already mentioned. The need for human control will be much lower, since detailed monitoring may be done by software which alerts human operators only when suspicious patterns are detected.

Possibilities for capital flight will be sharply reduced, even if this cannot be completely eradicated (capital controls in an electronic money environment should be a topic for further research).

5. Concluding remarks

Electronic money, applied with an MMT understanding, enables a revolution in macroeconomic control. But his insight will probably not be at the center of media hype and attention as electronic money becomes more widespread . The goal of this paper is to contribute to ensuring that the most important advantages of electronic money are not lost in the process.

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No, there need not be lack of credit with "100 % money"

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Abstract

This is a defense of "100% money" as in the famous Simons, Fisher et al Chicago Plan 1933 - 1936, recently supported by Kumhof and Benes in a paper. I argue on this point against another paper by Ann Pettifor, who holds that 100% reserve requirements means that society will be starved of credit. That said, I am in agreement with Pettifor on her other points, for instance her critique of the theory of the money multiplier, unregulated banks and neoclassical economics more generally.

The famous Fisher et al *Chicago Plan*, was recently re-examined, and in conclusion supported, by Kumhof and Benes (2012) in a paper[1], where they write in the abstract:

> At the height of the Great Depression a number of leading U.S. economists advanced a proposal for monetary reform that became known as the Chicago Plan. It envisaged the separation of the monetary and credit functions of the banking system, by requiring 100% reserve backing for deposits. Irving Fisher (1936) claimed the following advantages for this plan: (1) Much better control of a major source of business cycle fluctuations, sudden increases and contractions of bank credit and of the supply of bank-created money. (2) Complete elimination of bank runs. (3) Dramatic reduction of the (net) public debt. (4) Dramatic reduction of private debt, as money creation no longer requires simultaneous debt creation. We study these claims by embedding a comprehensive and carefully calibrated model of the banking system in a DSGE model of the U.S. economy. We find support for all four of Fisher's claims. Furthermore, output gains approach 10 percent, and steady state inflation can drop to zero without posing problems for the conduct of monetary policy.

Pettifor (2013:20) disagrees, and argues that 100% reserve banking will lead to lack of credit:

> The Kumhof and Benes proposal is indeed based on the monetarist ideas of the Chicago School, one that seeks to limit the quantity of money, and that would restore the role of banks to intermediaries between savers and borrowers. Only now the proposal is to eclipse the role of the private sector altogether, and only allow lending backed by a 100% reserve requirement. In other words, all banks or lenders would first have to mobilise 100% of the funds needed for lending. This would massively constrain the availability of credit.
> ...
> Limiting the quantity of credit is certainly one way of limiting employment. Thus monetarist theory and policies both tolerated and sustained a massive rise in unemployment in the 1930s and 1980s. The Kumhof and Benes proposal is no more than a revival of these policies: the 'barbaric relic' that was the gold standard.

My impression is that not only Pettifor, but also many proponents of Modern Monetary Theory, are either indifferent or hostile to the 100% reserve concept. While not defending DSGE modeling and all details in the Kumhof and Benes proposal, I don't understand why a 100% reserve system must mean that credit will be constrained in a harmful way. Since this author considers himself to belong to the MMT school, this is a disagreement that seems important to me.

[1] Click on an author name for referenced papers.

From now on I consider an economy that is run according to MMT principles, and where all money is base money (high-powered money, HPM):

Why can't banks – if they mean they have a worthwile and fairly safe lending opportunity – borrow from the Central Bank, if they need more money to lend than what they have gathered at the moment by selling bonds or offering time deposits to the public (from now on: "selling bonds")?

Such bank borrowing from the Central Bank implies that HPM will then grow somewhat as an effect of bank lending (not only though governemnet deficit spending as recommended by MMT), in contrast to today's situation where credit money is created directly through bank lending. But, since banks also gather money for their lending by selling bonds to the public, the amount of extra money created this way will constitute only a share of new loans given, not the whole as in today's system. And this share will be (100% safe) HPM, not credit money.

The Central Bank can then influence the amount of new bank lending by the interest rate and time to maturity they demand on loans given to banks, for instance through auction processes. And the Central Bank can give the banks a strong incentive to behave responsibly by tailoring loan conditions based on how they have performed, and in a final instance having the right to take them over and letting their owners take the loss if they become insolvent.

Additionally, in an MMT scenario, the government will run a persistent fiscal deficit (not by borrowing from the public but "borrowing" from the Central Bank - i.e. themselves - money out of thin air), spending new interest-free HPM into the system. This net inflow of HPM will give an extra incentive for the public to buy bonds from banks, thus helping banks gather the necessary HPM for their lending.

The amount of HPM created through deficit spending plus that created through banks borrowing from the Central Bank, can be controlled by fiscal and monetary policy to be sufficient, so that there is no harmful lack of credit, as Pettifor fears.

There will be no lack of flexibility and agility either, since licenced banks can always have ample credit lines to the Central Bank to access the necessary HPM in time, when giving a new loan. They can then grant a loan just as easily as they do today.

And banks that currently have to much HPM and see too few lending opportunities can save at the Central Bank, using a spectrum of available maturities and interest rates offered by the Central Bank for that purpose.

Related to this: in an an MMT scenario one can dispense with interbank lending. The Central Bank's observation of banks' demand for borrowing and saving there, and corresponding day-to-day conditions for both offered them by the Central Bank, will sufficiently do the job done by the interbank market today, but without any systemic risk.

This should be a perfectly robust monetary system.

How banks are able to create additional credit money

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1 Introduction

This brief note will try to explain, via a an imagined example, how a bank is able to create extra "credit money" when lending. When a bank extends a loan, a corresponding increase occurs in the borrowing depositor's account. This means that the bank creates money when lending, "out of thin air". In principle, if a bank was unconstrained by regulation, it could create as much extra money via this process as it wanted to, as long as there were willing borrowers.

Banks are, however, constrained in their lending by the BIS rules on minimum captal-asset requirements. But it will be seen that in spite of this, licensed banks can net create money as long as the required minimum capital/asset ratio is below 1, which it is by a wide margin.

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2 The model

All monetary entities are in nominal terms. We define the following variables and parameters:

D, M = assets, liabilities [\$]. D = bank loans, M = deposit money.

R = reserves = the Bank's deposit with the CB = high-powered money (HPM) [\$]. We assume that $R > 0$. The Bank's total financial assets are $D + R$ [\$].

$k_{\min}$ = the by BIS required minimum capital/asset ratio [ ].

r = loan repayment rate over a short given period on assets (= loans = debt) [ ].

i_D = interest rate over a short given period on assets [ ].

i_M = interest rate over a short given period on liabilities (= deposits = money) [ ]; $i_M < i_D$.

i_N = net interest rate over a short given period = $i_D - i_M$ [ ].

σ = share of net interest income that is left for banks after they have paid their expenses including wages [ ]; $0 < \sigma < 1$.

$i = \sigma i_N$, net interest after the bank has paid its expenses. This is what is available for accumulation. [ ]

x = the amount of new loan extended after the period [\$].

We assume that our bank goes through the following three stage process which occurs at the end of the assumed period:

1. The bank is about to receive interest and repayment on its loans, and is at the $k_{\min}$ limit
2. The bank receives these, and because of that it ends up above the $k_{\min}$ limit
3. The bank then extends a new loan x, exploiting the $k_{\min}$ limit by targeting it again. We will see that the bank creates net credit money while doing this, without breaching the $k_{\min}$ rule.

2.1 Stage 1

The bank awaits payments and is at the $k_{\min}$ limit. Note that risk weights shall only apply in the denominator. The capital-asset ratio is

$$k_{\min} = \frac{D + R - M}{D + 0 \cdot R} = \frac{D + R - M}{D} \;, \quad \text{or} \;\; D = \frac{D + R - M}{k_{\min}} \tag{1}$$

2.2 Stage 2

After receipt of interest and repayments for the period on D, the situation is

$$\begin{aligned} \frac{D - rD + R - (M - rD - \sigma(i_D D + i_M D))}{D - rD} &= \frac{D - rD + R - (M - rD - \sigma i_N D)}{D - rD} \\ &= \frac{D + R - (M - iD)}{D - rD} > k_{\min} \end{aligned} \tag{2}$$

Note that the bank interest income emerges in the form of reduced liabilities, not increased assets.. After receipt of interest and repayments for the period, the amount of deposits has decreased to

$$M - rD - \sigma(i_D D + i_M D) = M - rD - iD \tag{3}$$

Thus credit money is destroyed through interest and repayments of loans. Note that M is not only reduced by the repayments, but also by the interest payments. When the bank in the next round extends a loan, exploiting that it is somewhat above the $k_{\min}$ limit, the amount of money created has to be more than $rD + iD$ for net credit money creation to occur.

2.3 Stage 3

The bank now extends a loan x so that the capital/asset ratio becomes $k_{\min}$ again:

$$\frac{D + R + x - (M + x - iD)}{D - rD + x} = \frac{D + R - (M - iD)}{D - rD + x} = k_{\min} \tag{4}$$

We solve for x , exploiting that $(D + R - M)/k_{\min} = D$:

$$x = (r + \frac{i}{k_{\min}})D \tag{5}$$

For net money to be created through the process, x must be greater than $rD + iD$ as stated above. We get that net money created is

$$\Delta M = x - (rD + iD) = (r + \frac{i}{k_{\min}})D - rD - iD = (\frac{1 - k_{\min}}{k_{\min}})iD \tag{6}$$

Observe that the net new loan is iD larger than the net money amount created. This is reasonable, since the net interest income after expenses of the bank, allows it to extend loans without having to create money.

Note also that for $k_{\min} = 1$, which corresponds to a 100% reserve banking system with $M = R$, no net money will be created by lending.